How efficient are Kenyan hospitals? : an application of frontier analysis techniques

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How Efficient Are Kenyan Hospitals? An Application of Frontier Analysis Techniques

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A thesis submitted for the degree of Doctor of Philosophy

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Statement of originality

I hereby certify that to the best of my knowledge, the content of this thesis is my own work. This thesis has not been submitted for any degree or any other purposes. Any assistance received in preparing this thesis and sources have been acknowledged.

Julie Jemutai Kiprono

January 2016
Abstract

Efficiency is an important concern for health systems. This includes delivery of the health care, health financing, and investment on hospital facilities and the management of health facilities. Measurement of efficiency in health facilities is important to ensure maximum allocation and utilization of limited resources.

The aim of this study was to estimate efficiency in level IV Kenyan hospitals using data envelopment analysis and stochastic frontier analysis. Panel data were collected from 27 public and faith-based hospitals between 2008 and 2012. Data envelopment analysis (DEA), a non-parametric approach and stochastic frontier analysis (SFA) a parametric approach were applied to the data. Ownership as a factor of efficiency was assessed from the collected samples.

The results show evidence of technical inefficiencies across the hospitals. Based on DEA bootstrapped model, the efficiency scores was 0.7597 and 0.7751 for 2011 and 2012 data respectively. Using the cross sectional data set, SFA values were comparable to DEA with an average of 0.7919 and 0.7701 for the 2011 and 2012 data sets respectively. Based on the panel data, the SFA model gave a range of scores that were between 0.62 (Pitt and Lee and Battese and Coelli) and 0.85 (true effect models). There was no evidence of patterns in efficiency scores over time based on both DEA and SFA approaches. This data did not suggest a significant effect on efficiency based on hospital ownership.

In conclusion, this study shows presence of technical inefficiencies in Kenyan hospitals. It also provides a platform in exploring further the frontier techniques and incorporating ownership when measuring efficiency in Kenyan hospitals.
Dedication

George and Leon

Mum,
Christine Jebichii Kiprono

Dad,
David Kiprono Chepsaigut
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A lot of people have contributed to work in this thesis and supported me through this journey.

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Abbreviations/Nomenclature

AE  Allocative Efficiency
BC  Battese and Coelli
CRS  Constant Returns to Scale
DEA  Data Envelopment Analysis
DMU  Decision-making units
LMICs  Low and Middle Income Countries
MLE  Maximum Likelihood Estimates
MoH  Ministry of Health
OLS  Ordinary Least Squares
OOPs  Out-of-Pocket Payments
PL  Pitt and Lee
SFA  Stochastic Frontier Analysis
TFE  True Fixed Effects
TE  Technical Efficiency
TRE  True Random Effects
VIF  Variance Inflation Factors
VRS  Variable Returns to Scale
WHO  World Health Organization
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1 Introduction

1.1 Introduction

Health care expenditure represents a significant share of national income (World Health Organization, 2011a). The global health expenditure is estimated at US$ 6.5 trillion with spending per person per year of US$ 948 (World Health Organization, 2011a). Organization for Economic Co-operation and Developments (OECD) countries make up less than 20% of the global population, but have a higher expenditure on health, which accounts for 12.4% of the gross domestic product (GDP) (World Health Organization, 2011a). Despite high burden of disease and health needs, particularly in low and middle-income countries (LMICs), health care resources are more limited than in high-income countries. For instance, in the World Health Organisation (WHO) African and South East Asian regions, only 6.5% and 3.7% of the GDP is spent on health respectively. In LMICs, the general government expenditure on health is 39.6% of the total health expenditure (World Health Organization, 2011a). Most of the LMICs rely heavily on out-of-pocket (OOP) payments, which accounts for up to 48% of the total health spending (World Health Organization, 2014b). The out-of-pocket (OOP) payments are a major barrier to accessing health services especially for the poor.

Although the resources are limited in some countries more than others, most health care systems fail to fully exploit the resources available either due to mismanagement, corruption and poor procurement (World Health Organization, 2010). Inefficiency is presented in different ways with some countries achieving better outcomes with their money than others and still a major gap exists between what they achieve and what they would potentially achieve with the same resources (World Health Organization, 2010). Other than looking for money for health, policy makers also have a keen interest in finding
efficient ways of using the limited resources available.

1.1.1 Why measure efficiency?

The importance of efficiency measurement is embedded in economics definitions that centers on the concept of scarcity (Robbins, 1935). Increasing emphasis on measuring inefficiency, is reflected in a number of studies (Fried, Lovell, & Schmidt, 2008) and a larger study that was conducted in 191 countries in measuring health system performance (Tandon, Murray, Lauer, & Evans, 2000). It is estimated that about 20-40% of resources spent on health are wasted globally (World Health Organization, 2010). Wastage occurs in all countries irrespective of their economic status. In the USA, for example, more than half of the US$600 - $850 billion spent on health care per year, is wasted (World Health Organization, 2011a). The waste covers areas in medicine (pricing, poor quality, inappropriate and ineffective use); health care services (inappropriate hospital size, admissions and length of stay); health workers (poor mix and unmotivated staff); medical errors; inefficient mix of interventions and health system leakages related to corruption and fraud (Chisholm & Evans, 2010). These factors together with the limited nature of resources, in the context of increasing health care needs, have led to a growing interest by policy makers in examining efficiency.

While efficient use of resources is important in all countries, it is particularly important in resource poor countries. Efficiency measurement is an important step in evaluating resource utilization and management in health systems. Efficiency is defined differently by different organizations. A policy maker views efficiency as the extent to which objectives are achieved in relation to consumed resources (Jacobs, Smith, & Street, 2006). The Institute of Medicine (IOM), defines efficiency as avoiding waste including waste of
equipment, supplies, ideas and energy (Berwick, 2002; Institute of Medicine (U.S.), 2001). There is no clear consensus on the definition of efficiency in health care.

Overall, efficiency in health care can be viewed as a relation between resource inputs (labour, capital, equipment) and intermediate outputs (e.g. outpatient and inpatient services) or final outcomes (quality of care, life) (Palmer & Torgerson, 1999). Definition of efficiency in the context of this thesis and theoretical development of efficiency definition is described in section 2.2.2.

1.1.2 Overview of the main approaches to measuring efficiency

Most studies that estimate efficiency scores in health care use frontier analysis techniques (Aigner, Lovell, & Schmidt, 1977; Charnes, Cooper, & Rhodes, 1978; Meeusen & van den Broeck, 1977). There are a number of studies that have utilised the frontier methods in high income countries (mostly referred to as developed countries) (Hollingsworth, Dawson, & Maniadakis, 1999; Worthington, 2004).

The non-parametric frontier approach does not require a functional form of the frontier to be pre-established but calculated from sample observations. The most common method is data envelopment analysis (DEA) developed by Charnes, Cooper and Rhodes (Charnes et al., 1978; Farrell, 1957). In DEA, a mathematical programming model is applied to observed data in constructing the frontier and calculating the efficiency scores relative to the constructed frontier.

The parametric approach incorporates both inefficiency and measurement error when estimating the frontier. It however requires assumptions to be made on the functional form
and distribution of the error term. The most common technique is the stochastic frontier analysis (SFA) that was developed in the 1970s (Aigner et al., 1977; Meeusen & van den Broeck, 1977).

DEA is a data-driven approach; therefore joining a set of ‘best’ performing hospitals forms the frontier. While using SFA, the shape and location of the frontier is guided by economic theory. DEA is a more flexible approach, as it does not require assumption of the underlying functional form. Due to this flexibility, DEA is a more common approach among researchers (Hollingsworth, 2003; Kiadaliri, Jafari, & Gerdtham, 2013; Worthington, 2004). The analysis steps of the two approaches, their advantages and disadvantages are discussed further in Chapter 2.

1.1.3 Challenges of measuring efficiency in health care

In certain cases, there are some issues on reliability of efficiency results when employing data from health services. Newhouse criticism of efficiency in health care raised key aspects that are still in question today (Newhouse, 1994). One of the key problems is that the ideal output (improved health status or quality of life) is difficult to measure (Kooreman, 1994; Worthington, 2004). However, defining health output as intermediate outcomes such as average length of stay, outpatient visits and admissions is still acceptable (Palmer & Torgerson, 1999) (Medeiros & Schwierz, 2015).

The other challenge in measuring efficiency in health care is defining inputs such as capital. This is because they are rarely measured and therefore labour inputs are commonly used. Omitted variables may lead to bias in the efficiency results and in such cases number of beds has been used as a proxy of capital input in previous studies (Jacobs et al., 2006;
The issues surrounding ownership in health care may also have an effect on efficiency. The effect of excess revenues in non-public hospitals makes them look more attractive in meeting hospital demands (Worthington, 2004). However, this is still a perception in most African countries with only few studies showing that either public hospitals perform better than non-public ones (Maredza, 2012; Masiye, 2007; Masiye, Kirigia, & Emrouznejad, 2006) or the vice versa (Jehu-Appiah et al., 2014). This thesis used data from public and faith-based hospitals to explore the effect of ownership on estimated efficiency levels.

The issues surrounding efficiency estimation techniques not only affects health care but also different industries. When choosing a non-parametric approach, which is largely data driven, one should be confident that the data has minimal bias and errors and that the organization has a well defined production process. This is because in this approach, random noise is included as part of the estimated level of inefficiency. The parametric approach requires an assumption on the functional form and if misspecified can lead biases in the results. Distributional assumption of the error component of random noise and inefficiency need to be made. This study explores different assumptions of the two frontier approaches and is discussed in detail in Chapters 2, 4 and 5.

1.2 Overview of the Kenyan health system

1.2.1 Socio-Economic Profile

Kenya has an estimated area covering 582,646km² and 80% of this land is arid or semi-arid. As of 2009 population census, Kenya had a population of approximately 40 million people (Kenya National Bureau of Statistics, 2009) with projected increase to

The 2010-2014 reported gross national income (GNI) per capita of Kenya is $1730 compared to an average of $2594 in the African region and global average of $12018 (World Health Organization, 2011a). In 2012, Kenya spent US$ 1.9 billion in health, which was approximately 4.7% of its gross domestic product (GDP) (World Health Organization, 2014b). The total health expenditure (THE) per capita in Kenya was US$ 66.6 in 2012/2013 (Ministry of Health Kenya, 2015c) with government spending and household spending of 31.2% and 32% of THE respectively (Ministry of Health Kenya, 2015c).

1.2.2 Health Profile: Key indicators

Kenya faces challenges given inadequate resources specifically in the health care system. There are approximately 1.8 physicians and 7.9 nurses and midwives per 10000 people in Kenya compared to 2.6 physicians and 12 nurses and midwives per 10000 people in the African region (World Health Organization, 2014a).

The average life expectancy of Kenya is 60 years (World Health Organization, 2011b). The health indicators have made minimal changes over the years. In the most recent report (Kenya National Bureau of Statistics et al., 2015b) maternal mortality ratio was 362 deaths per 100,000 live births in the year 2014 compared to 488 deaths per 100,000 live births in 2008. There is still room for improvement considering that the global maternal mortality rate is approximately 210 per 100,000 live births (World Health Organization, 2012). The

1.2.3 Organization of the Kenyan health system

Over the years, Kenya has taken several important steps in overcoming development obstacles and ensuring improvement in the socio-economic status. Some of these steps include the development of the Kenya Health Policy Framework, Health Strategic Plans, Vision 2030, enactment of Constitution 2010 and the Millennium Development Goals (MDGs) (Ministry of Health Kenya, 2014). However implementation of these strategies is still and often dependent on the prevailing political climate and goodwill.

Kenya Health Policy Framework (KHPF) was initially developed and approved by the Government of Kenya (GoK) in 1994. It outlined the strategic plan for the Kenyan health sector in developing and managing health services. In 1997, the Ministerial Reform Committee (MRC) was formed to oversee the implementation of the framework that covered up to the year 2010. This policy was aimed at responding to issues in health sector expenditure, utilization of resources, management information systems and increasing burden of disease.

During the period prior to the enactment of the new constitution in 2010, preventive and curative services were provided at six levels of care as shown in Figure 1.1. The data used in this study were obtained from individual level 4 hospitals between 2008 and 2012. This was the period before the implementation of the new structure of the health system as described in the 2012-2013 health policy (Ministry of Health Kenya, 2014)
The Constitution of Kenya 2010 was adopted in August 2010 that introduced a different governance structure with a national government and 47 counties. This was a major shift from highly centralized form of governance that had led to a weak, unresponsive, inefficient and inequitable distribution of health services in the country (Ndavi, Ogola, Kizito, & Johnson, 2009). The Constitution introduced a devolved system, which aims to improve efficiency and accessibility of services to all. Responsibility of service delivery is assigned to the county level while developing policies governing health care, regulation of health services, technical assistance to counties, health care delivery at the national referral hospitals and capacity building are mainly the responsibility of the national government. Although the transition process is currently underway, some challenges have been experienced in the implementation of the Constitution due to the complexity of the framework and readiness of the counties to deliver the services.
At the county level, there are four financing sources with one mainly from revenues generated by the counties through taxes (Ministry of Health Kenya, 2015b; Republic of Kenya, 2015). The other three sources from national government are:

- Equitable block grant with the counties assured of receiving at least 15% of the national government revenue. This is allocated to the counties using a formula developed by Commission of Revenue Allocation (CRA) that include population (45%), basic equal share 25%), poverty (20%), land mass area (8%) and fiscal responsibility (2%) (Commission on Revenue Allocation, 2015)
- Conditional grants from national government to support Level 5 hospitals, free maternal health care and user fees removal
- Equitable share specifically for marginalized communities that represents 0.5% of the national revenue

The 1994-2010 KHPF review showed an increase in non-communicable diseases and violence related conditions. The negative impact of communicable diseases still remains significant. This implies that new policies should address such challenges. In order to achieve this, the current health policy (2012-2030) is designed to respond to communicable diseases, non-communicable diseases and violence related conditions and to ensure attainment of standard health as outlined in Figure 1.2.
The overall aim of the current policy is to attain universal health coverage of critical services that positively contribute to the realization of the overall policy goal. The policy objectives is to eliminate communicable conditions, halt and reverse the rising burden of non-communicable conditions, reduce the burden of violence and injuries, provide essential health care, minimize exposure to health risk factors and strengthen collaboration with other sectors that have an impact on health. The tier system (levels of care) in the current policy (2012-2030) includes community, primary care, primary referral and tertiary referral services as shown in Figure 1.3.
The health sector comprises of the public system and the private sectors. The public system is mainly run by Ministry of Health and parastatal organizations while the private system mainly involves the private for-profit, non-governmental organizations and the faith based facilities. There are currently approximately 9896 health facilities countrywide with the public sector accounting for 48% of these facilities. The different levels, types and ownership of the health facilities are shown in Table 1.1.
Table 1.1: Levels, types and ownership of facilities

<table>
<thead>
<tr>
<th></th>
<th>Government-owned</th>
<th>Privately-owned</th>
<th>NGO</th>
<th>Faith-based</th>
<th>Total, n (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hospitals</td>
<td>297</td>
<td>126</td>
<td>7</td>
<td>81</td>
<td>511 (5.2)</td>
</tr>
<tr>
<td>Health Centers</td>
<td>827</td>
<td>69</td>
<td>23</td>
<td>174</td>
<td>1093 (11)</td>
</tr>
<tr>
<td>Dispensaries</td>
<td>3471</td>
<td>192</td>
<td>41</td>
<td>614</td>
<td>4318 (43.6)</td>
</tr>
<tr>
<td>Others</td>
<td>149</td>
<td>3355</td>
<td>270</td>
<td>200</td>
<td>3974 (40.2)</td>
</tr>
<tr>
<td>Total</td>
<td>4744</td>
<td>3742</td>
<td>341</td>
<td>1069</td>
<td>9896</td>
</tr>
<tr>
<td>N (%)</td>
<td>(47.9)</td>
<td>(37.8)</td>
<td>(3.4)</td>
<td>(10.8)</td>
<td></td>
</tr>
</tbody>
</table>

Total number of beds and cots 64276

Health care quality and efficiency is important for the overall economy as well as the health care sector. All the health policies developed in Kenya have always emphasized on efficient utilization of resources as a key objective. In this study, data was collected based on a period before implementation of the new constitution and provides baseline estimates of efficiency in the individual hospitals. If further data is collected and analysed post implementation, this study provides a platform for carrying out a pre and post estimation of efficiency in selected Kenyan hospitals.

1.3 Objectives and contribution

The overall aim of this study is to measure efficiency using frontier analysis techniques in Kenyan hospitals. Data between 2008 and 2012 were obtained from individual hospitals in public and faith-based hospitals.

The specific objectives of this thesis are:

i. *To estimate technical and scale efficiency in Kenyan hospitals*
In order to achieve this, two main frontier analysis techniques are considered. This includes a non-parametric method, data envelopment analysis (DEA), and a parametric approach, stochastic frontier analysis (SFA). The advantages and disadvantages of each technique are highlighted in the thesis. Out of all studies published using hospital data from African countries, only two estimated efficiency using SFA in rural health districts (Ramanathan & Chandra, 2003) and specialized surgeon clinics (Koch & Slabbert, 2012). This is a significant gap especially if there exists measurement error in data in which case DEA approach would be very sensitive. SFA incorporates measurement error when estimating efficiency.

The study further discusses the different model assumptions in DEA providing comparable results to assess. Further discussions on the choice of functional form and distribution of the one-sided error term in the SFA approach are presented in thesis in order to provide new knowledge in the area of efficiency measurement in developing countries.

**ii. To estimate efficiency in Kenyan hospitals over time using data from 2008 to 2012**

Estimating efficiency over time provides more information regarding whether the levels were constant or changed in an upward or downward direction. Published studies using Kenyan data in measuring efficiency used cross sectional data sets (Kirigia, Emrouznejad, & Sambo, 2002; 2004). Although collecting cross sectional data involve less time and resource constraints in terms of data availability, panel data have considerable advantages over cross section data. Panel data relaxes some of the strong assumptions for efficiency analysis when using cross sectional data and therefore possible to disentangle explanatory variables and efficiency terms (Coelli, Rao, & Battese, 2005; Jacobs et al., 2006). With
panel data sets one can also obtain consistent estimates and changes of the efficiency level over time can be assessed.

This study aim is to estimate efficiency over time in 20 panels (quarterly structure for a period of 5 years). This is unique in developing countries context and analyses of data with more observations provide insight on variations in efficiency in selected Kenyan hospitals.

iii. To compare efficiency estimates obtained from DEA and SFA models

DEA is a more flexible method as it does not require prior assumptions on the functional form but it is sensitive to data and assumes that the distance form the frontier is solely inefficiency. SFA incorporates measurement error having an advantage over DEA despite the complexity of measurement due to prior assumptions on the functional form. In Africa, one study used both DEA and SFA in estimating efficiency in health districts (Ramanathan & Chandra, 2003). They did not however, compare the estimates from the two methods. This thesis contributes to the knowledge on how DEA and SFA efficiency estimates compare using data from selected Kenyan hospitals.

iv. To determine the effect of ownership on efficiency in Kenyan hospitals

One of several factors that drive inefficiency is ownership (Worthington, 2004). Published research on efficiency estimation by ownership in Kenya is unavailable. This study explores how ownership of the hospitals affect estimated efficiency. It is often assumed that non-government owned hospitals perform better than government owned although there is literature that suggests that this varies as discussed in section 2.4. This study explores the effect of ownership on estimated efficiency levels in Kenyan hospitals. It also
compares estimated efficiency levels of public (government-funded) and faith-based hospitals.

As highlighted in the specific objectives of this thesis, there are some new questions that this study addresses in a developing country context. Below is a summary of the key contributions that this thesis aims to make:

- Use of panel data set to provide more information on estimated efficiency of selected Kenyan hospitals over time
- Use of primary data collected from individual hospitals other than secondary data available at the national level
- Estimating efficiency using stochastic frontier analysis (SFA), which has only been used in two published studies in Africa (Koch & Slabbert, 2012; Ramanathan & Chandra, 2003)
- Compare efficiency estimates between DEA and SFA. Currently, there are no existing literature in Africa
- Obtain efficiency estimates using different assumptions and model specification of DEA and SFA
- Compare estimated efficiency by ownership type using data from public and faith-based hospitals
- Provide data and estimated levels of efficiency in hospitals before the devolution of health services to county level in Kenya.

1.4 Structure of the thesis

Chapter 2 provides a theoretical description of the efficiency measurement techniques specifically DEA and SFA. It introduces the different assumptions in frontier techniques as
well as their respective advantages and disadvantages. An overview of empirical literature in both developed and developing countries is outlined.

Chapter 3 provides an overview of the data, which includes the source and types of data that were collected. The choice of study sites and sampling procedures is described. Inputs and outputs that were selected from a set of hospital variables and choice of variables is discussed. Finally it describes the challenges that were encountered during the data collection phase of the study and how some of them were mitigated.

An application of the DEA technique is discussed in Chapter 4. Calculation of relative technical efficiency using both input and output oriented models under the assumptions of both constant and variable returns to scale is discussed. Scale efficiency results and analysis of relative efficiency by ownership is discussed in this chapter.

Chapter 5 discusses the application of the SFA technique in a section of Kenyan hospitals from data described Chapter 3. The choice of the functional form and distribution of the one-sided error term is argued in this chapter. Hospital efficiency estimates are then obtained using panel data and presented over time. This chapter shows the use of different SFA panel data under time-invariant and time-varying models.

The efficiency estimates derived from DEA and SFA approaches are discussed in Chapter 6. In frontier analysis technique, the choice of one method can have an advantage over the other and therefore choosing a particular method for efficiency measurement is dependent on the research questions and sensitivity of the methods to the available data.
Chapter 7 highlights sensitivity analysis of both the DEA and SFA approaches. In order to check for robustness of the efficiency estimates, validity of the findings was carried out. Results from the different combinations of variables and model assumptions are assessed to check for stability and consistency.

Chapter 8 provides a summary of the main findings from the study. The contributions and policy implication of the study results are highlighted with emphasis on future research. The general limitations of the study are also highlighted.
2 Literature Review

2.1 Introduction

This chapter reviews both the theoretical and empirical literature pertaining to efficiency analysis. Theoretical definition and types of efficiency are highlighted in this chapter. The theoretical literature review section discusses the various techniques for measuring efficiency, their strengths and weaknesses and provides a justification for methods employed in this study. Specifically two frontier techniques, DEA and SFA, are discussed. In addition, model specifications and assumptions of DEA and SFA are discussed in detail.

The empirical literature review studies conducted in developed and developing countries using frontier techniques in measuring efficiency. Gaps in the literature are identified and ways of handling these gaps in the thesis context are discussed in this chapter.

2.2 Production theory

2.2.1 Production process

In economic theory, production is the transformation of inputs into outputs. This transformation takes place in a production function, \( y = f(x) \). The process is dynamic and therefore technical change is expected to take place. This process of transformation can be denoted as:

\[
T = \{(y, x): x \text{ can produce } y\}
\]  

(2.1)

This means that a set of inputs in the production process needs to be sufficient in order to produce a vector of outputs. This also defines the set of inputs that is insufficient to
produce y, which defines the limits of the producers' ability. The boundary of this set is the production frontier, which relates to the maximum possible outputs for a given set of inputs. The production function is therefore defined by the isoquant, which forms the boundary for the inputs requirements set as shown in the equation below:

\[ L(y) = \{x: (y, x) \in \mathcal{Y}\} \]

(2.2)

The outputs set on the other hand is the defined as set feasible outputs for every input vector x.

\[ P(x) = \{y: (y, x) \in \mathcal{Y}\} \]

(2.3)

The production process in health care is illustrated in Figure 2.1 below:

![Figure 2.1: Production process in health care](image)

Inputs and outputs in the production process are defined as factors of production. There are common factors of production used in literature. Capital, labour and materials are
examples of factors of production widely used. In some cases, it is complex to measure or identify inputs and outputs. Proxy variables are alternatively chosen to represent such cases.

2.2.2 Theoretical definitions of efficiency

There are mainly two types of efficiencies; technical and allocative, which were originally defined by Michael J. Farrell (Farrell, 1957). Technical efficiency (TE) is considered when input use is minimized in the production of a given output (input-oriented) or maximizing output in a given input vector (output-oriented). Allocative efficiency (AE) on the other hand is considered when optimal combination of inputs is chosen to produce a given set of outputs. These definitions were highly influenced by Koopman’s formal definition and Debreu’s definition and measurement of technical efficiency in the 1950s (Debreu, 1951; Koopmans C, 1951). An organization that is both technically and allocative efficient is considered to have achieved total economic efficiency.

The analysis of efficiency carried out by Farrell (1957) can be explained in Figure 2.2 below:
Figure 2.2: Technical and Allocative efficiencies (Farrell, 1957)

YY' – Isoquant: Minimum combination of inputs per unit of output needed to produce a unit of output. A point along the isoquant is considered technically efficient while any point above such as P is technically inefficient.

RP – the distance measures technical inefficiency of producer P. Hence, technical inefficiency of P is expressed as RP/OP and technical efficiency (TE) is 1-(RP/OP) = OR/OP

CC' – Budget / isocost line: Combination of all inputs, which cost the same amount

SR – measures allocative inefficiency of producer P. Hence allocative inefficiency of producer P is expressed as SR/OR and allocative efficiency as 1-(SR/OR) = OS/OR.

Assuming a constant returns to scale (CRS), the technology set is defined by the isoquant YY'. In this case every unit along the isoquant is considered technically efficient while any point above it such as point P is considered technically inefficient. Therefore, the distance RP' measures technical inefficiency of P. Allocative efficiency on the other hand, is measured by the distance SR (Murillo Zamorano, 2004). The total economic efficiency (EE) is:
In order to measure allocative efficiency, data on input prices and costs have to be available. In most cases, these data are not readily available. It is also unrealistic for a decision-making unit (DMU) such as a hospital to achieve 'full' allocative efficiency because distributing care to patients due to right combination of inputs is secondary to providing high quality of care. Due to these reasons, the discussions and results from this thesis focus mainly on technical efficiency.

2.3 Methods of measuring hospital efficiency

Measuring efficiency of the production process is an important step considered by policy makers. There are two main approaches of measuring efficiency, parametric and non-parametric methods. Parametric methods assume a particular functional form, while non-parametric avoid the distributional assumptions. Another way of categorizing the methods of measuring efficiency is by either stochastic or deterministic nature of the model. Stochastic frontiers are randomly determined and allow statistical noise while deterministic frontiers do not.

Parametric models have been developed since the early 1920s with ordinary least squares (OLS) method as the pioneer (Cobb & Douglas, 1928). The other parametric methods include parametric mathematical programming (PP) (Aigner and Chu, 1968 Timmer, 1971), corrected ordinary least squares (COLS) (Winsten, 1757, Greene, 1980) and stochastic frontier analysis (SFA) (Aigner 1977, Meeusen and Vanden Broeck 1977). PP and COLS have a deterministic nature while SFA has a stochastic nature.
The initial non-parametric model developed was the convex non-parametric least squares (CNLS) (Hildreth, 1954 Hanson and Pledger, 1976). Data envelopment analysis (DEA) initially proposed by Farell (1957) and further developed (Charnes 1978) is another form of non-parametric model but with a deterministic nature. Recently developed non-parametric method is the stochastic data envelopment analysis (SDEA), which is more of a stochastic nature. Table 2.1 summarizes the different techniques of measuring efficiency in different sectors of the economy including health care.

<table>
<thead>
<tr>
<th>Parametric</th>
<th>Non-Parametric</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Central Tendency</strong></td>
<td><strong>Deterministic</strong></td>
</tr>
</tbody>
</table>

The growing interest in measuring efficiency in health care is attributed to concerns about limited resources, costs of health care and demand for accountability of resource use in the health systems. Governments also have interest in assessing efficiency in the health facilities to ensure efficient use of scarce resources. The focus of efficiency analysis in all organizations is referred to as decision-making unit (DMU). In health care, examples of DMUs are the whole health system, hospitals, health centres, specialized individual
physicians or health facility departments. Hospitals as a DMU are the main focus of this review.

The central tendency methods (Table 2.1) have major disadvantages in that they assume all deviation from the frontier is solely due to noise indicating all organizations are efficient. The CNLS method is difficult to solve due to its quadratic programming nature. Similar to OLS, COLS and PP methods require large datasets in order to obtain reliable results. These techniques are highly sensitive to functional form if error is not interpreted adequately. Since SDEA, is a more recent method of measuring efficiency and still under development, it was not included in this thesis. Therefore, in the subsequent work discussed in this review and thesis, the emphasis is on data envelopment analysis (DEA) and stochastic frontier analysis (SFA) as techniques for measuring hospital efficiency. Reasons for this choice are highlighted in section 2.6.

Box 1 highlights the key concepts of efficiency analysis.
Box 1: Key Concepts

Health system outputs are the intermediate results of activities taken by the health systems. Examples include number of outpatient visits, inpatients visits, deliveries and immunizations.

Health system outcomes are the final outcomes attributed to health gains. Examples include improvement in quality of life and life expectancy.

Health system inputs are resources that the health system uses in order to produce outputs or outcomes. Inputs can be divided into physical inputs such as number of beds and equipment, financial inputs such as expenditure and human resources such as doctors and nurses.

Decision-making unit (DMU) is the main organizational focus of efficiency analysis. It is the entity that controls the production process. Examples in health care include the health system (Ministry of Health), hospitals or specific physician practices.

Technical efficiency is the considered when input use is minimized in the production of a given output (input-oriented) or maximizing output in a given input vector (output-oriented)

Allocative efficiency is the optimal combination of inputs is chosen to produce a given set of outputs

Scale efficiency is the measure of the DMU’s size of operations that is optimal so that any modifications to its size will render the DMU less efficient.

Overall efficiency is a combination of technical, allocative and scale efficiency.
2.3.1 Data Envelopment Analysis (DEA)

DEA is a linear programming method used to measure relative efficiencies of a DMU. This method was first introduced by Charnes, Cooper and Rhodes (CCR) and is driven by the data available and not based on any econometric theory (Charnes et al., 1978). In health care, efficient hospitals (TE or AE = 100%) are considered as best practicing hospitals and are called ‘frontiers’. DEA model identifies efficient hospitals for each inefficient hospital to act as a comparator in order to identify the gaps that need to be filled in the inefficient hospitals. This means the efficiency scores are relative measures. This method is best applied on data that has no random error and in a well-defined production process. This is particularly a challenge in health care setting, as it does not have a well-defined production process and might lead to biased results.

There are two main stages when using a DEA model. First, it identifies a frontier based on the hospitals that most minimizes input or maximizes output (fully efficient). Then secondly, it assigns an efficiency score to each hospital as compared to the efficient hospitals. This means the inefficient hospitals are then ‘enveloped’ by the efficient frontier.

The efficiency of a hospital producing one health service output from one health system input is obtained by dividing the quantity of that output by the quantity of an input. However, hospitals have several outputs and inputs indicating that efficiency needs to be expressed as the weighted sum of outputs divide by the weighted sum of inputs.

The DEA method has been of interest in health care because of several advantages. First, it can handle multiple inputs and multiple outputs such as hospital setting. DEA is also less
complex as it does not require an assumption of a functional form related to inputs and outputs. DEA provides inefficient hospitals with peers that are considered efficient and therefore gaps can be identified.

However, DEA has its own limitations. This technique is prone to measurement error, as it does not distinguish between technical inefficiency and statistical noise. Also since DEA is compared to the best practice, there could be under or over estimation if the best practice in the real sense is biased. Since DEA is data-driven, the location and shape of the frontier is defined by data indicating that the DMU that uses less input to produce the same output can be considered more efficient (Jacobs et al., 2006).

2.3.1.1 Formulation of DEA

According to Chames et al, efficiency in DEA can be defined by the ratio of weighted sum of measure of outputs divided by the weighted sum of inputs (Chames et al., 1978) as shown:

\[
\text{Max} \ h_j = \frac{\sum_{r=1}^{r} u_r y_{rio}}{\sum_{i=1}^{m} v_i x_{io}}
\]

(2.5)

where \( y \) is the amount of output \( r \) produced by hospital \( j \) and \( x \) is the amount of input \( i \) used by hospital \( j \).

\( u_r \) – Weight given to output \( r \)
\( v_i \) – Weight given to input \( i \)

Subject to
\[
\frac{\sum_{j=1}^{n} u_j y_{fj}}{\sum_{i=1}^{m} v_i x_{ii}} \leq 1
\]

\(j=1,\ldots,n\); \(n\) is the number of hospital
\(r=1,\ldots,s\); \(s\) is the number of outputs
\(i=1,\ldots,m\); \(m\) is the number of inputs
\(u_r, v_i \geq 0\)

The linear programming model is solved for each DMU and the output of the model include an efficiency score value between 0.0 and 1.0 for each DMU.

### 2.3.1.2 Returns to scale assumption

The envelopment surface of DEA differs depending on the scale assumption of the model. Two scale assumptions are normally employed: constant returns to scale (CRS) and variable returns to scale (VRS). Returns to scale refers to change in output as the same proportion of inputs changes. Charnes, Cooper and Rhodes initially assumed CRS (Charnes et al., 1978). The CRS assumption implies that an increase in inputs results in a proportionately greater increase in outputs. CRS assumes that the hospitals operate at the most productive level. This model was extended to accommodate a more flexible VRS model (Banker, Charnes, & Cooper, 1984). In the VRS model, there is an assumption that there are economies and diseconomies of scale and that not all hospitals operate at the optimal scale. VRS envelops data more tightly and the inefficient DMUs are compared only to DMUs of similar size. This is a major advantage over CRS assumption.

The VRS model measures pure technical efficiency and scale efficiency. Scale efficiency measures for a mix of inputs how a maximum output attained or how the DMU is close to an optimal scale (Fried et al., 2008). Scale efficiency is calculated by dividing technical

28
efficiency under CRS by technical efficiency under VRS. This thesis explores both returns to scale assumptions and efficiency estimates compared.

2.3.1.3 Input vs. output oriented models

In DEA, efficiency of DMUs can be examined using either input or output orientation. In an input oriented model, a given level of output is held constant, and minimizes inputs. In the output-oriented models, proportional augmentation of output is explored while inputs are held constant. The technical efficiency estimated by the two oriented models are the same under CRS but different when VRS is assumed. The researcher chooses the orientation depending on the production process of the particular DMU. This thesis explored both orientation models and the reasons for this choice is outlined in Chapter 4.

2.3.2 Stochastic Frontier Analysis (SFA)

Stochastic frontier (SF) models were initially suggested by (Aigner et al., 1977; Meeusen & van den Broeck, 1977) and they have become popular models over the years. SF model is based on the idea that the frontier represents the maximum possible output and any deviations from this represent individual inefficiency. SF models are stochastic in nature thus allowing for composite error term that captures any noise or random error and a one-sided disturbance term that represents inefficiency. The inefficiency component, \( u_i \), is strictly positive and typically assumes a half-normal distribution (although other distributions are possible). The inefficiency term reflects that if \( u_i \geq 0 \), the hospital will not produce at a maximum attainable level. The random error/noise term, \( \eta_i \), is assumed to be independently and identically distributed (i.i.d) normal variables with zero means and variances.
Given that the distribution of the inefficiency term is non-normal, the total error term is asymmetrical and non-normal. Therefore estimation by ordinary least squares (OLS) provides consistent parameter estimates except for the intercept. Before obtaining maximum likelihood estimates (MLE), OLS residuals are tested for negative skewness. If the OLS estimates are the same as MLE, there is positive skewness and therefore no technical inefficiency. If there is negative skewness, then the OLS estimates are used as starting values in the maximum likelihood routine. OLS does not also provide an estimate for hospital specific efficiency. SF model is a parametric method meaning that it requires a prior assumption of the functional form and various types of functional forms are discussed in sections 0 and 2.3.2.2.

The basic formulation of the SF model is as described below:

\[ y_i = f(x_i; \beta) \cdot TE_i \]  

(2.6)

where \( f(x_i; \beta) \) is the production frontier and TE is the technical efficiency.

\( y_i \) – Observed outcome/output \( i^{th} \) hospital

\( x_i \) – Vector of inputs of \( i^{th} \) hospital

\( \beta \) - Vector of unknown parameters

In order to add the statistical noise to the model, the stochastic production frontier is written as:

\[ y_i = f(x_i; \beta) \cdot TE_i \cdot \exp(r_i) \]  

(2.7)

This can be written in the below form:
\[ y_i = \beta' x + \nu_i - u_i \]  \hspace{1cm} (2.8)

where \( \nu_i \) and \( u_i \) are error terms, \( \nu_i \) caused by stochastic noise and \( u_i \) is the degree of inefficiency. This component is key to SF model. The first step of SF model analysis is to obtain the parameter \( \beta \) estimates. At this point estimates of the parameters of the distributions of the errors terms, \( \sigma_\nu \) and \( \sigma_u \) are obtained. There are various assumptions made on the inefficiency term, \( u_i \). These forms include Cobb-Douglas, Translog and CES functional forms.

### 2.3.2.1 Choosing Functional forms of SFA

Estimation of the stochastic production frontier requires specification of the functional form. The two most common forms are Cobb-Douglas and Transcendental Logarithmic (Translog). The choice of functional form has an implication on the shape of the isoquants, elasticities of demand and factor substitution. They impose restrictions and therefore have an influence of the efficiency measures.

Cobb-Douglas production function has a universally smooth and convex isoquants. The implication of this is that it makes strong assumptions on demand elasticities and factor shares as constant for given input prices. This is not straightforward in the production side (Greene, 2008). The Cobb Douglas function is represented as:

\[
\log y = \beta' x + \nu - u \]  \hspace{1cm} (2.9)
The Cobb Douglas functional form has strong assumptions that demand elasticities and factor shares are constant for given input prices and Allen elasticities of factor substitution are all -1 (Greene, 2008).

These implications have motivated the use of a more flexible functional form, Translog. The Translog equations of a SF model is represented as:

\[ \ln y_i = \alpha + \beta' \ln x_i + \beta' (\ln x_i)^2 + \beta' (\ln x_i \ln x_i) + v_i - u_i \]  

(2.10)

The Translog model relaxes the restrictions on elasticities of demand and factor substitution. A side effect of the Translog form is that they are not monotonic and are globally convex. Imposing an appropriate curvature is a challenge in this form. The Translog form is also complicated by multicollinearity. Some of the solutions in dealing with multicollinearity as discussed in literature are obtaining more data, dropping some of the variables suspected of causing the problem, using principle component analysis or ridge regression estimation (Greene, 2003). Greene suggested that in most cases there is no need to address multicollinearity since these attempts tend to force theory or assumptions on the data.

Although this study mainly discussed the two commonly used functional forms (Cobb-Douglas and Translog), there are other form discussed in literature. This includes constant elasticity of substitution (CES), generalised Leontief, normalised quadratic and its variants. They are however rarely used in assessing efficiency in healthcare.
2.3.2.2 Choosing distribution for the one-sided error term

The model shown above in the SFA formulation places a normal-half normal distribution on the inefficiency term. In efficiency measurement, the error term, \( r_i - u_i \) needs to be separated. The JLMS estimator created by (Jondrow, Lovell, Materov, & Schmidt, 1982), showed that for the half-normal case, the expected value of \( u_i \) given the error term is:

\[
E[u_i | e_i] = \frac{\sigma \lambda}{1 + \lambda^2} \left[ \frac{\phi(e_i \lambda / \sigma)}{\Phi(e_i \lambda / \sigma)} - \frac{e_i \lambda}{\sigma} \right]
\]

where \( \phi(\cdot) \) is the density of standard normal distribution

\( \Phi(\cdot) \) is the cumulative density function

\( \lambda = \sigma_u / \sigma_v \)

\( e_i = r_i - u_i \)

\( \sigma^2 = \sigma_u^2 + \sigma_v^2 \)

Given \( E[u_i | e_i] \), technical efficiency can be calculated for each producer as:

\[
TE_i = 1 - E[u_i | e_i]
\]

Both half-normal and exponential distributions have a mode at zero. In a truncated normal distribution of the one-sided error term, the assumption zero mean restriction is relaxed. In this case, the one-sided error term is obtained by truncating at zero the distribution of a variable with a non-zero mean (Stevenson, 1980). In JLMS estimator, the \( e_i \lambda / \sigma \) in the truncated distribution is replaced with
\[ \gamma_i = \frac{e_i \lambda}{\sigma} + \frac{\mu_i}{\sigma \lambda} \] (2.13)

Gamma distribution adds an additional parameter to the exponential distribution (Greene, 1990). It produces an unbiased but inconsistent estimate of the error term \( \gamma_i \) using maximum simulated likelihood.

### 2.3.2.3 Stochastic Frontier Analysis for panel data

In a panel data set it is possible to estimate efficiency for each hospital over time while in cross sectional data one can only measure in a specific period in time. The main disadvantage with cross sectional data is that technical efficiency cannot be separated from hospital specific effects that are not related to inefficiency. Panel structure relaxes the assumption in cross sectional data that inefficiency is independent of the inputs and avoids distributional assumptions. The panel model is in the form:

\[ \gamma_{it} = \alpha \beta + \beta X_{it} + \nu_{it} - \mu_{it} \] (2.14)

The time dimension of the inefficiency term has to be defined before estimating the model.

### 2.3.2.4 Time-Invariant inefficiency

A model with time-invariant inefficiency is a case in which the inefficiency term \( \mu \) is kept constant over time for each hospital. This is in the form:
\[ y_{it} = \alpha_{0} + \beta x_{it} + \nu_{it} - u_{i} \]

(2.15)

Alternatively, the intercept can be eliminated by defining \( \alpha_{t} = \alpha_{0} - u_{i} \) and have a standard panel data model:
\[ y_{it} = \alpha_{t} + \beta x_{it} + \nu_{it} \]

(2.16)

The term, \( \nu \) is assumed to be independent and identically distributed. In order to ensure consistency of the within and parameter estimates \( \beta, \nu \) are also assumed to be uncorrelated with the inputs \( x \). This is derived from the OLS estimation under fixed effects model (within) and random effect model (estimators of parameter vector \( \beta \)).

The initial use of panel data in SF models was by Pitt and Lee who interpreted the random effects as inefficiency rather than heterogeneity (Pitt & Lee, 1981). A similar interpretation was used by Schmidt and Sickles but in a fixed effects model (P. Schmidt & Sickles, 1984). A main disadvantage of these models is that any unobserved, time-invariant, firm specific heterogeneity is treated as inefficiency. (Cornwell, Schmidt, & Sickles, 1990) and (Battese & Coelli, 1992) extended the random effects model to include time-invariant inefficiency.

2.3.2.5 Fixed Effects Model

In the fixed effects (FE) model, the inefficiency term \( u_{i} \) and the intercept are treated as fixed. There are no assumptions in the FE model made on the inefficiency term or on the correlation between the inefficiency term with regressors and the statistical noise, \( \nu_{i} \). Using OLS on the model, the within estimator is derived. In this case, the estimate of the
intercept terms $\alpha_i$ is available and therefore the hospital specific inefficiencies can be estimated:

$$\hat{\alpha} = \max (\hat{\alpha}_i)$$  \hspace{1cm} (2.17)

$$\hat{\alpha}_i = \hat{\alpha} - \hat{\alpha},$$  \hspace{1cm} (2.18)

The above equation means that the frontier is normalized in terms of best hospital in the sample. The FE model however has some drawbacks in cases where time-invariant regressors are included. The regressors appear as inefficiency since the fixed effects, $\mu_i$, captures both time-invariant inefficiency and regressors. Estimating a random effects model solves this setback.

### 2.3.2.6 Random Effects Model

In a random effects (RE) model, the inefficiency term is assumed to be independent of the regressors, and therefore time-invariant regressors are included in the model. Rewriting the model equation with $\alpha^* = \alpha - \mu$, where $\mu = \mathbb{E}(u_i)$, we have:

$$y_{it} = \alpha^* + \beta x_{it} + v_{it} - u_{it}^*$$  \hspace{1cm} (2.19)

where $u_{it}^* = u_i - \mu$

The above model can be estimated using generalized least squares (GLS) method, which is consistent as $N$ approaches infinity. The main advantage of RE model is that it allows time-invariant variables in the specification.
The main advantage of panel data is that it avoids strong assumptions in specification and estimation of SF functions. Maximum likelihood techniques can be applied to obtain precise estimates of efficiency. The models discussed so far considered inefficiency as time-invariant. With data with long panels, it is more reasonable to allow inefficiency to vary over time. Time varying inefficiency can also be estimated using fixed, random or maximum likelihood techniques.

2.3.2.7 Time-Varying Inefficiency

The assumption of constant levels of efficiency over time is not ideal especially if data are observed over long periods. Cornwell, Schmidt and Sickles were the first to propose a model to account for time varying inefficiency within the SF panel data model (Cornwell et al., 1990). When the assumption of a time invariant inefficiency term is relaxed, the model is in the form:

\[ y_{it} = \alpha_{it} + \beta x_{it} + \nu_{it} \]

where \( \alpha_{it} = \alpha_t - \mu_{it} \) and \( \mu_{it} \geq 0 \) \hspace{1cm} (2.20)

If the intercept parameters are estimated then the technical inefficiency term is:

\[ \tilde{\mu}_{it} = \bar{\alpha}_t - \alpha_{it} \]

where \( \bar{\alpha}_t = \max (\alpha_{it}) \) \hspace{1cm} (2.21)

Another approach of incorporating changes of efficiency over time is to assume that the effect is the same for all the hospitals. A way of doing this is to separate inefficiency into two; one for each hospital, \( \mu_i \) and the secondly is all the hospitals but for each period, \( \gamma_t \) (Greene, 1993).
The other assumption is for the change in efficiency to be the same in all hospitals but with different magnitudes (Y. H. Lee & Schmidt, 1993). Lee and Schmidt outline that this model can be use fixed effects estimator or random effects model by generalized least squares (GLS).

Maximum likelihood techniques can also be used in assessing time varying efficiency. Kumbakhar suggested a model that assumes a half-normal distribution on the technical inefficiency component and vary systematically with time (Kumbhakar, 1990).

Battese and Coelli suggested an alternative to Kumbhakar 1990 model, where the technically inefficiency was assumed to have an exponential distribution (Battese & Coelli, 1992).

2.3.2.8 Heterogeneity in Stochastic Frontier Models

Heterogeneity can be measured in the inefficiency component that is attributed to time varying effects in panel data sets. However, there is additional heterogeneity that should be incorporated in efficiency analysis models. Heterogeneity can be categorized into observable and unobservable heterogeneity.

Observable heterogeneity is reflected in the measured variables. These variables might shift the production function or the inefficiency distribution or might scale them in the form of heteroscedasticity (Greene, 1993).

Unobservable heterogeneity enters the model in terms of effects and this is problematic in panel data sets but worse in cross sectional data sets as it is difficult to control for the
effects. Unfortunately also, data on such effects are rarely available or poorly measured. Greene developed models that capture unobservable heterogeneity using both fixed and random effects estimators known as 'true' effect models of the SF model (Greene, 2004; 2005a; 2005b).

This thesis explored results from different panel data models including the true effect models. Exploring the specific heterogeneity in the measured variables was not carried due to limited data.

2.4 Theory of hospital behaviour

In healthcare delivery, there are three main types of ownership: public, private for-profit and private not-for-profit. The three types have much in common such as similar resources, same regulations, employ professionals trained in similar manner and governed by same professional and ethical obligations (Horwitz, 2007). In public hospitals however, government owns and administer healthcare delivery using mostly public funds. On the other hand, shareholders or investors own the for-profit hospitals. They distribute some surplus or profits to the owners. The not-for-profit hospitals have a different structure in which members such as religious organizations, communities or non-governmental organizations, own the hospitals. They cannot distribute surplus to those who control.

From a theoretical point of view, there are several reasons and factors that may have impact on the efficiency of hospitals based on their ownership structures (Sloan, 2000). Research on health services mostly relies on agency theory, property-rights theory or public choice theory to describe the behaviour of mixed ownership. The three theories explain common reasoning that private ownership (both for and non-profit) perform better.
than public ownership due to difference in the objectives, incentives and control mechanisms (Tiemann, Schreyögg, & Busse, 2012)

Agency theory assumes that the managers (the agent), seek to maximize their own utility rather than the organization or its owners (principals). Consequently, the owners are faced with principal-agent dilemma. The agency theory assumes that private for-profit hospitals are better able to address this dilemma due to existence of a market for ownership rights, threat of takeover, threat of bankruptcy and managerial labour market (Villalonga, 2000). In public or not-for-profit ownership, the income of individuals (for example physicians) is rarely tied to hospital’s performance creating little incentive to enforce efficient behaviour (Tiemann et al., 2012).

The property rights theory emphasizes two essential elements: 1) rights to control the firm and 2) rights to the organization’s income (Hansmann, 1988). The difference between private for-profit and public and non-profit hospitals is that latter ownerships do not distribute their financial surplus to those in control (Hansmann, 1980). In the for-profit hospitals, some of the surplus is assigned to individuals and this provides a way to monitor their activities. With this, the property theory assumes that private for-profit ownership has higher efficiency compared to other types of ownerships.

The public choice theory is based on the assumption that politicians impose their objectives on the public organizations and this may lead to higher efficiency. Early public choice theorists indicated that the role of government should be limited and the most desirable way is to involve private for-profit organizations in order to increase competition and theoretically increase efficiency (Crowell, 2008).
Although according to agency and property rights theory assumptions that for-profit hospitals have higher efficiency, this might not be necessarily the fact in healthcare as there are other objectives other than profits. This can either be patient welfare, research, among others and they still face barriers such as technology and regulations. They are also faced with another challenge in which they might have fewer resources to spend on care because of taxes and emphasis on high investment return.

The public and not-for-profit hospitals main objective would ideally be to maximize the welfare of the community compared to for-profit hospitals that are profit-maximizers. The government mainly imposes the public hospitals goals, which is to serve the poor and are considered to fill the unmet needs for medical services (Alam, Elshafie, & Jarjoura, 2008). Not-for-profit hospitals main interest is also in the public but they can also respond to private or public market failures by devoting more resources to serving the needy or maximize the quality and quantity of service delivery at the expense of profits (Horwitz, 2007; Newhouse, 1970). (Deneffe & Masson, 2002) developed a model that to identify the objective function of not-for-profit hospitals and showed that they not only emphasize of social welfare but also profit (Alam et al., 2008).

Research findings on the effect of ownership differ and this might be driven by the different mixes of outputs the three types of hospitals produce (Ozcan, Luke, & Haksever, 1992). Most for profit hospitals tend to be relatively small in sizes and therefore not provide complex, tertiary services. This can lead to higher efficiencies as it minimizes complexity of output-mix. However, higher efficiencies can also be achieved when there is greater volume and mix of outputs given as a set of inputs hence will favour the more
complex non-profit hospitals (Ozcan et al., 1992). In this case, the government hospitals may seem least efficient compared to the other two types of ownership. However, government hospitals are also capable of performing better by producing higher volume and diverse outputs relative to a limited input set (Ozcan et al., 1992).

2.5 Empirical Literature of Hospital Efficiency Studies

Studies measuring hospital efficiency have been in existence since the 1980s and literature has since grown. Most of the studies are conducted in developed countries. The first studies that measured hospital efficiency were conducted in US hospitals (Nunamaker, 1983; Sherman, 1984) mainly examining the appropriateness of frontier models in health care. Considering broad scope of health services, efficiency measurement has been conducted in different type of health facilities. The review of studies in both developed and developing countries show existence of inefficiencies in the health systems implying potential scope of improvements. A recap in understanding the estimated efficiencies, frontier models produces scores that range between 0.0 (technically/allocative inefficient) and 1.0 (fully efficient).

Sections 2.5.1 and 2.5.2 summarizes the efficiency scores obtained from different studies. Given that the studies were conducted in different countries and at different levels of health care using different model assumptions and specifications, sections 2.5.1 and 2.5.2 provides only a summary of literature and their results. Section 2.5.3 highlights factors that affect efficiency as shown in literature.
2.5.1 Studies in developed countries using frontier-based techniques


Although most of the studies have been conducted in the US, frontier methods have also been applied in other developed countries. The measures of efficiency varied across all the countries. A study that examined productivity in acute Norwegian hospitals using DEA, found an average score ranging from 0.93 to 0.94 for various choices of outputs (Magnussen, 1996). There have been several studies that examined efficiency in the Greek
health care system. One was conducted in general hospitals in rural and urban regions and efficiency scores ranged from 0.67 and 0.86 (Athanassopoulos, Gounaris, & Sissouras, 1999). The other two studies conducted in Greece were done in primary health care facilities using the traditional DEA (Zavras, Tsakos, Economou, & Kyriopoulos, 2002) and in hospitals using a bootstrap DEA approach (Kounetas & Papathanassopoulos, 2012). Two studies conducted in Spain using SFA showed different results with one indicating average efficiency of 0.72 (Wagstaff, 1989) and 0.42 (Wagstaff & López, 1996) on the other study. This could be explained by the use of different sample hospitals and data. In a study conducted in the English NHS hospitals, they compared various research methods and the consistency and robustness of DEA and SFA (Jacobs, 2001). The OLS average score ranged between 0.541 and 0.611 and SFA ranged from 0.88 to 0.90. Although the study showed differences between the two techniques, they highlighted that they both have their own strength and weaknesses. Another study conducted in Finnish hospitals compared DEA and SFA showed that the choice of method depends on various factors (Linna & Hakkinen, 1998).

Other than the few literature highlighted in the above section, there are several review studies that have been conducted that summarizes most of the studies that examined efficiency in health care (Hollingsworth, 2003; Hollingsworth et al., 1999; Moshiri, Aljunid, & Amin, 2010; Worthington, 2004). In the reviews, they showed that public hospitals have higher mean efficiency of 0.95 compared to not-for-profit hospitals with a mean score of 0.824 (Hollingsworth, 2003). The average efficiency score from hospitals in the USA was 0.834 compared to Europe (UK, Finland, Greece, Spain, Austria, Norway, Belgium and France) with a score of 0.892.
2.5.2 Studies in developing countries using frontier-based techniques

Improving efficiency in health care is a priority in all countries but recently increased attention is seen in developing countries. Although expenditure in health is much lower compared to the rest of the world, any inefficiency in the health system leads to waste of resources. Policy makers are keen in improving efficiency in order to yield better value for money and improve health care delivery to the community.

Frontier-based techniques are becoming a growing interest in researchers measuring efficiency in developing countries. Initially, few statistical analyses to measure efficiency were carried out in developing countries (Anderson, 1980; Bitran-Dicowsky & Dunlop, 1989; Dor, 1987). They mainly used average or various form of total cost function. ("The cost and efficiency of public and private health care facilities in Ogun State, Nigeria," 1993) not only examined the cost structure of health services but also estimated efficiency. These methods have been rapidly changing and the frontier techniques are more common in the current literature.

In Latin America, several studies have been conducted using both SFA and DEA. A study in Chile measured efficiency in public health centres from 259 municipalities (Ramírez-Valdivia, Maturana, Mendoza-Alonzo, & Bustos, 2015). They found average efficiency score of 0.6837 and 0.5446 for the urban and rural facilities respectively in the DEA model and 0.7089 and 0.6583 respectively for the SFA model and this is attributed to lower income in rural compared to the urban municipalities. Twenty six public hospitals in Brazil were also assessed to measure performance using financial and non-financial rates and showed what indicators such as financial and operational can be used in performance analysis (Guerra, de Souza, & Moreira, 2012). Another study conducted in 30 teaching
hospitals in Brazil showed that the inefficient hospitals had mean score 0.81 and 0.84 for hospitals with beds ≥300 and <300 respectively. This implies the potential to improve efficiency in the teaching hospitals (Ozcan et al., 2009). In a study that employed a bootstrapping technique of DEA in for-profit hospitals in Brazil, showed that efficiency varied depending on the conditions of the accreditation and specialization of the hospital (Araújo, Barros, & Wanke, 2013). In one study that accounted for quality of care in hospital performance measurement in Costa Rica, showed that hospital performance was mainly driven by improved quality increases (Arocena & García-Prado, 2007). They defined quality as the number of re-admissions. In rural health posts, (Hernández & San Sebastián, 2014) assessed technical efficiency using DEA in 3 Guatemala rural health posts. The average efficiency score was 0.78 and 0.75 in 2008 and 2009 respectively.

There are other several studies conducted in other developing countries. Two studies in China that assessed efficiency using DEA showed that average efficiency improved over time. (N. Zhang, Hu, & Zheng, 2007) showed positive relationship between population density and efficiency but negative relationship between proportions of public health expenditure and efficiency. In the other study, average efficiency scores were 0.697, 0.748 and 0.790 in 2010, 2011 and 2012 respectively (Cheng et al., 2015). They also showed a positive relationship between efficiency and bed occupancy rate, ratio of beds to nurses and ratio of nurse to physicians. There are several studies conducted in Iran using DEA (Goudarzi et al., 2014; Lotfi et al., 2014; Shahhoseini, Tofighi, Jaafaripooyan, & Safiaryan, 2011; Yusefzadeh, Ghaderi, Bagherzade, & Barouni, 2013). A systematic review of efficiency measurement studies in Iran showed a pooled mean TE estimate of 0.846 (Kiadaliri et al., 2013) meaning that in general hospitals could improve performance by 15%. The results varied in the different studies but suffered from similar
methodological challenges, for example, lack of data on quality of care and case mix. They also showed that there were no differences in DEA and SFA results from studies conducted in Iran. Another study conducted in district, sub-divisional and state general hospitals in India showed an overall mean efficiency of 0.728 (Dutta, Bandyopadhyay, & Ghose, 2014) compared a score of 0.90 in study conducted in district hospitals in another state (Ram Jat & San Sebastián, 2013). Both studies used DEA in measuring efficiency. A larger study that measured technical efficiency and determined factors affecting TE was conducted in 10 Economic Cooperation Organization (ECO) countries (Ravangard, Hatam, Teimourizad, & Jafari, 2014). The countries included were Iran, Turkey, Azerbaijan, Pakistan, Afghanistan, Kyrgyzstan, Tajikistan, Kazakhstan, Turkmenistan and Uzbekistan. They used two approaches with different inputs and outputs in the model and obtain an average score of 0.497 and 0.563 in the first and second approach respectively. They also showed that GDP per capita and health expenditure per capita had significant relationship with efficiency in the health systems.

Studies conducted in Africa on the other hand have been developing in recent years. Most of the studies employed DEA in the assessment of efficiency in different types and levels of hospitals. Table 2.2 summarizes the different efficiency measurement studies conducted in Africa. The studies varied in terms of sample size, data structure (cross-sectional, panel or multiple cross-sections), type of hospitals (government-owned, private not-for-profit and for-profit), input and output variables and the frontier techniques used. Majority of the studies employed DEA for measuring efficiency. Overall, the efficiency scores in health centres in the studies reviewed were slightly higher than those conducted in hospitals.
There are only two studies that used SFA approach for efficiency measurement. (Ramanathan & Chandra, 2003) study used both DEA (discussed above) and SFA in measuring technical efficiency in 22 health districts in Botswana. They estimated individual efficiency scores relative to various ailments. They found three health districts (all urban or semi-urban) performed better in treating patients from most ailment groups. Hukuntsi, Chobe and Kgalagadi, which are rural health districts, had lowest ranks. They however, did not carry out direct comparison of DEA and SFA. The other study that used SFA was a study in specialized surgeon clinics (Koch & Slabbert, 2012). They found that the average efficiency score was 0.50, which is much lower than previous studies conducted in Africa. This suggested that the private surgical clinics were less efficient than most hospitals in South Africa.

2.5.3 Ownership and efficiency in health care

There are several factors that determine efficiency in health care. The most common determinant that has been examined in literature is the ownership type. The perception has been that non-public facilities run more efficiently than public facilities. This is because the 'excess' recurrent expenditure makes the non-public hospitals attractive to meet the high demands of the health system. Some studies confirmed that indeed non-public hospitals were more efficient than the public/government hospitals (Czypionka, Kraus, Mayer, & Rohrling, 2014; Herr, Schmitz, & Augurzky, 2011; Maredza, 2012; Masiye, 2007; Masiye et al., 2006). However, there are several studies that have disputed this claim in their respective settings (Bosmans & Fecher, 1995; Herr, 2008; Jehu-Appiah et al., 2014; Ozcan et al., 1992; Roh, Moon, & Jung, 2013; Valdmanis et al., 2008) and showed that public hospitals perform better than the non-public hospitals. In a systematic review of efficiency measurement in hospitals (Hollingsworth et al., 1999), the study showed that
public hospitals had the highest mean efficiency score of 0.96 compared with not-for-profit hospitals (score of 0.80). Other factors than ownership highlighted in literature are in relation to size and capacity, geographical location and specialization.

2.5.4 Overall recommendations from studies conducted in Africa

The efficiency measurement studies conducted in Africa as summarized in Table 2.2, proposed varied recommendations depending on the data used and methods employed. These include:

- **Inputs:** there were recommendations in reducing excess inputs especially staffing levels and beds either by transferring to other lower levels (e.g. health centres) from hospitals or terminating contracts for some of staff cadres.

- **Outputs:** there was emphasis on the need to increase hospitals services in order to improve efficiency. Also due to lack of data, most of the studies did not use health final outcomes or quality-adjusted outputs. Including outputs adjusted for quality or case-mix was highlighted as one of the areas for future research in studies conducted in Africa.

- **Environmental factors:** including other factors that might affect efficiency was recommended in the studies.

- **Ownership:** exploring the effect of ownership on efficiency by including for-profit and not-for-profit hospitals was highlighted in the studies that were conducted in public hospitals.

- **Other levels of hospitals:** there recommendation in examining efficiency in other levels of health care e.g. health centres, health posts etc.

- **Panel data:** use of panel data to examine efficiency over time was highlighted as areas of future research in the cross sectional studies. They also emphasized on the use of Malmquist index to analyse productivity change.

- **Allocative efficiency:** due to lack of data on prices, all the studies as highlighted in Table 2.2 were not able to assess allocative efficiency. This is still a major gap in studies conducted in Africa.
Overall in all the studies conducted in Africa as summarized in Table 2.2, there was emphasis on better data and health information systems in order to ensure routine and high quality data for monitoring and developing nation-wide performance framework.
<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
<th>Country</th>
<th>Data Year</th>
<th>Number of facilities</th>
<th>Type of health facility</th>
<th>Input/output variables (n)</th>
<th>Method</th>
<th>Measures</th>
<th>Mean technical efficiency score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(Kirigia, Lambo, &amp; Sambo, 2000)</td>
<td>South Africa</td>
<td>1995/1996</td>
<td>55</td>
<td>Public hospitals</td>
<td>Doctors, nurses, paramedics, technicians, administrative staff, general staff, labour provisioning staff, other staff, beds (9 inputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.906</td>
</tr>
<tr>
<td>2.</td>
<td>(Kirigia, Sambo, &amp; Scheel, 2001)</td>
<td>South Africa</td>
<td>1995/1996</td>
<td>155</td>
<td>Public Clinics</td>
<td>Number of nurses, number of general staff (2 inputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.730</td>
</tr>
<tr>
<td>3.</td>
<td>(Zere, McIntyre, &amp; Addison, 2001)</td>
<td>South Africa</td>
<td>1992/1993 to 1996/1997</td>
<td>86</td>
<td>Level I, II and III hospitals</td>
<td>Recurrent expenditure, Beds (2 inputs)</td>
<td>DEA</td>
<td>Tobit model</td>
<td>Level I - 0.828 Level II - 0.825 Level III - 0.820</td>
</tr>
<tr>
<td>4.</td>
<td>(Kirigia et al., 2002)</td>
<td>Kenya</td>
<td>2000</td>
<td>54</td>
<td>Public district hospitals</td>
<td>Medical officers/pharmacists/dentists, clinical officers, nurses, administrative staff, technicians/technologists, other staff, subordinate staff, pharmaceuticals, non pharmaceutical supplies, maintenance of equipment, vehicles and buildings, food and rations (11 inputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.956</td>
</tr>
<tr>
<td>5.</td>
<td>(Ramanathan &amp; Chandra, 2003)</td>
<td>Botswana</td>
<td>1997</td>
<td>22</td>
<td>Public health districts</td>
<td>Number of hospitals, number of clinics, number of health posts, number of beds, doctors, nurses, other staff (7 inputs)</td>
<td>SFA</td>
<td>DEA</td>
<td>Technical efficiency</td>
</tr>
<tr>
<td>6.</td>
<td>(Kirigia et al., 2004)</td>
<td>Kenya</td>
<td>-</td>
<td>32</td>
<td>Public health</td>
<td>Clinical officers and nurses</td>
<td>DEA</td>
<td>Technical</td>
<td>0.766</td>
</tr>
<tr>
<td>No.</td>
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<td>Country</td>
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<td>Number of facilities</td>
<td>Type of health facility</td>
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<td>Method</td>
<td>Measures</td>
<td>Mean technical efficiency score</td>
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<td>7.</td>
<td>(Osei et al., 2005)</td>
<td>Ghana</td>
<td>2000</td>
<td>34</td>
<td>Public district hospitals and health centres</td>
<td>Physiotherapist, occupational therapist, public health officers and dental technologist, laboratory technician and technologist, administrative staff, non-wage expenditures, number of beds (6 inputs) Diarrhoeal, malaria, STI, UTI, intestinal worms and respiratory disease visits, antenatal and FP visits, immunizations, other general outpatient visits (4 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>District - 0.815 HCs – 0.91</td>
</tr>
<tr>
<td>8.</td>
<td>(Renner, Kirigia, Zere, &amp; Barry. 2005)</td>
<td>Sierra Leone</td>
<td>2000</td>
<td>37</td>
<td>Public Peripheral health units</td>
<td>Technical staff, sub-ordinate staff (2 inputs) Antenatal and post natal care, deliveries, nutrition/growth monitoring visits, FP visit, &lt;5 years immunized and pregnant women, health education sessions (6 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.78</td>
</tr>
<tr>
<td>9.</td>
<td>(Masiye et al., 2006)</td>
<td>Zambia</td>
<td>-</td>
<td>40</td>
<td>Public and private Health Centres</td>
<td>Number of Clinical officers, nurses, support staff (3 inputs) Number of outpatient visits (1 output)</td>
<td>DEA</td>
<td>Technical, allocative efficiency</td>
<td>Public – 0.56 Private – 0.70</td>
</tr>
<tr>
<td>10.</td>
<td>(Zere et al., 2006)</td>
<td>Namibia</td>
<td>1997/1998 - 2000/2001</td>
<td>30</td>
<td>District hospitals</td>
<td>Recurrent expenditure, number of beds, staff (3 inputs) Outpatient visits, inpatient days (2 outputs)</td>
<td>DEA</td>
<td>Technical efficiency</td>
<td>97/98 – 0.716 98/99 – 0.743 99/00 – 0.627 00/01 – 0.669</td>
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<tr>
<td>11.</td>
<td>(Kibarbe &amp; Koch, 2007)</td>
<td>South Africa</td>
<td>2004</td>
<td>14</td>
<td>Public hospitals</td>
<td>Beds, Doctors, Nurses (3 inputs) Outpatient visits, inpatient days, admission, surgeries (4 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>Varied combinations of inputs and outputs Single output – 0.636 – 0.903</td>
</tr>
<tr>
<td>No.</td>
<td>Author(s)</td>
<td>Country</td>
<td>Data Year</td>
<td>Number of facilities</td>
<td>Type of health facility</td>
<td>Input/output variables (n)</td>
<td>Method</td>
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<td>Mean technical efficiency score</td>
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<td>12</td>
<td>Kirigia, Emrouznejad, Vaz, Bastiene, &amp; Padayachy, 2007)</td>
<td>Seychelles</td>
<td>2001-2004</td>
<td>17</td>
<td>Public Health centres</td>
<td>Total number of doctor hours, total number of nurses hours (2 inputs) Number of patients dressed, domiciliary cases treated, PFMAPIS (3 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency Mahnquist index</td>
<td>2001 – 0.93 2002 – 0.92 2003 – 0.92 2004 – 0.96</td>
</tr>
<tr>
<td>13</td>
<td>Masiye, 2007)</td>
<td>Zambia</td>
<td>2003</td>
<td>52</td>
<td>Public and Mission hospitals</td>
<td>Non labour costs, doctors, nurses/COs/lab techs/radiographers/pharmacists, administrative/other staff (4 inputs) Ambulatory care, inpatients, MCH, lab tests/X rays/theatre operations (4 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>Overall – 0.67 Public – 0.63 Mission – 0.73</td>
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<td>14</td>
<td>Akazili, Adjuik, &amp; Jehu-Appiah, 2008)</td>
<td>Ghana</td>
<td>2004</td>
<td>89</td>
<td>Public health centres</td>
<td>Non clinical staff, clinical staff, beds and cots, expenditure on drugs and supplies (4 inputs) General outpatient visits, antenatal visits, deliveries, children immunized, FP visits (5 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.748</td>
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<tr>
<td>15</td>
<td>Kirigia, Emrouznejad, &amp; Cassoma, 2008)</td>
<td>Angola</td>
<td>2000-2002</td>
<td>28</td>
<td>Municipal hospitals</td>
<td>Doctors and nurses, expenditure on pharmaceutical and non pharmaceutical supplies, beds (3 inputs) Outpatient visits, inpatient admissions (2 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency Mahnquist index</td>
<td>2000 – 0.662 2001 – 0.658 2002 – 0.675</td>
</tr>
<tr>
<td>16</td>
<td>Marschall &amp; Flessa, 2009)</td>
<td>Burkina Faso</td>
<td>2004</td>
<td>20</td>
<td>Health centres</td>
<td>Personnel cost, building area, depreciation of equipment, vaccination costs (4 inputs) General consultation and nursing care, deliveries, immunization, special services (4 outputs)</td>
<td>DEA</td>
<td>Tobit model</td>
<td>0.91</td>
</tr>
<tr>
<td>17</td>
<td>Ismail, 2010)</td>
<td>Sudan</td>
<td>2007</td>
<td>15</td>
<td>Public hospitals</td>
<td>Number of hospitals, number of health centres, beds, physicians, ancillary staff (5 inputs) Outpatients, inpatients (2 inputs)</td>
<td>DEA</td>
<td>Technical efficiency</td>
<td>0.935</td>
</tr>
<tr>
<td>18</td>
<td>Sebastian &amp; Lenuna, 2010)</td>
<td>Ethiopia</td>
<td>2000</td>
<td>60</td>
<td>Health posts</td>
<td>Number of extension health workers, voluntary health workers (2 inputs) Health education sessions, antenatal care visits, deliveries, FP visits, diarrhoeal cases treated, number of visits carried by community health workers, total new</td>
<td>DEA</td>
<td>Tobit model</td>
<td>0.57</td>
</tr>
</tbody>
</table>

53
<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
<th>Country</th>
<th>Data Year</th>
<th>Number of facilities</th>
<th>Type of health facility</th>
<th>Input/output variables (n)</th>
<th>Method</th>
<th>Measures</th>
<th>Mean technical efficiency score</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>(Tlotlego, Nonvignon, Sambo, Asbu, &amp; Kirigia, 2010)</td>
<td>Botswana</td>
<td>2006-2008</td>
<td>21</td>
<td>Public district and primary hospitals Mission hospital (2) Private hospital (1)</td>
<td>Clinical staff, beds (2 inputs) Outpatient visits, inpatient days (2 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency Malmquist productivity index</td>
<td>2006 – 0.704 2007 – 0.742 2008 – 0.763</td>
</tr>
<tr>
<td>20</td>
<td>(Ichoku, Fonta, Onwujekwe, &amp; Kirigia, 2011)</td>
<td>Nigeria</td>
<td>2009</td>
<td>200</td>
<td>Public Private hospitals</td>
<td>Beds, doctors, pharmacists, nurses, other staff, expenditure on drugs, expenditure on power, expenditure on equipment (8 inputs) Outpatients, inpatients, lab tests (3 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.72 Results not reported by ownership</td>
</tr>
<tr>
<td>21</td>
<td>(Kirigia, Sambo, Mensah, Mwikisa, &amp; Asbu, 2011b)</td>
<td>Benin</td>
<td>2003-2007</td>
<td>23</td>
<td>Zone Public hospitals</td>
<td>Doctor/physician hours, nurses/midwives hours, laboratory, x-ray, anaesthetists, paramedics and assistants hours, non-salary running costs, beds (5 inputs) Outpatient visits, admissions (2 outputs)</td>
<td>DEA</td>
<td>Malmquist productivity index</td>
<td>Mean technical change – 0.757</td>
</tr>
<tr>
<td>22</td>
<td>(Kirigia, Sambo, &amp; Renner, 2011a)</td>
<td>Sierra Leone</td>
<td>2008</td>
<td>79</td>
<td>Public peripheral health units</td>
<td>Number of community health officers/ MCH aides/state controlled community health nurses, number of support staff (2 inputs) Outpatient/MCH/FP/Immunization visits, vector control activities, health education sessions (3 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.692</td>
</tr>
<tr>
<td>23</td>
<td>(Marschall &amp; Flessa, 2011)</td>
<td>Burkina Faso</td>
<td>2005</td>
<td>25</td>
<td>Primary care facilities</td>
<td>Personnel cost, building area, depreciation of equipment, vaccination costs (4 inputs) General consultation and nursing care, deliveries, immunization, special services (4 outputs)</td>
<td>Two-stage DEA Tobit model</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.85</td>
</tr>
<tr>
<td>24</td>
<td>(Koch &amp; Slabbert, 2012)</td>
<td>South Africa</td>
<td>2007</td>
<td>58</td>
<td>Specialist surgeon clinics</td>
<td>Nurses, administrators, orthopaedics, vascular surgeons, other surgeons (5 inputs) Total patients, new patients, surgeries (3 outputs)</td>
<td>SFA</td>
<td>Technical efficiency</td>
<td>0.50</td>
</tr>
<tr>
<td>25</td>
<td>(Maredza, 2012)</td>
<td>Zimbabwe</td>
<td>2006-2008</td>
<td>100</td>
<td>Public Profit hospitals</td>
<td>Beds, doctors, nurses (3 inputs) Inpatient days, discharges (2 outputs)</td>
<td>Two-stage Technical efficiency</td>
<td>Public – 0.503 For-profit – 0.614</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>Author(s)</td>
<td>Country</td>
<td>Data Year</td>
<td>Number of facilities</td>
<td>Type of health facility</td>
<td>Input/output variables (n)</td>
<td>Method</td>
<td>Measures</td>
<td>Mean technical efficiency score</td>
</tr>
<tr>
<td>-----</td>
<td>-----------</td>
<td>---------------</td>
<td>--------------------</td>
<td>----------------------</td>
<td>-------------------------</td>
<td>--------------------------------------------------------------------------------------------</td>
<td>-------------------------</td>
<td>------------------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>26</td>
<td>Zamo-Akono, Ndjokou, &amp; Son-Namack, 2013</td>
<td>Cameroon</td>
<td>2001/02 2002/03</td>
<td>108</td>
<td>Peripheral health centres</td>
<td>Health care givers, nurse assistants, other medical staff, administrative staff, beds (5 inputs) Consultations, deliveries (2 outputs)</td>
<td>DEA Tobit model</td>
<td>Scale efficiency</td>
<td>0.7098</td>
</tr>
<tr>
<td>27</td>
<td>Kirigia &amp; Asbu, 2013</td>
<td>Eritrea</td>
<td>2007</td>
<td>19</td>
<td>Public Community Hospital</td>
<td>Physicians/doctors, nurses/midwives, laboratory technicians, beds and cots (4 inputs) Outpatient visits, discharges (2 outputs)</td>
<td>Two-stage DEA Tobit model</td>
<td>Scale efficiency</td>
<td>0.967</td>
</tr>
<tr>
<td>28</td>
<td>Nannyonjo &amp; Okot, 2013</td>
<td>Uganda</td>
<td>2008/09 2009/10</td>
<td>44</td>
<td>Public hospitals</td>
<td>Number of staff at local government, financial resources, management system, number of staff at health facilities, number of beds, number of equipment (6 inputs) Management indicators (3), service delivery indicators (7) (Total 10 outputs)</td>
<td>Two-stage DEA</td>
<td>Technical Efficiency Scale efficiency</td>
<td>0.92</td>
</tr>
<tr>
<td>29</td>
<td>Sede &amp; Ohemeng, 2013</td>
<td>Nigeria</td>
<td>2000-2008</td>
<td>24</td>
<td>Public hospitals</td>
<td>Beds, doctors, nurses, other staff (4 inputs) Admissions, outpatients, surgeries, deliveries (4 outputs)</td>
<td>DEA</td>
<td>Technical Efficiency Scale efficiency</td>
<td>0.84</td>
</tr>
<tr>
<td>30</td>
<td>Jebu-Appiah et al., 2014</td>
<td>Ghana</td>
<td>2005</td>
<td>128</td>
<td>Public Mission Quasi-government Private hospitals</td>
<td>Beds, clinical staff, non clinical staff, expenditure (4 inputs) Inpatient days, outpatient days, deliveries, laboratory services (4 outputs)</td>
<td>Two-stage DEA Tobit model</td>
<td>Technical Efficiency Scale efficiency</td>
<td>0.7035 Mission – 0.6859 Private – 0.5583 Quasi – 0.83</td>
</tr>
<tr>
<td>31</td>
<td>Bwana &amp; Raphael, 2015</td>
<td>Tanzania</td>
<td>2009-2013</td>
<td>16</td>
<td>Private not-for-profit hospitals</td>
<td>Beds, doctors, nurses, non-medical (4 inputs) Inpatients, discharges, outpatients (3 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>2009 – 0.5750 2010 – 0.5950 2011 – 0.6079 2012 – 0.5274 2013 – 0.5691</td>
</tr>
<tr>
<td>32</td>
<td>Kinyanjui, Gachanja, &amp; Muchai, 2015</td>
<td>Kenya</td>
<td>-</td>
<td>30</td>
<td>Faith-Based hospitals</td>
<td>Medical officers and specialists, nurses, beds and cots, other workers (4 inputs) Inpatients and outpatients (2 outputs)</td>
<td>DEA</td>
<td>Technical efficiency Scale efficiency</td>
<td>0.779</td>
</tr>
</tbody>
</table>
2.6 Gaps in Literature

2.6.1 Geographical

As described in the empirical literature, most of efficiency measurement studies in health care have been conducted in developed countries. There is need to explore more of these aspects in developing countries and especially in Africa. In Kenya alone, there are only two published studies that examined efficiency in health care (Kirigia et al., 2002; 2004) and since then some of the recommendations in the study have not been explored. This thesis examines efficiency measurement in Kenyan hospitals using both DEA and SFA in public and faith-based hospitals.

Efficiency measurement in Kenyan health facilities used central data from the national level (Kirigia et al., 2002). These types of data are prone to error due to the various steps taken to transfer data to the national level and has been shown to be of poor quality (Kihuba et al., 2014). In this study, data were collected from individual hospitals, which provided an opportunity to check for errors and seek clarification from the original source or the health records officers.

2.6.2 Methods/Analysis techniques

Data envelopment analysis (DEA) has been widely used in efficiency measurement in health care in Africa. Considering the various challenges with obtaining and compiling data from this setting, there is a need to separate any measurement error and consider how this influences efficiency. Stochastic frontier analysis separates inefficiency and measurement error/noise, while the DEA lumps them together. This thesis explores both DEA and SFA by applying data from Kenyan hospitals.
This study is also the first in Kenya to look at the various methodological assumptions and specifications of the SF model. In this study, efficiency is estimated using both cross sectional and panel data and therefore provides a platform to examine the different methodological aspects of the frontier analysis techniques.

One of the areas that have not been explored in Kenya is hospital efficiency measurement by ownership. Data from obtained from public and faith-based hospitals in Kenya are analysed and discussed in this thesis. The effect of ownership on efficiency and the differences between the two ownership types are also discussed in this thesis.

2.7 Conclusion

This chapter’s aim was to highlight the main theoretical and empirical review of measurement of efficiency. Although the empirical review is not exhaustive, the summarized studied in this chapter gives an overview of the area of study and applications of the frontier techniques applied in different settings. SFA and DEA approaches are employed in this thesis with further exploration of different model assumptions and factors that determine efficiency in Kenyan hospitals.
3 Overview of Data: Sources and Methods

3.1 Introduction

This chapter presents the data collection approach. Sample size and inclusion criteria for the select hospitals are discussed. Also discussed are the various inputs and outputs that were applied in the thesis and the challenges and limitations of the study.

3.2 Study setting

The study was conducted in Nairobi, Central, Nyanza and Coast Provinces. Provinces and districts were the initial administrative set up before the implementation of the new Kenyan constitution in 2013 that devolved functions to the current county set up (refer to Chapter 1). Geographical regions were selected to represent the poverty levels in the Kenya. The constituency poverty report (Kenya National Bureau of Statistics, 2007) classified 2 provinces (Coast and North Eastern) as extremely poor (poverty incidence > 55%), 4 provinces (Eastern, Rift Valley, Nyanza and Western) as poor (45%-54%) and 2 provinces (Nairobi and Central) as non-poor (<45%). Therefore, a total of 4 provinces were selected purposively representing each poverty level. These were Nairobi, Central, Nyanza and Coast Provinces. Nairobi was selected primarily because it largely urban with few district hospitals. The hospitals were then randomly selected from both rural and urban regions using the health facilities master list (Ministry of Health Kenya, 2011).

Data were collected from both public (government-funded) and non-profit faith-based hospitals in Kenya. These types of hospitals were chosen for several reasons: 1) Public hospitals serve the majority of the population, estimated as 58% of all outpatient visits (Ministry of Health Kenya, 2015a); 2) Measuring efficiency is important for ensuring
accountability and efficiency of public spending in the health sector 3) Faith-based hospitals are mainly non-profit and aim to provide services for the poor. Before the devolution, some faith based facilities, particularly facilities in remote rural areas, received support from the government largely in the form of supplies and commodities and employment of health workers. Lastly, collecting data from both public and faith-based hospitals provides an opportunity to estimate efficiency scores by ownership, as it is often perceived that private-not-for profit facilities are more efficient than public.

Only the Level IV former district hospitals (currently known as County hospitals) were selected for the study. This was to ensure more homogenous of hospitals recruited in the study. Each district had at least one public level IV hospital, which provides primary care to the population in that particular region. After collecting data from the individual hospitals, some of the observations were missing from particular hospitals. In such cases, the gaps were later filled by additional data obtained from the Ministries of Health and Finance and records and accounts departments from overseeing faith-based organization bodies. Overall the data collected from the selected hospitals were between the period of 2008 and 2012.

3.3 Data Sources

Data collected in this study were obtained from different sources. The hospital activity data (number of visits and procedures) were obtained from the individual hospitals recorded in MoH facility forms (Workload form MoH 717, service delivery form MoH 105) (refer to Appendix C, Appendix D and Appendix E on sample MoH forms). The records were in monthly format with both outpatient and inpatient workload information. For any data not available at the facilities, the data were obtained from similar forms and database (Kenya
health information systems, DHIS) from the Ministry of health. This was the same for both public and faith-based hospitals.

Staffing levels data were obtained from the various human resource departments in the hospitals (refer to Appendix G on sample record form). There was an emphasis in most hospitals that there were no significant changes in the numbers over time because if there was any turnover of staff, the position was replaced in most cases.

The expenditure data were obtained in the different finance and accounting departments in the individual hospitals. The forms and structure of the data varied between the public and faith based hospitals (refer to Appendix F for a sample of finance form in a public hospital). Most of the data from public hospitals were in monthly format but for faith-based the data were in annual format. Data not available from the individual hospitals were obtained from records submitted at the national level. This was either from the MoH or the specific faith-based organization bodies that oversee the hospitals (Christian Health Association of Kenya (CHAK) or Kenya Conference of Catholic Bishops (KCCB)).

### 3.4 Sampling

Sample size calculation was done using Banker and Morey (Banker & Morey, 1989) method described by the equation

\[ n \geq 3 (m+s) \]  \hspace{1cm} (3.1)

where \( n \) is the number of hospitals, \( m \) is the number of inputs and \( s \) is the number of outputs.
This equation does not incorporate distribution of inefficiencies and covariate structure of factors therefore a multi-stage random selection process was used in addition to estimate the sample size for the study. A total of 52 hospitals were included in the sample with at least 31 district public hospitals and 23 faith-based hospitals.

3.5 Data collection process

A pilot study was conducted to evaluate the status, availability and format of data from two hospitals (see summary in Appendix A). The data collection tool was guided by previous studies conducted in hospitals across Africa and a WHO/African regional office efficiency questionnaire (World Health Organization, 2000).

After identifying the type of data available at the hospitals, the thesis study was designed to ensure as much of variables identified in the pilot study were collected. The process of data collection for this thesis is outlined in the flow diagram Figure 3.1.
**PILOT STUDY:** Assess status, format and availability of data

- Developed a proposal and submitted to national scientific committee
- Submitted proposal to national ethical review committee
- Approval from MoH
- Approval from various faith-based umbrella organizations
- Obtained approval from each hospital
- Data collection from individual hospitals
- Data collection from other sources: Ministry of Health, Ministry of Finance, Accounting departments for overseeing organizations (faith-based)

Figure 3.1: Data approval and collection process

### 3.6 Data Sets

In general not all data were collected from some of the hospitals either due to accessibility challenges or lack of data. The flowchart describes the total number of hospitals sampled (refer to Appendix B for the list and location of hospitals) and in which hospitals data were collected and analysed (Figure 3.2).

Data used for the different frontier methods are discussed in the analysis chapters. Sub sample data sets were created for analysis in order to compare DEA and SFA.
Sampled hospitals \( (n = 52) \)

- Excluded due to lack of approval \((n = 9)\)
  - Complete refusal due to hospital policy \((n = 1)\)
  - Silent refusal: internal ethics approval that takes long, no feedback or has to be approved by head that was not available \((n = 8)\)

Approval granted \((n = 43)\)

- Excluded due to physical accessibility and slow response on approval (time constraints) \((n = 5)\)
  - Physical constraints \((n = 3)\)
  - Time constraints \((n = 2)\)

Accessed hospitals \((n = 38)\)

- Excluded due to lack of all data \((n = 2)\)

Hospitals included in initial data cleaning and management \((n = 36)\)

- Excluded due to lack of specific key variables i.e. staffing levels and finance data \((n = 9)\)

Hospitals included in final data analysis \((n = 27)\). There were 432 observations and this is because some observations were dropped due to lack of output data.

Figure 3.2: A summary of the data collection process from the sampled hospitals
3.7 Selection of variables (inputs and outputs)

The pilot study identified possible inputs and outputs that were collected routinely in the hospitals were identified. Ideally, outputs should be measured in relation to increasing patient health status by using final outcomes. However, since this is technically complex to measure, the outputs collected were mainly intermediate outputs. These were activity-based such as number of visits, hospital stays and procedures. The inputs were both physical (number of staff disaggregated and capital input approximated by number of beds) and monetary (expenditure). All the inputs and outputs data collected from the hospitals are summarized in Table 3.1. This study did not include other environmental factors due to challenges in obtaining this data.

Estimates of outpatient and inpatient services were obtained from costing studies conducted in Kenya. ABCE Kenya study (Institute for Health Metrics and Evaluation (IHME), 2014) and the Kenya health sector costing study (Flessa, 2011) estimated average outpatient visit cost to be Kshs. 835 ($10), cost per admission was Kshs.12970 ($153) and average cost per inpatient bed-day was Kshs. 2818 ($34) for district-level hospitals. This thesis employed multiple outputs (total outpatients and admissions) in form of an index calculated using the Fisher’s index (Fisher, 1922) and several inputs (doctors, nurses, clinical officers, other health workers, expenditure and number of beds). The other variables as highlighted in Table 3.1 were not included in the final analysis due to lack of complete data in most of the hospitals.
Table 3.1: Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Data format</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Outputs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outpatients</td>
<td>Total number of patients seen at the outpatient. This includes general and special outpatient services. Examples of special outpatient services are dental units, maternal and child health, STI/HIV clinics, psychiatry, orthopaedic, eye units and ear, nose and throat (ENT) clinics.</td>
<td>Monthly</td>
</tr>
<tr>
<td>Admissions</td>
<td>Number of total patients admitted to the hospital.</td>
<td>Monthly</td>
</tr>
<tr>
<td><strong>Inputs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Staffing levels</td>
<td>Total number of staff in the hospital i.e. number of medical doctors, clinical officers, nurses, community health workers, laboratory staff and support staff for example administrative, transport and maintenance</td>
<td>Quarterly</td>
</tr>
<tr>
<td>Beds</td>
<td>Total number of recorded beds within a hospital</td>
<td>Monthly</td>
</tr>
<tr>
<td>Recurrent expenditure</td>
<td>Total amount of the budget spent on hospital services including wages and salaries for staff at faith-based hospitals and casual workers at public hospitals.</td>
<td>Quarterly</td>
</tr>
<tr>
<td><strong>Other Variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type of hospital</td>
<td>Information regarding whether the hospital is a public/government hospital or a non-profit faith based hospital</td>
<td>-</td>
</tr>
</tbody>
</table>

Although data on discharges, surgeries, deliveries, radiology services, lab tests, length of hospital stay were collected for some of the hospitals, they were not enough to include in the final analysis.

### 3.8 Computation of expenditure

This section describes the expenditure data used in the analysis. The general description of the sources of funding and budget allocation in the Kenyan health system is described in Chapter 1. Both recurrent and development expenditure data were collected in this study. Data on hospital own generated revenue from user fees (collections) were also obtained from the facilities.
The main finance data used in the analysis were recurrent expenditure. This is because they were consistent in all the hospitals. In cases where recurrent expenditure was completely missing, the gaps were substituted by amount of user fees collections as a proxy measure. The final analysis used non-staff costs, which is the total expenditure without the staff costs (salaries and wages). The reason for including only non-staff expenditure is due to high correlation between total expenditure and staffing level. This was producing biased results in the frontier model outputs. In this thesis, the term expenditure refers to total recurrent expenditure without salaries and wages (non-staff costs).

There were differences in the reporting of non-staff costs in public and faith-based hospitals. The itemized version of the finance data was similar in all the public hospitals with limited items on the list. For the faith-based hospitals, there were a lot more items in the financial list including some administrative costs that were might not directly link to patient care. These differences were however noted in some of the hospitals and not all of them (for faith-based).

3.9 Challenges and limitations

When conducting a study on efficiency measurement in healthcare, there are several challenges in finding accurate, quality data that are comparable over time. There are also issues with lack of data and the need to deal with missing data. This section highlights some of the challenges and limitation encountered during data collection this study.
3.9.1 Missing data

Missing data poses a challenge for measuring efficiency especially using frontier analysis techniques. This might lead to inconclusive and invalid results. Challenges of missing data were addressed by adopting various strategies including:

- **Missing hospital-level data:** There were cases where some of the selected hospitals did not have any of the required data properly stored (e.g. recorded documents scattered in the archives room) or were lost. In such cases where all the data were not available, the hospitals were excluded from analysis. For some selected hospitals, gaining access was difficult and was a major barrier. Such hospitals were considered to have missing data since no data was collected from the facility.

- **Missing observations (monthly, quarterly or annually):** This issue was common in several hospitals. Most of the data collected were in monthly format but in some cases there were only quarterly and annual format available depending on the variable and type of hospital. Since the activity data were collected in monthly format and all analysis was carried out in quarterly format, the data were aggregated to generate the quarterly data. Some hospital had missing months even after counterchecking with other sources. For these scenarios, average was calculated from available months. In cases where only annual data were available, the data were disaggregated into quarters equally or based on any other quarterly data available.

- **Missing variables:** there were cases in which some of the input and output variables were missing in particular hospitals. As much data for these variables were obtained from all sources and for cases where the key inputs and outputs were not available, the particular observations were excluded from the analysis.
3.9.2 Hospital differences

Although the hospitals were selected at district level, there were inconsistencies in data recording of some of the variables. This was more so in the different types of ownership. Although public hospitals have standard government forms for recording data, faith based hospitals have a different system of entering and storing information especially for finance data. These differences were a challenge in synthesizing and ensuring that data collected had similar interpretations across all hospitals. If there are systematic differences in the two types of hospitals, these are reflected in the analysis.

3.9.3 Limitations of variables

The ideal outcome measurement is the final outcomes on patient’s quality of care. However, this is complex to measure and intermediate outputs are used instead in this analysis. The inputs and outputs were limited to those that were available and routinely collected in the hospitals. There were few options due to status of data management in Kenya. Future analyses that capture other inputs and outputs including quality of care will be key in this type of health system analysis.

3.9.4 Lack of real-time data

The data collected for this study was based on retrospective routinely collected hospital data from 2008 to 2012. This is a limitation due to lack of data for recent years. Collecting data prospectively, although might be more expensive, will be ideal in carrying out analysis on real time data and will also deal in capturing as much information as possible with less challenge of missing and poor quality data.
3.10 Conclusion

This chapter outlined the data that were used for this thesis. There were some challenges and limitations with the data and recognizing this not only explains the nature of analysis but also highlights the issues with hospital data in Kenya and developing countries as a whole. More work is needed in developing better tools for collecting and storing data generated from facilities. This will provide better quality, variety, scope and larger sample sizes of data for future studies.

Chapters 5 and 6 discuss the various analysis techniques used in the context of this thesis and the results from the efficiency measurement of the sampled Kenyan hospitals.
4 Application of Data Envelopment Analysis on Kenyan Hospital Data

4.1 Introduction

This chapter outlines efficiency measurement of cross sectional hospital data using the DEA approach. The theoretical development of DEA is described in Chapter 2. This chapter also examines in detail the DEA bootstrapping modelling order to assess the confidence intervals around the estimates.

Efficiency estimation efficiency by ownership is also presented and discussed in this chapter. This study collected data from both public and faith-based hospitals to show the effect of ownership on efficiency. The discussion in this chapter will present the technical efficiency levels of selected public and faith-based hospitals in Kenya. Truncated regression analysis will be used to assess the effect of ownership on efficiency.

4.2 Model specification

In DEA, a hospital is considered fully efficient if the score is equal to 1.0 and all of the slacks are zero. This means that the fully efficient hospital(s) is (are) located on the frontier. One of the main advantages of DEA over SFA is that the approach can incorporate multiple outputs easily. Since, the results from the DEA model are compared with the SFA approach (Chapter 5), analysis using single output is explored. The analysis framework employs single output (derived from an index total outpatients and total admissions (refer to Chapter 3)) and seven inputs (doctors, nurses, clinical officers, other health workers, other staff, expenditure and beds). Multiple output approach of outpatients and admissions is also discussed.
A limitation with DEA is that it does not allow a panel data structure. Therefore, the framework in this chapter is an analysis of subset of the data collected. Two cross sectional samples of the aggregated data are analysed and this consists of data from the years 2011 and 2012. This implies that the DEA results are of two separate cross-sections i.e. 2011 and 2012. Malmquist index can, however, be used to assess productivity change and this is highlighted in the areas of future research (section 8.4.2).

Input-oriented model assumes that the hospitals have limited control over the outputs while the output-oriented model assumes that the hospital management has no control over the inputs. The input-oriented model was used in this study because hospital management have control over the use of inputs such as ensuring staffing levels and beds are maintained to a certain level (English, Claudio F, Isaac, & Smith, 2006; World Health Organization, 1998) unlike in health centres where the main objective is to increase the number of people seeking treatment through outreach programmes to communities (World Health Organization, 1998). Previous studies conducted in hospitals in Africa also assumed input-oriented model (Kinyanjui et al., 2015; Kirigia et al., 2002; Maredza, 2012; Masiye, 2007; Nannyaonjo & Okot, 2013; Sede & Ohemeng, 2013; Zere et al., 2001; 2006). Sensitivity analysis was conducted to explore how sensitive the results were when input and output-oriented models were used (section 7.2.2).

4.3 Results

This section outlines the results from DEA for both input and output oriented models under the CRS and VRS assumptions. Efficiency measurement using DEA was analysed with LIMDEP version 10 (Greene, 1995), Stata statistical software version 11.2 (StataCorp, 2011) and R: A language for statistical computing (R Development Core Team, 2008).
4.3.1 Descriptive statistics

A summary of the descriptive statistics from the sampled 27 hospitals is indicated in Table 4.1. The mean values of the variables did not vary significantly between the two cross sections (2011 and 2012).

Table 4.1: Descriptive statistics of inputs and outputs in the cross sectional data

<table>
<thead>
<tr>
<th>Variable</th>
<th>Year</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Outpatients</td>
<td>2011</td>
<td>66413</td>
<td>39233.91</td>
<td>11505</td>
<td>145035</td>
</tr>
<tr>
<td></td>
<td>2012</td>
<td>61853</td>
<td>35157.11</td>
<td>12044</td>
<td>137993</td>
</tr>
<tr>
<td>Total admissions</td>
<td>2011</td>
<td>5256</td>
<td>3889.83</td>
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</tr>
<tr>
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<td>743</td>
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<tr>
<td>Doctors</td>
<td>2011</td>
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</tr>
<tr>
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<td>2012</td>
<td>9</td>
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</tr>
<tr>
<td>Nurses</td>
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<tr>
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<td>58</td>
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</tr>
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<td>2012</td>
<td>9</td>
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</tr>
<tr>
<td>Other IIWs</td>
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<td>10.74538</td>
<td>4</td>
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</tr>
<tr>
<td></td>
<td>2012</td>
<td>23</td>
<td>11.15083</td>
<td>4</td>
<td>44</td>
</tr>
<tr>
<td>Expenditure</td>
<td>2011</td>
<td>18662140</td>
<td>13638960</td>
<td>1719564</td>
<td>47586296</td>
</tr>
<tr>
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<td>2012</td>
<td>21293357</td>
<td>16599583</td>
<td>1574634</td>
<td>53578044</td>
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<tr>
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<td>2011</td>
<td>113</td>
<td>72.49156</td>
<td>11</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>2012</td>
<td>113</td>
<td>70.33603</td>
<td>10</td>
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</tr>
</tbody>
</table>

The correlation between the output and input variables are shown in Table 4.2 and Table 4.3. Generally there was strong correlation between all the variables except for other staff variable that had weak correlation with the other variables and was not statistically significant for both data sets.
Table 4.2: Correlation between output and input variables using 2011 data set

<table>
<thead>
<tr>
<th></th>
<th>OPD</th>
<th>ADM</th>
<th>Doctors</th>
<th>Nurses</th>
<th>COs</th>
<th>HWs</th>
<th>Other staff</th>
<th>Exp</th>
<th>Beds</th>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Admissions</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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</tr>
<tr>
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<td>0.9021</td>
<td>0.7922</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COs</td>
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<td>0.8202</td>
<td>0.7806</td>
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<td>1</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Other HWs</td>
<td>0.5881</td>
<td>0.7323</td>
<td>0.6205</td>
<td>0.8506</td>
<td>0.7343</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other staff*</td>
<td>0.0917</td>
<td>0.3722</td>
<td>0.1519</td>
<td>0.3520</td>
<td>0.3520</td>
<td>0.3197</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Expenditure</td>
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<td>0.7241</td>
<td>0.6997</td>
<td>0.6970</td>
<td>0.7197</td>
<td>0.6143</td>
<td>0.5066</td>
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<td></td>
</tr>
<tr>
<td>Beds</td>
<td>0.5700</td>
<td>0.8382</td>
<td>0.7218</td>
<td>0.8380</td>
<td>0.7728</td>
<td>0.7368</td>
<td>0.5452</td>
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</tr>
</tbody>
</table>

*Only the correlation between other staff variable and the other variables not statistically significant

Table 4.3: Correlation between output and input variables using 2012 data set

<table>
<thead>
<tr>
<th></th>
<th>OPD</th>
<th>ADM</th>
<th>Doctors</th>
<th>Nurses</th>
<th>COs</th>
<th>HWs</th>
<th>Other staff</th>
<th>Exp</th>
<th>Beds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outpatients</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Admissions</td>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doctors</td>
<td>0.6604</td>
<td>0.7225</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nurses</td>
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<td>0.77</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COs</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other HWs</td>
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<td>0.6734</td>
<td>0.8337</td>
<td>0.6933</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other staff*</td>
<td>0.0130</td>
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<td>0.0976</td>
<td>0.2670</td>
<td>0.2593</td>
<td>0.1637</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Expenditure</td>
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<td>0.6763</td>
<td>0.5272</td>
<td>0.5629</td>
<td>0.5382</td>
<td>0.5359</td>
<td>0.4180</td>
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</tr>
<tr>
<td>Beds</td>
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<td>0.8222</td>
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<td>0.8391</td>
<td>0.7657</td>
<td>0.7007</td>
<td>0.4490</td>
<td>0.6354</td>
<td>1</td>
</tr>
</tbody>
</table>

*Only the correlation between other staff variable and the other variables not statistically significant

4.3.2 Choice of output in the DEA approach

Data envelopment analysis handles multiple outputs and multiple inputs with ease as compared to SFA. In this study, incorporating both outpatients and admissions as outputs in the DEA model is possible. However, since the results are compared with the SFA model, the output index might be ideal for comparison purposes.

If a variable returns to scale (VRS) is assumed in an input-oriented model, the efficiency scores varied depending on the output variable in the model. Table 4.4 shows the summary
of the efficiency scores with single output (outpatients, admissions or a derived output index) and multiple output approach. The data set used in this table was from 2011 dataset assuming that there are no significant differences between the two data sets. Table 4.4 is for illustrative purposes on the use of different types of outputs in this study.

Outpatients as single output in the model gives lower efficiency scores compared to admissions as single output. The latter has more hospitals on the frontier (16 of the 27 hospitals). The multiple outputs approach (incorporating both outpatients and admissions as outputs in the DEA model) indicates higher efficiency levels and that more hospitals were considered 'fully' efficient compared to the single output models.

Table 4.4: Efficiency scores using DEA Input-Oriented with VRS assumption using 2011 data

<table>
<thead>
<tr>
<th>Efficiency Range</th>
<th>Outpatients as output n (%)</th>
<th>Admissions as output n (%)</th>
<th>Multiple outputs DEA n (%)</th>
<th>Output index n (%)§</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2&lt;= E&lt;0.3</td>
<td>1 (3.7)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.3&lt;= E&lt;0.4</td>
<td>2 (7.4)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>0.4&lt;= E&lt;0.5</td>
<td>4 (14.8)</td>
<td>-</td>
<td>-</td>
<td>1 (3.7)</td>
</tr>
<tr>
<td>0.5&lt;= E&lt;0.6</td>
<td>3 (11.1)</td>
<td>1 (3.7)</td>
<td>1 (3.7)</td>
<td>3 (11.1)</td>
</tr>
<tr>
<td>0.6&lt;= E&lt;0.7</td>
<td>3 (11.1)</td>
<td>2 (7.4)</td>
<td>1 (3.7)</td>
<td>2 (7.4)</td>
</tr>
<tr>
<td>0.7&lt;= E&lt;0.8</td>
<td>4 (14.8)</td>
<td>4 (14.8)</td>
<td>4 (14.8)</td>
<td>2 (7.4)</td>
</tr>
<tr>
<td>0.8&lt;= E&lt;0.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>3 (11.1)</td>
</tr>
<tr>
<td>0.9&lt;= E&lt;1</td>
<td>-</td>
<td>4 (14.8)</td>
<td>1 (3.7)</td>
<td>2 (7.4)</td>
</tr>
<tr>
<td>E == 1</td>
<td>10 (37.0)</td>
<td>16 (59.3)</td>
<td>20 (74.1)</td>
<td>14 (51.9)</td>
</tr>
<tr>
<td>Mean (SD)</td>
<td>0.7088 (0.2580)</td>
<td>0.9140 (0.1474)</td>
<td>0.9323 (0.1318)</td>
<td>0.8641 (0.1794)</td>
</tr>
</tbody>
</table>

*DEA incorporating multiple outputs (outpatients and inpatients)

§DEA using a single output (index of outpatients and admissions) derived from Fishers method

The output index (single output derived from outpatients and admissions using Fishers method as described in Chapter 3) balances between the single output results and the multiple output approach. Carrying out a Student’s t-test to compare the mean of multiple output approach and single output index approach indicate that there are no significant
differences between the two methods (p=0.1365). Therefore, for this chapter the output index of outpatient and admissions is used as the output of the DEA model.

### 4.3.3 Input-oriented efficiency scores

In the 2011 data, the average efficiency score for the input-oriented model is 0.7771 with a standard deviation (SD) of 0.2124 under the CRS assumption and 0.8641 with SD of 0.1794 under the VRS assumption. A total of 14 of the 27 hospitals lie on the frontier under the VRS assumption and only 6 under the CRS assumption. The average scale efficiency was 0.8983 with a standard deviation of 0.1573. There were also only 6 hospitals that were scale efficient. Hospitals 2, 5, 6, 9, 11 and 24 were lie on the frontier under both CRS and VRS assumption and they also had a scale efficiency score of 1.0.

Using the 2012 cross section data, the average efficiency score was 0.7624 in the CRS and 0.8721 in the VRS. Twelve hospitals in the 2012 compared to 14 hospitals in the 2011 data were on the frontier under VRS assumption and only 5 hospitals in the CRS assumption. The average scale efficiency was 0.8730 with SD of 0.1534.

Table 4.5 and Figure 4.1 summarize the efficiency scores under various assumptions in the input-oriented model using both datasets.
<table>
<thead>
<tr>
<th></th>
<th>CRS_TE</th>
<th></th>
<th>VRS_TE</th>
<th></th>
<th>SCALE</th>
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<td>0.9651</td>
</tr>
<tr>
<td>2</td>
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<td>1.0000</td>
<td>1.0000</td>
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<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>3</td>
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<td>1.0000</td>
<td>1.0000</td>
<td>0.6320</td>
<td>0.8043</td>
</tr>
<tr>
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<td>0.2168</td>
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<td>0.7059</td>
<td>0.3040</td>
<td>0.3071</td>
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<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>6</td>
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<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
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<td>0.7224</td>
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<tr>
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<td>1.0000</td>
<td>0.9314</td>
</tr>
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<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
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</tr>
<tr>
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<td>0.8438</td>
<td>0.6682</td>
<td>0.9579</td>
<td>0.9990</td>
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</tr>
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<td>1.0000</td>
</tr>
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</table>

E = 165141265

Mean (SD) Mean (SD) Mean (SD) Mean (SD) Mean (SD) Mean (SD)
76
The variable returns to scale (VRS) is the preferred model specification as it is more flexible than CRS. VRS assumes that there are economies and diseconomies of scale and that not all hospitals operate at optimal scale. Therefore, it measures both pure technical efficiency and scale efficiency. The summary of results for the output-oriented model is outlined in Appendix L and shows that there were no significant differences between the input and output oriented models.
Table 4.6: Ranking for individual hospitals under the VRS assumption

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Input Oriented</th>
</tr>
</thead>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
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</tr>
<tr>
<td>3</td>
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<td>4</td>
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</tr>
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<td>25 24</td>
</tr>
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<tr>
<td>21</td>
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</tr>
<tr>
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</tr>
<tr>
<td>23</td>
<td>19 1</td>
</tr>
<tr>
<td>24</td>
<td>1 15</td>
</tr>
<tr>
<td>25</td>
<td>1 1</td>
</tr>
<tr>
<td>26</td>
<td>1 1</td>
</tr>
<tr>
<td>27</td>
<td>23 1</td>
</tr>
</tbody>
</table>

Table 4.6 shows the hospital individual ranks under the VRS assumption. Most of the hospitals have ranks within a similar for both 2011 and 2012 datasets. However, some hospitals ranked differently depending on the data set. Table 4.6 shows that hospital 7 was laying on the frontier with the 2011 dataset but ranked 17 in the 2012 dataset. Other hospitals that dropped in the ranking in the 2012 dataset compared to the 2011 one, were hospitals 12, 14 and 24. The hospitals that exhibit the reverse results i.e. were on the frontier in the 2012 dataset and not in the 2011 one were hospitals 23 and 27.
4.3.4 DEA with Bootstrapping

DEA with bootstrapping can be used when dealing with relatively small sample sizes (Simar & Wilson, 1998; 2000). The true efficiency frontier is unknown and the construction of the frontier is based on best-observed practice. The measures of efficiency in DEA are sensitive to sampling variation. Bootstrapping is a technique that is used to measure the variation in sampling of the obtained frontier (Moran & Jacobs, 2013). This procedure allows correction of sample bias and statistical inference methods can be used in generating confidence intervals. The steps for bootstrapping the DEA scores are shown in Figure 4.2.

The variable returns to scale (VRS) assumption was used and input-oriented model employed for the 2011 and 2012 data set to develop efficiency measures with bootstrapping. Figure 4.3 shows the bootstrapped DEA scores with confidence intervals for the 27 hospitals. The graph has been ordered from hospitals with higher corrected efficiency scores to the lowest. The hospitals with higher efficiency levels had wider confidence intervals compared to the hospitals with lower efficiency levels. The hospitals ranked at the bottom tend to have tighter confidence intervals. These results are consistent with previous studies that corrected the DEA efficiency scores using bootstrapping technique (Moran & Jacobs, 2013; H.-O. Nguyen, Nguyen, Chang, Chin, & Tongzon, 2016).
Step 1
From the original data set of \( n \) DMUs, compute the DEA efficiency scores \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_n] \).

Step 2
From the original efficiency scores \( \hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_n] \), generate a random sample \( \hat{\theta}_n^* = [\hat{\theta}_1^*, \hat{\theta}_2^*, ..., \hat{\theta}_n^*] \).

Step 3
Compute \( X_h^* = [(\hat{\theta}_1 / \hat{\theta}_h) x_1, (\hat{\theta}_2 / \hat{\theta}_h) x_2, ..., (\hat{\theta}_n / \hat{\theta}_h) x_n] \) where \( x_1, x_2, ..., x_n \) are the columns of the input matrix \( X \) shown in (15).

Step 4
For each \( X_h^* \) and \( Y \), re-compute the DEA scores \( \hat{\theta}_h^* = [\hat{\theta}_1^*, \hat{\theta}_2^*, ..., \hat{\theta}_n^*] \).

Step 5
Repeat Steps 2-4 \( B \) times and then calculate the bias-corrected efficiency scores and confidence intervals using equations (22)-(24).

Figure 4.2: Bootstrapped DEA scores: Adapted from Simar and Wilson (1998)
Corrected efficiency scores with Cls - 2011 data

Corrected efficiency scores with Cls - 2012 data

Figure 4.3: Efficiency scores with confidence intervals corrected for bias – 2011 and 2012 data

4.3.5 Ownership as a determinant of efficiency

In the input-oriented model under the VRS assumption out of the 27 hospitals, 14 lie on the frontier in the 2011 data with the uncorrected estimates. Out of these 14 hospitals, 4 of the hospitals were faith-based hospitals and the remaining 10 were public hospitals. In the 2012 data, 4 of 12 hospitals that lie on the frontier were faith-based hospitals. Table 4.7 shows the average efficiency scores for the two types of hospitals. There were no significant differences between the two types of hospitals using different models. This might be driven by the small sample size in both types of hospitals. If a larger sample size was used, the results might be different.
Table 4.7: Mean (SD) efficiency scores by ownership

<table>
<thead>
<tr>
<th>DMU</th>
<th>Input Oriented</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2011</td>
<td>2012</td>
<td></td>
</tr>
<tr>
<td>Public hospitals</td>
<td>0.8552</td>
<td>0.8529</td>
<td></td>
</tr>
<tr>
<td>(n=20)</td>
<td>(0.1916)</td>
<td>(0.2082)</td>
<td></td>
</tr>
<tr>
<td>Faith-based hospitals</td>
<td>0.8895</td>
<td>0.9269</td>
<td></td>
</tr>
<tr>
<td>(n=7)</td>
<td>(0.1488)</td>
<td>(0.1167)</td>
<td></td>
</tr>
<tr>
<td>P-Value</td>
<td>0.6723</td>
<td>0.3840</td>
<td></td>
</tr>
</tbody>
</table>

When the ownership variable was incorporated in the DEA bootstrapped model as an uncontrolled input, more hospitals had wider confidence intervals and higher efficiency scores (Figure 4.4).

![Ownership included as input - 2011 data](image1)

![Ownership included as input - 2012 data](image2)

Figure 4.4: Corrected efficiency scores with ownership included as an input

The results of the corrected efficiency scores and the uncorrected varied indicating that there could be bias. Summary of the statistics by ownership of the different models is shown in Table 1.
Table 4.8: Mean efficiency scores for the 2011 data set by ownership (corrected and uncorrected for bias)

<table>
<thead>
<tr>
<th>Ownership</th>
<th>Mean</th>
<th>SD</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without ownership as an input</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncorrected</td>
<td>Public</td>
<td>0.8552</td>
<td>0.1916</td>
</tr>
<tr>
<td></td>
<td>Faith-based</td>
<td>0.8895</td>
<td>0.1488</td>
</tr>
<tr>
<td>Corrected</td>
<td>Public</td>
<td>0.7527</td>
<td>0.1556</td>
</tr>
<tr>
<td></td>
<td>Faith-based</td>
<td>0.7788</td>
<td>0.1075</td>
</tr>
<tr>
<td>Ownership as an input</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uncorrected</td>
<td>Public</td>
<td>0.9379</td>
<td>0.0807</td>
</tr>
<tr>
<td></td>
<td>Faith-based</td>
<td>1.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Corrected</td>
<td>Public</td>
<td>0.8958</td>
<td>0.0678</td>
</tr>
<tr>
<td></td>
<td>Faith-based</td>
<td>0.9423</td>
<td>0.0037</td>
</tr>
</tbody>
</table>

The other methods of dealing with environmental variables such as ownership factor, a two-stage approach can be used where DEA efficiency scores obtained in the first stage were used as dependent variable again the ownership variable in the second stage using tobit regression model (details of the model described in Appendix H). In DEA, efficiency estimates are serially correlated and the error term in second stage is correlated with the regressors. Simar and Wilson method (Simar & Wilson, 2007) was used in running a truncated regression of the corrected efficiency estimates. Table 4.9 shows the results from the truncated regression model. This means that being a public hospital had a negative impact on efficiency. However, the ownership factor is not statistically significant indicating that it might not have an influence on efficiency in the sample hospitals in this study.
Table 4.9: Truncated regression model by ownership

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Err.</th>
<th>P-Value</th>
<th>[95% Confidence Interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public hospital</td>
<td>-0.0414</td>
<td>0.0923</td>
<td>0.6540</td>
<td>-0.2223 0.1395</td>
</tr>
<tr>
<td>Constant</td>
<td>0.8691</td>
<td>0.1728</td>
<td>0.0000</td>
<td>0.5304 1.2078</td>
</tr>
<tr>
<td>2012</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public hospital</td>
<td>-0.1242</td>
<td>0.1605</td>
<td>0.439</td>
<td>-0.4388 0.1904</td>
</tr>
<tr>
<td>Constant</td>
<td>1.1092</td>
<td>0.3402</td>
<td>0.001</td>
<td>0.4424 1.7759</td>
</tr>
</tbody>
</table>

4.4 Conclusion

Applying data envelopment analysis on data from Kenyan hospitals shows that there are some inefficient hospitals. Efficiency measurement is based on relative efficiency with a high efficiency score indicating that resources are well managed relative to the other hospitals in the sample data. This study used two cross sectional datasets and the results showed no significant differences between the two datasets. On average the efficiency scores for 2011 and 2012 dataset under VRS assumptions were 0.8641 and 0.8721 respectively in the input-oriented model. When DEA bootstrapped model was used the mean efficiency scores used was 0.7597 and 0.7751 for 2011 and 2012 respectively.

There were wide confidence intervals in the DEA bootstrapped model for the hospitals that had higher efficiency scores. The hospitals with lower efficiency scores had tighter confidence intervals.

The ownership factor does not significantly influence efficiency in the sample data from Kenyan hospitals. There were no significant differences in efficiency between the two types of hospitals regardless of model assumptions.
5 Application of Stochastic Frontier Model on Kenyan Hospital Data

5.1 Introduction

This chapter gives an overview of the stochastic frontier equation that is used to estimate technical efficiency in Kenyan hospitals. Computing the efficiency measures involves estimating the unknown production frontier. Production units also referred to as decision-making units (DMUs) such as firms, hospitals, regions etc. are assumed to reach the frontier when they produce the maximum possible output for a given set of inputs. With improved technology, better economy and better resources, the DMUs can potentially become less inefficient and therefore reaching the frontier. The frontier can possibly shift indicating technical progress or the DMUs can move along the frontier by changing the quantity of inputs. The frontier estimation however falls short of this because only the frontier of the observed DMUs is estimated.

The methodological framework is also highlighted with clear description of the inputs and outputs used in the stochastic frontier model. As discussed in Chapter 3, inputs and outputs are similar in the sampled hospitals but vary in terms of proportions due to the different types and structure of hospitals although all the hospitals are at the district level. Measuring efficiency is important in exploring the performance of the different hospitals. If the efficiency is not consistent in the selected hospitals, then there is indication that resources are either over or under utilized.

The chapter also explores various methodological approaches when using stochastic frontier analysis for a panel dataset in order to determine which best fits the hospital data. Cobb-Douglas and Translog production functions are examined and the form that fits the
data is selected. Varying distributions of the one-sided error term are also be reviewed and discussed in this chapter.

5.2 Methodological framework

Cross-sectional stochastic frontier analysis assumes that the technical inefficiency is independent of the inputs and assumptions of the error terms distributions. Using panel data can solve these limitations. Panel data allow relaxation of the assumption of independence and avoidance of distribution assumptions. Also with panel data set, estimates of efficiency levels of each hospital and observation can be obtained.

The first step in defining the SF framework is to identify the inputs and outputs. This thesis selected multiple-outputs multiple-inputs structure. One of the main advantages of DEA over SFA is that its non-parametric nature makes it easy in handling multiple outputs and multiple inputs. Stochastic frontier analysis on the other hand is restricted, as it cannot directly use multiple outputs in a production function. In health care, various types outputs have been selected in various studies but the most considered is the measure of services offered. In a hospital setting, there are several services offered indicating that when estimating efficiency, multiple outputs multiple inputs specifications would be preferable.

There are several methods that have been developed to incorporate multiple outputs in a SF production function. Distance functions can be used to estimate a multi-outputs production function in cases where price data is not available. Input distance function estimates minimal proportional contraction of the input vector given outputs and output distance functions estimates maximal proportional expansion of output vector given an input vector. Input distance function is commonly used especially where the DMUs have
more control over inputs and outputs. The main disadvantage of distance functions is that the explanatory variables might be correlated with the composite error term. One output (or input in an input distance function) is chosen and plays an asymmetric role i.e. becomes the denominator for all other outputs (or inputs). There is no particular test for choosing the one output (or input) and also the results might be biased since the explanatory variables are not independent of the output/input selected as the dependent variable.

When data on the cost of services are available, an output index derived from the quantity and cost of services can be used. Aggregating the outputs into one output using index number methods can be used in frontier estimation. Index number can be used for comparison over time, space or both and also measure changes over time and across DMUs.

A multiple-output multiple-input framework was implemented. The output considered in this study was an output index consisting of the number of outpatients and number of admission (refer to Chapter 3). The inputs included in this study were non-staff recurrent expenditure, number of beds and staffing levels (number of doctors, nurses, clinical officers and other health workers).

The stochastic frontier model is represented by the equation:

\[ y = \beta'x + v - u \]  
(5.1)

Production or cost model is normally based on Cobb-Douglas, Translog or other form logarithm model:
\[
\log y = \beta' x + \nu - u
\]  

(5.2)

The Cobb-Douglas stochastic frontier production function of a panel data set is of the form:

\[
\ln y_{it} = \alpha + \beta' \ln x_{it} + v_{it} - u_{it}
\]  

(5.3)

The Translog equation of a stochastic frontier model is represented by:

\[
\ln y_{it} = \alpha + \beta' \ln x_{it} + \beta'' (\ln x_{it})^2 + \beta'' (\ln x_{it} \ln x_{it}) + v_{it} - u_{it}
\]  

(5.4)

where \( i = 1, \ldots, N; \ t = 1, \ldots, T \)

Translog production function is however often complicated by multicollinearity due to the possible correlation between the explanatory variables (squares and cross productions of inputs). One of the ways to deal with multicollinearity is to present it in one of its reduced forms. Dropping some of the variables either by maintaining the squares or the cross products was applied. It is however not advisable to drop variables especially if they are important. A likelihood test can be used to compare the full model and the reduced forms. In this study, whether to maintain the squares or the cross products, the full model seem to be a better fit. Therefore, dropping the variable might not be an ideal method in this case.

The other alternative is to check the variance inflation factors (VIF). VIF gives an index measure of how much the variance of a coefficient increases if the predictors (regressors) are correlated (multicollinear). When VIF is greater than 10, then the coefficients are poorly estimated. So when the VIF is greater than 10, the variables are either eliminated or combined to form one variable. This method has been however faulted indicating that the
threshold needs to be re-evaluated because with higher VIFs as high as 40 do not by themselves discounts the results of the regression analysis (O'brien, 2007).

The third method would be to center the independent variables on the mean before computing the squares and the cross products. Centering increases interpretability of the coefficients but produces negative values on the independent variables it is not ideal when estimating efficiency using a SF model.

Although multicollinearity might lead to unreliable and unstable estimates, there are cases in which it can be safely ignored. Kennedy and Greene suggest two options; do nothing or incorporate additional information in the analysis (Greene, 2003; Kennedy, 2003). Since the standard errors are not elevated and the overall regression is not affected, this study did not deal with multicollinearity.

5.3 Results

This section outlines the results from stochastic frontier analysis (SFA) using time-invariant and time-varying models. Efficiency measurement using SFA model was analysed using LIMDEP version 10 (Greene, 1995), Stata statistical software version 11.2 (StataCorp, 2011) and R: A language for statistical computing (R Development Core Team, 2008).

5.3.1 Descriptive statistics

There were 27 hospitals in the sampled panel data set. The data set was unbalanced with a total of 432 observations. Since the data was on quarterly format for the period of 5 years (2008-2012), some hospitals had data for all the quarters (N=20) and some were much less
due to lack of data. The data were unbalanced due to missing data at different levels. For example some hospitals had missing data on some of the quarters while some had data missing within some of the observations. There were some hospitals that had missing data for particular years such as 2008 and 2009 therefore data were only available for 3 years, which means that only 12 observations available for that hospital.

Table 5.1 gives the descriptive statistics of the input and output variables that were used in the SF model.

Table 5.1: Descriptive Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total outpatients</td>
<td>16736</td>
<td>8940.507</td>
<td>2343</td>
<td>39640</td>
</tr>
<tr>
<td>Total admissions</td>
<td>1369</td>
<td>956.98</td>
<td>124</td>
<td>4149</td>
</tr>
<tr>
<td>Doctors</td>
<td>9</td>
<td>7.134653</td>
<td>1</td>
<td>28</td>
</tr>
<tr>
<td>Nurses</td>
<td>62</td>
<td>40.45357</td>
<td>12</td>
<td>161</td>
</tr>
<tr>
<td>Clinical officers</td>
<td>10</td>
<td>5.073105</td>
<td>2</td>
<td>23</td>
</tr>
<tr>
<td>Other health workers (HWs)</td>
<td>23</td>
<td>10.97326</td>
<td>4</td>
<td>48</td>
</tr>
<tr>
<td>Other Staff</td>
<td>33</td>
<td>18.55637</td>
<td>2</td>
<td>87</td>
</tr>
<tr>
<td>Number of beds</td>
<td>119</td>
<td>73.1175</td>
<td>10</td>
<td>299</td>
</tr>
<tr>
<td>Expenditure</td>
<td>4231091</td>
<td>3449871</td>
<td>141730.5</td>
<td>15384979</td>
</tr>
</tbody>
</table>

There was significant correlation between the different outputs (outpatients and admissions) and the inputs with much stronger correlation between admissions and inputs as compared to outpatients (Table 5.2)
Table 5.2: Correlation between input and output variables

<table>
<thead>
<tr>
<th></th>
<th>OPD</th>
<th>ADM</th>
<th>Doctors</th>
<th>Nurses</th>
<th>COs</th>
<th>HWs</th>
<th>Other staff</th>
<th>Exp</th>
<th>Beds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outpatients</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Admissions</td>
<td>0.7237</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doctors</td>
<td>0.6789</td>
<td>0.7686</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nurses</td>
<td>0.7422</td>
<td>0.9021</td>
<td>0.7922</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COs</td>
<td>0.7789</td>
<td>0.8202</td>
<td>0.7806</td>
<td>0.8755</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other HWs</td>
<td>0.5881</td>
<td>0.7323</td>
<td>0.6205</td>
<td>0.8506</td>
<td>0.7343</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other staff</td>
<td>0.1831</td>
<td>0.3327</td>
<td>0.169</td>
<td>0.3822</td>
<td>0.4004</td>
<td>0.2646</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenditure</td>
<td>0.5444</td>
<td>0.7241</td>
<td>0.6997</td>
<td>0.6970</td>
<td>0.7197</td>
<td>0.6143</td>
<td>0.4808</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beds</td>
<td>0.5700</td>
<td>0.8382</td>
<td>0.7218</td>
<td>0.8380</td>
<td>0.7728</td>
<td>0.7368</td>
<td>0.7008</td>
<td>0.7641</td>
<td>1</td>
</tr>
</tbody>
</table>

OPD: Outpatients, ADM: Admissions, COs: Clinical officers, HWs: health workers

The staffing levels inputs shown in Figure 5.1 indicate a decline in the number of staff in the different cadres over time. The decline however is marginal i.e. by 4 doctors, COs or other health workers by end of 2012 but the decline trend was significant. The average drop in the number of nurses from beginning of 2008 and end of 2012 is by 20 and significant (p=0.001). The number of beds also declined over time (Figure 5.2) but this was not significant (p=0.115). Expenditure on the other hand significantly increased over time (Figure 5.2). This might be due to the substantial increase in total government spending on health and inflation. The other variable included as a determinant of inefficiency is the ownership type.
The outputs on the other hand varied over time and had no clear trend. Figure 5.3 shows an oscillating pattern with high peaks at the third quarter of each year. The peak coincides
with the beginning of the financial year (July), which might be driving the pattern seen. There is a sharp decline in the last quarter 2012 and this was due to a major doctors’ and nurses’ strike in the months of November and December, which impacted negatively on services provision in all public hospitals.

![Graph showing quarterly hospital workload over a period of 2008-2012](image)

Figure 5.3: Output variables (outpatients and admissions)

### 5.3.2 Skewness of the OLS residuals and sign of parameter estimates

The SF model can be estimated using maximum likelihood. The first step is estimated by ordinary least squares (OLS). There is need to check the skewness of the OLS residuals before proceeding with maximum likelihood estimation (MLE). This is because if the OLS residuals are skewed positively, the maximum likelihood estimator is basically OLS for the slopes and $\sigma_u^2$ and zero for $\sigma_u^2$. Running a stochastic frontier analysis on the thesis data using either outpatients or admissions as the single output variable, the OLS residuals are right skewed. However, with the output index as a derivation of multiple outputs, the results were stable without any issues on the skewness of the OLS residuals with the pooled data.
From Table 5.3, there is also an issue with coefficient sign. For the CD function, the significant negative coefficient on the other health workers could be due to systematic differences. Since the data were collected from public and faith-based hospitals, the definition of other staff consisted of different people depending on the hospital. The negative coefficient in the beds variable might indicate that it is not a binding factor in assessing efficiency. One of the possible explanations is that past decisions could influence the results seen in the models. For example, a decision was made to build and equip a particular hospital but the beds are not utilized as much currently. The other staff variable although has a negative coefficient is not significant.

The Translog function on the other hand, has more variables (specifically note the squares and cross products) with negative and significant coefficients. The standard errors were also much higher compared to the Cobb-Douglas function (Table 5.3). A likelihood ratio test showed that the Translog was a better fit than the Cobb-Douglas ($\chi$ with 28 degrees of freedom was 385.39 with p-value <0.0001). The full Translog form was also preferred compared to either a model with squares variables only or cross products.

Although, the full Translog form was preferred to Cobb Douglas by running the LR test, there were still major challenges using the Translog form with the panel data from Kenyan hospitals. Cobb-Douglas form on the other hand was stable with varying distribution assumptions. As indicated in Chapter 2, one of the ways of solving the skewness problem is increasing the sample size. This is a challenge especially in setting where data is not easily accessible. The skewness problem in this thesis might be due to small sample size. In order to examine whether this issue can be solved using alternative approaches, the specification of the model was once again checked. Re-specifying the model by either
using a different functional form or alternative distribution assumption is another alternative. Running a Cobb-Douglas functional form using the panel data, the skewness issue was resolved. In this thesis, the Cobb-Douglas production function was used for this analysis and the discussion henceforth is based on this functional form.

In order to estimate efficiency, the Cobb-Douglas production function was assumed for this data set:

\[ \ln y_{it} = \alpha + \beta' \ln x_{it} + \nu'_{it} - u_{it} \]

\( i=1, \ldots, N; \quad t=1, \ldots, T \)

Output vector \((y)\) = output index of outpatients and admissions

Input vector \((x)\) = doctors, nurses, clinical officers, other health workers, expenditure and beds

\( \beta = \) Unknown parameter

\( \nu = \) Noise/measurement error

\( u = \) Inefficiency

OLS estimates are compared with White's robust estimates in order to check for assumption of homoscedasticity and the results are discussed in Appendix I. Normality of the least squares was also checked and results are summarized in Appendix J. Half normal, exponential, truncated and gamma distribution assumptions of the one-sided error term are explored in section 5.3.3 using data from Kenyan hospitals.
<table>
<thead>
<tr>
<th>Output Index</th>
<th>Estimate</th>
<th>SE</th>
<th>Estimate</th>
<th>SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-3.208***</td>
<td>0.289</td>
<td>6.653**</td>
<td>2.958</td>
</tr>
<tr>
<td>Doctors</td>
<td>0.098***</td>
<td>0.032</td>
<td>0.683</td>
<td>0.684</td>
</tr>
<tr>
<td>Nurses</td>
<td>0.687***</td>
<td>0.058</td>
<td>6.727***</td>
<td>0.935</td>
</tr>
<tr>
<td>C0s</td>
<td>0.284***</td>
<td>0.054</td>
<td>-2.290**</td>
<td>1.031</td>
</tr>
<tr>
<td>Other HWs</td>
<td>-0.142***</td>
<td>0.051</td>
<td>-2.733**</td>
<td>1.096</td>
</tr>
<tr>
<td>Other staff</td>
<td>-0.025</td>
<td>0.032</td>
<td>1.437**</td>
<td>0.696</td>
</tr>
<tr>
<td>Expenditure</td>
<td>0.070***</td>
<td>0.021</td>
<td>-1.494***</td>
<td>0.386</td>
</tr>
<tr>
<td>Beds</td>
<td>-0.146***</td>
<td>0.038</td>
<td>-3.011***</td>
<td>0.918</td>
</tr>
<tr>
<td>Doctors²</td>
<td>-0.316**</td>
<td></td>
<td>0.128</td>
<td></td>
</tr>
<tr>
<td>Nurses²</td>
<td>-1.293***</td>
<td></td>
<td>0.425</td>
<td></td>
</tr>
<tr>
<td>C0s²</td>
<td>3.006***</td>
<td></td>
<td>0.463</td>
<td></td>
</tr>
<tr>
<td>Other HWs²</td>
<td>-0.559</td>
<td></td>
<td>0.288</td>
<td></td>
</tr>
<tr>
<td>Other staff²</td>
<td>-0.509***</td>
<td></td>
<td>0.121</td>
<td></td>
</tr>
<tr>
<td>Expenditure²</td>
<td>0.092***</td>
<td></td>
<td>0.031</td>
<td></td>
</tr>
<tr>
<td>Beds²</td>
<td>0.163</td>
<td></td>
<td>0.211</td>
<td></td>
</tr>
<tr>
<td>Doctors*Nurses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doctors*C0s</td>
<td>-0.706***</td>
<td></td>
<td>0.165</td>
<td></td>
</tr>
<tr>
<td>Doctors*HWs</td>
<td>0.139</td>
<td></td>
<td>0.151</td>
<td></td>
</tr>
<tr>
<td>Doctors*Others</td>
<td></td>
<td></td>
<td>-0.089</td>
<td>0.080</td>
</tr>
<tr>
<td>Doctors*Expenditure</td>
<td></td>
<td></td>
<td>-0.151***</td>
<td>0.044</td>
</tr>
<tr>
<td>Doctors*Beds</td>
<td>-0.007</td>
<td></td>
<td>0.092</td>
<td></td>
</tr>
<tr>
<td>Nurses*C0s</td>
<td>-1.588***</td>
<td></td>
<td>0.323</td>
<td></td>
</tr>
<tr>
<td>Nurses*HWs</td>
<td>0.408</td>
<td></td>
<td>0.277</td>
<td></td>
</tr>
<tr>
<td>Nurses*Others</td>
<td>0.393***</td>
<td></td>
<td>0.147</td>
<td></td>
</tr>
<tr>
<td>Nurses*Expenditure</td>
<td></td>
<td></td>
<td>-0.156**</td>
<td>0.070</td>
</tr>
<tr>
<td>Nurses*Beds</td>
<td>0.139</td>
<td></td>
<td>0.247</td>
<td></td>
</tr>
<tr>
<td>C0s*HWs</td>
<td>0.036</td>
<td></td>
<td>0.238</td>
<td></td>
</tr>
<tr>
<td>C0s*Others</td>
<td>-0.599***</td>
<td></td>
<td>0.147</td>
<td></td>
</tr>
<tr>
<td>C0s*Expenditure</td>
<td></td>
<td></td>
<td>0.228***</td>
<td>0.079</td>
</tr>
<tr>
<td>C0s*Beds</td>
<td>0.438**</td>
<td></td>
<td>0.212</td>
<td></td>
</tr>
<tr>
<td>HWs*Others</td>
<td>0.397***</td>
<td></td>
<td>0.113</td>
<td></td>
</tr>
<tr>
<td>HWs*Expenditure</td>
<td></td>
<td></td>
<td>0.184**</td>
<td>0.073</td>
</tr>
<tr>
<td>HWs*Beds</td>
<td>-0.423**</td>
<td></td>
<td>0.168</td>
<td></td>
</tr>
<tr>
<td>Others*Expenditure</td>
<td></td>
<td></td>
<td>-0.109**</td>
<td>0.050</td>
</tr>
<tr>
<td>Others*Beds</td>
<td>0.138</td>
<td></td>
<td>0.166</td>
<td></td>
</tr>
<tr>
<td>Expenditure*Beds</td>
<td></td>
<td></td>
<td>0.099</td>
<td>0.067</td>
</tr>
</tbody>
</table>

### Noise and Inefficiency distributions

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>2.29136***</td>
<td>0.0567</td>
<td>2.11029***</td>
<td>0.26080</td>
</tr>
<tr>
<td>σ</td>
<td>0.47621***</td>
<td>0.0009</td>
<td>0.29873***</td>
<td>0.00058</td>
</tr>
<tr>
<td>σ_u</td>
<td>0.43646</td>
<td></td>
<td>0.26995</td>
<td></td>
</tr>
<tr>
<td>σ_v</td>
<td>0.19048</td>
<td></td>
<td>0.12792</td>
<td></td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-116.0993</td>
<td></td>
<td>76.59633</td>
<td></td>
</tr>
</tbody>
</table>

***, **, * => Significance at 1%, 5% and 10% level

§Dependent variable (output index) is the derived Fisher’s index from cost and quantity of outpatients and admissions.

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5.3.3 Varying distribution of the one-sided error term

The first step in the SF model is to obtain the ordinary least squares (OLS) estimates then maximum likelihood is estimated for the Cobb-Douglas stochastic production frontier. There are four main distribution types that can be employed in SF model: half-normal, exponential-normal, truncated-normal and gamma distributions. Table 5.4 shows the maximum likelihood estimates of the pooled Cobb-Douglas production model.

Table 5.4: Pooled Stochastic Cobb-Douglas Production Model with Different Distribution Assumptions of the One-Sided Error Term

<table>
<thead>
<tr>
<th></th>
<th>Half-Normal</th>
<th>Exponential</th>
<th>Truncated</th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate (SE)</td>
<td>Estimate (SE)</td>
<td>Estimate (SE)</td>
<td>Estimate (SE)</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.20759*** (0.2891)</td>
<td>-3.11831*** (0.26839)</td>
<td>-3.11855*** (0.26842)</td>
<td>-3.13979*** (0.28046)</td>
</tr>
<tr>
<td>Doctors</td>
<td>0.09847*** (0.03197)</td>
<td>0.11593*** (0.03123)</td>
<td>0.11587*** (0.03124)</td>
<td>0.12329*** (0.02926)</td>
</tr>
<tr>
<td>Nurses</td>
<td>0.68686*** (0.05776)</td>
<td>0.61575*** (0.05789)</td>
<td>0.61591*** (0.05789)</td>
<td>0.59349*** (0.05701)</td>
</tr>
<tr>
<td>COs</td>
<td>0.28409*** (0.05437)</td>
<td>0.30458*** (0.05121)</td>
<td>0.30454*** (0.05122)</td>
<td>0.31147*** (0.06198)</td>
</tr>
<tr>
<td>Other HWs</td>
<td>-0.14183*** (0.05099)</td>
<td>-0.06605 (0.049563)</td>
<td>-0.06618 (0.04957)</td>
<td>-0.04483 (0.05449)</td>
</tr>
<tr>
<td>Other staff</td>
<td>-0.02527 (0.03248)</td>
<td>0.00311 (0.03129)</td>
<td>0.00305 (0.0313)</td>
<td>0.01278 (0.03049)</td>
</tr>
<tr>
<td>Expenditure</td>
<td>0.07007*** (0.02099)</td>
<td>0.06108*** (0.01997)</td>
<td>0.06110*** (0.01997)</td>
<td>0.05912*** (0.02006)</td>
</tr>
<tr>
<td>Beds</td>
<td>-0.14639*** (0.0377)</td>
<td>-0.18131*** (0.26839)</td>
<td>-0.18126*** (0.03627)</td>
<td>-0.19298*** (0.03649)</td>
</tr>
<tr>
<td>Noise and inefficiency distributions</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lambda )</td>
<td>2.29136*** (0.05675)</td>
<td>1.32192*** (0.02439)</td>
<td>49.7144*** (15.7071)</td>
<td>0.17519 (0.000)</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>0.43646*** (0.26388)</td>
<td>0.26638*** (0.03129)</td>
<td>10.1067 (0.0313)</td>
<td>0.11091 (0.03049)</td>
</tr>
<tr>
<td>( \sigma_v )</td>
<td>0.19048*** (0.02051)</td>
<td>0.20151*** (0.01997)</td>
<td>0.20149*** (0.01997)</td>
<td>0.633097 (0.02006)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>-116.0993 (0.0377)</td>
<td>-112.2850 (0.26839)</td>
<td>-112.2880 (0.03627)</td>
<td>-110.52715 (0.03649)</td>
</tr>
</tbody>
</table>

Efficiency scores

<table>
<thead>
<tr>
<th></th>
<th>Mean (SD)</th>
<th>Mean (SD)</th>
<th>Mean (SD)</th>
<th>Mean (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.7237 (0.1383)</td>
<td>0.7831 (0.1408)</td>
<td>0.7828 (0.1408)</td>
<td>0.8091 (0.1428)</td>
</tr>
<tr>
<td>Min-Max</td>
<td>0.351-0.932</td>
<td>0.311-0.940</td>
<td>0.312-0.940</td>
<td>0.309-0.953</td>
</tr>
</tbody>
</table>

***, **, * => Significance at 1%, 5% and 10% level

The estimates of the parameters do differ significantly in the varying distribution types. All the parameters were significant at 1% level for the four distribution types except for other...
health workers and other staff variables. Other health workers parameter was however significant in the half-normal model and had a negative coefficient in all distribution assumptions. The parameter for beds also had negative coefficient in all the distribution assumption.

The mean score under different distribution assumption ranged between 0.7237 and 0.8091 as shown in Table 5.4. The scores under the exponential and truncated distribution assumptions were not significantly different but half-normal assumption was significantly lower than the other three assumptions and gamma distribution had a significantly higher mean score compared to the other three distribution assumptions. However, there was strong and significant correlation between the scores in the different distribution assumptions (Table 5.5).

Table 5.5: Spearman correlation of efficiency scores derived from different distribution assumptions

<table>
<thead>
<tr>
<th></th>
<th>Half-Normal</th>
<th>Exponential</th>
<th>Truncated</th>
<th>Gamma</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-Normal</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exponential</td>
<td>0.9725</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Truncated</td>
<td>0.9729</td>
<td>0.9999</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Gamma</td>
<td>0.9557</td>
<td>0.9952</td>
<td>0.9951</td>
<td>1</td>
</tr>
</tbody>
</table>

There are no a priori reasons for choosing one distributional form over another. If theoretical implications are considered, half-normal and exponential distributions are avoided because they have a mode of zero. This implies that most inefficiency effects are near zero and efficiency effects are near one (Coelli et al., 2005). The truncated normal and gamma models allow different shapes of the distributions but are not flexible because they are complex to compute.
The distribution assumptions of the one-sided error term in this study were strongly correlated (Table 5.5) indicating the estimates are robust to distributional choice.

5.3.4 Hospital efficiency estimates using SFA panel data

The ordinary least square (OLS) estimates of the Cobb-Douglas production function were first obtained and then used to obtain the maximum likelihood estimates of the SF model. The maximum likelihood estimates for the parameters of the production functions with time-invariant and time-varying model parameters are presented in Table 5.6.

Overall, there are technical inefficiencies in hospital production using data from selected Kenyan hospitals. The lambda value \( \lambda \) can be used to derive the percentage of total error variance due to inefficiency \( (\lambda^2/\lambda^2 + 1) \). As derived from the \( \lambda \) in Table 5.6, the percentage of the total variation due to inefficiency is 93.8%, 95.5%, 95.7%, 94.3% and 94.2% in the Schmidt and Sickles, Pitt and Lee, Battese and Coelli (1992 model), true random effects and true fixed effects model respectively.
Table 5.6: Maximum Likelihood estimates for parameters of stochastic frontier production functions with time-invariant and time-varying models

<table>
<thead>
<tr>
<th>Variable</th>
<th>Time-Invariant Models</th>
<th>Time-Varying Models</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Schmidt and Sickles Fixed Effects</td>
<td>Pitt and Lee Random Effects</td>
</tr>
<tr>
<td>Constant</td>
<td>-1.99141*** (0.31485)</td>
<td>-2.01668*** (0.58955)</td>
</tr>
<tr>
<td>Doctors</td>
<td>-0.07065 (0.04986)</td>
<td>-0.00518 (0.04555)</td>
</tr>
<tr>
<td>Nurses</td>
<td>0.11026 (0.13289)</td>
<td>0.34468*** (0.09167)</td>
</tr>
<tr>
<td>COs</td>
<td>0.08310 (0.083573)</td>
<td>0.07567 (0.07526)</td>
</tr>
<tr>
<td>Other HWs</td>
<td>-0.06815 (0.10590)</td>
<td>0.04029 (0.08904)</td>
</tr>
<tr>
<td>Other staff</td>
<td>0.21227*** (0.07842)</td>
<td>0.17751 (0.13835)</td>
</tr>
<tr>
<td>Expenditure</td>
<td>0.03582** (0.01656)</td>
<td>0.03588 (0.01641)</td>
</tr>
<tr>
<td>Beds</td>
<td>-0.04923 (0.07209)</td>
<td>-0.08428 (0.05398)</td>
</tr>
</tbody>
</table>

Noise and Inefficiency distributions

| λ      | 3.87874*** (0.11554) | 4.63091*** (0.011554) | 4.73563*** (0.03888) | 4.05690*** (0.50229) | 4.02590*** (0.70556) |
| σ_u    | 0.61125 | 0.72942 | 0.74335 | 0.24102 | 0.23149 |
| σ_v    | 0.15759 | 0.15751 | 0.15697 | 0.05941 | 0.05750 |
| Log Likelihood | 124.7639 | 126.4179 | 148.1047 | 231.84503 |

***, **, * => Significance at 1%, 5% and 10% level

The time-invariant models (Pitt and Lee random effects and Schmidt and Sickles fixed effects) have two major drawbacks. They both assume that there is constant inefficiency over time and this is unrealistic in long panel data sets. Secondly, they also include any time-invariant hospital-specific unobserved heterogeneity and therefore tend to overestimate inefficiency. The Schmidt and Sickles FE model estimates a negative coefficient on the doctors, other health workers and beds. This indicates a negative
relationship of these inputs with the outputs but not significant. The Pitt and Lee model also indicates similar negative relationship between doctors and beds variables with the outputs but also not significant. In the time-invariant models, the higher parameter estimates of nurses and other staff indicate a stronger relationship with the outputs. However, this is only significant in the Pitt and Lee model.

Attempts to relax the invariance assumption about $u_i$ have been approached using the time varying models (Battese and Coelli, true fixed and random effects). Battese and Coelli model is most frequently used in empirical literature. The inefficiency component varies through time but the variation is systematic with respect to time. It also mixes firm effects and inefficiency. From Table 5.6, the estimates from Battese and Coelli model seem similar to the time-invariant Pitt and Lee model and also seen in the distribution Figure 5.4. This is an on-going question in literature on the extent to which this model actually moves away from the time-invariant Pitt and Lee model (Carroll, Newman, & Thorne, 2007). The beds variable also has a negative coefficient in the BC model and this is also seen in all the different models. It is only the nurses’ parameter estimate that is significant in the BC model and also a higher magnitude compared to the other inputs. This shows that nurses are the main drivers of hospital performance and play a key role in the hospital activity processes.

The true random effects (TRE) model compared to the time varying models, separates the time invariant unobserved heterogeneity from the time varying inefficiency. The time invariant effects are captured by the hospital-specific constant and any persistent inefficiency not included in the inefficiency term might lead to underestimation of inefficiency. True fixed effects (TFE) model is an extension of the TRE model and
overcomes the bias if any correlation between unobserved heterogeneity and explanatory variables exist.

The parameter estimates vary across the different models as shown in Table 5.6. In the Pitt and Lee model (time-invariant) and Battese and Coelli model (time-varying), only the parameter estimates for nurses were statistically significant. This was also the case with the TRE and TFE but more parameters were significant in latter models.

![Density distribution of different models](image)

Figure 5.4: Distribution of different models

5.3.5 Technical efficiency scores for hospitals in Kenya

The technical efficiency scores from Table 5.7 and parameter estimates from Table 5.6 differ across the different time-invariant and time varying models mainly due to the underlying assumptions. Pitt and Lee and Battese and Coelli models had similar scores (0.617 and 0.621 respectively) but still much lower than the true fixed and random effects models. The true random and fixed effect models have similar distributions (Table 5.7) and
efficiency scores of 0.835 and 0.849 respectively. The similar scores in the two models indicate that a bias resulting from any correlation between the hospital specific effects and explanatory variables does not influence the estimates. The difference seen in the mean estimates of the different models is due to the presence of unobserved heterogeneity in Pitt and Lee (PL) and Battese and Coelli (BC) models and its exclusion in the TRE and TFE models.

Table 5.7: Mean efficiency estimates for pooled, time-invariant and time-varying models

<table>
<thead>
<tr>
<th>Model</th>
<th>Mean</th>
<th>Std.Dev.</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooled model</td>
<td>0.722932</td>
<td>0.138430</td>
<td>0.350825</td>
<td>0.932486</td>
</tr>
<tr>
<td>Pitt and Lee</td>
<td>0.616568</td>
<td>0.210924</td>
<td>0.170971</td>
<td>0.965696</td>
</tr>
<tr>
<td>Battese and Coelli</td>
<td>0.621137</td>
<td>0.211211</td>
<td>0.160447</td>
<td>0.968236</td>
</tr>
<tr>
<td>True Random Effects</td>
<td>0.835271</td>
<td>0.103653</td>
<td>0.431372</td>
<td>0.978662</td>
</tr>
<tr>
<td>True Fixed Effects</td>
<td>0.849764</td>
<td>0.081017</td>
<td>0.715467</td>
<td>0.982200</td>
</tr>
</tbody>
</table>

The correlation between BC and PL was strong and significant \((r=0.9990\text{ with } p<0.0001)\) and similar strong correlation between TRE and TFE \((r=0.9313, p<0.0001)\). However, the correlation between the former and latter models differed and was not strong. Correlation between BC model and TRE was \(0.037\text{ with } p=0.4430\) and TFE was \(0.079\text{ with } p=0.099\). PL model on the other hand also had a correlation on \(0.037\text{ with } TRE (p=0.4459)\) and \(0.079\text{ with } TFE (p=0.099)\).

The hospital specific efficiency scores and ranking shown in Table 5.8 are estimated using the pooled Cobb-Douglas production function. In the top five hospitals according to the ranking, there were 3 public hospitals and 2 faith-based hospitals. This was also the case with the bottom five hospitals.
In the pooled SFA data set as shown in Table 5.8, the mean efficiency scores of public hospitals was 0.738 with SD of 0.098 while the faith-based hospitals had a mean of 0.684 with SD of 0.171. There were no significant differences between the two types of hospitals using the pooled SFA data. Although the ranks in Table 5.8 are based on mean efficiency of each hospital, the efficiency scores vary over time within each hospital (Figure 5.5).

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Mean</th>
<th>SE</th>
<th>Confidence Interval</th>
<th>Rank</th>
<th>Type of hospital</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7238576</td>
<td>0.0176022</td>
<td>0.6870159 to 0.7606994</td>
<td>16</td>
<td>Public</td>
</tr>
<tr>
<td>2</td>
<td>0.8549105</td>
<td>0.0160485</td>
<td>0.8186063 to 0.8912146</td>
<td>3</td>
<td>Faith Based</td>
</tr>
<tr>
<td>3</td>
<td>0.6673279</td>
<td>0.0140586</td>
<td>0.637903 to 0.6967528</td>
<td>21</td>
<td>Faith Based</td>
</tr>
<tr>
<td>4</td>
<td>0.3841203</td>
<td>0.0088115</td>
<td>0.3656777 to 0.402563</td>
<td>27</td>
<td>Faith Based</td>
</tr>
<tr>
<td>5</td>
<td>0.8412795</td>
<td>0.0095595</td>
<td>0.8212713 to 0.8612877</td>
<td>5</td>
<td>Public</td>
</tr>
<tr>
<td>6</td>
<td>0.5985035</td>
<td>0.0149884</td>
<td>0.5630615 to 0.6339455</td>
<td>24</td>
<td>Public</td>
</tr>
<tr>
<td>7</td>
<td>0.6747167</td>
<td>0.0097034</td>
<td>0.6541464 to 0.6952869</td>
<td>20</td>
<td>Public</td>
</tr>
<tr>
<td>8</td>
<td>0.7569154</td>
<td>0.0306447</td>
<td>0.6927754 to 0.8210554</td>
<td>12</td>
<td>Public</td>
</tr>
<tr>
<td>9</td>
<td>0.8168257</td>
<td>0.0155423</td>
<td>0.7842953 to 0.8493561</td>
<td>6</td>
<td>Public</td>
</tr>
<tr>
<td>10</td>
<td>0.7476319</td>
<td>0.0218586</td>
<td>0.6981842 to 0.7970796</td>
<td>13</td>
<td>Public</td>
</tr>
<tr>
<td>11</td>
<td>0.8961007</td>
<td>0.0099759</td>
<td>0.8744139 to 0.9180575</td>
<td>1</td>
<td>Public</td>
</tr>
<tr>
<td>12</td>
<td>0.7302086</td>
<td>0.0224211</td>
<td>0.6826779 to 0.777393</td>
<td>15</td>
<td>Public</td>
</tr>
<tr>
<td>13</td>
<td>0.7863606</td>
<td>0.0257659</td>
<td>0.7324321 to 0.8402892</td>
<td>10</td>
<td>Public</td>
</tr>
<tr>
<td>14</td>
<td>0.7918408</td>
<td>0.009604</td>
<td>0.7709936 to 0.8126881</td>
<td>9</td>
<td>Public</td>
</tr>
<tr>
<td>15</td>
<td>0.5890155</td>
<td>0.0185182</td>
<td>0.5482571 to 0.6297739</td>
<td>25</td>
<td>Faith Based</td>
</tr>
<tr>
<td>16</td>
<td>0.7129918</td>
<td>0.0181613</td>
<td>0.6749798 to 0.7510037</td>
<td>18</td>
<td>Public</td>
</tr>
<tr>
<td>17</td>
<td>0.7377658</td>
<td>0.0149162</td>
<td>0.7059727 to 0.769558</td>
<td>14</td>
<td>Public</td>
</tr>
<tr>
<td>18</td>
<td>0.5995687</td>
<td>0.0212204</td>
<td>0.5543386 to 0.6447989</td>
<td>23</td>
<td>Public</td>
</tr>
<tr>
<td>19</td>
<td>0.851429</td>
<td>0.0120965</td>
<td>0.8248049 to 0.8780352</td>
<td>4</td>
<td>Faith Based</td>
</tr>
<tr>
<td>20</td>
<td>0.6306841</td>
<td>0.0176101</td>
<td>0.5890429 to 0.6723253</td>
<td>22</td>
<td>Faith Based</td>
</tr>
<tr>
<td>21</td>
<td>0.6775617</td>
<td>0.0135192</td>
<td>0.6489023 to 0.7062211</td>
<td>19</td>
<td>Public</td>
</tr>
<tr>
<td>22</td>
<td>0.7668047</td>
<td>0.0154161</td>
<td>0.7345385 to 0.7990709</td>
<td>11</td>
<td>Public</td>
</tr>
<tr>
<td>23</td>
<td>0.8081946</td>
<td>0.0202184</td>
<td>0.7648305 to 0.8515587</td>
<td>7</td>
<td>Public</td>
</tr>
<tr>
<td>24</td>
<td>0.4966502</td>
<td>0.0211556</td>
<td>0.4521002 to 0.5412001</td>
<td>26</td>
<td>Public</td>
</tr>
<tr>
<td>25</td>
<td>0.8856952</td>
<td>0.0101274</td>
<td>0.8644983 to 0.9068922</td>
<td>2</td>
<td>Public</td>
</tr>
<tr>
<td>26</td>
<td>0.8079669</td>
<td>0.0181053</td>
<td>0.7651548 to 0.8507791</td>
<td>8</td>
<td>Faith Based</td>
</tr>
<tr>
<td>27</td>
<td>0.7180135</td>
<td>0.0150281</td>
<td>0.685982 to 0.750045</td>
<td>17</td>
<td>Public</td>
</tr>
</tbody>
</table>
5.3.6 Efficiency measurement over time

Examining individual efficiency score over time can be assessed and this is done using the time varying models. Figure 5.6 shows the technical efficiency score over time using the Battese and Coelli model. There is no particular trend other than fluctuation over the different quarters. However, the mean efficiency score of the last quarter of 2012 (mean=0.565, SD=0.208) is generally lower and but not significant than first quarter of 2008 (mean=0.666, SD=0.201, p=0.1396). Although there is fluctuation over time, the mean efficiency scores are all within closer range in this model. The lowest score in the last quarter of 2012 (mean=0.565, SD=0.208) and highest in 2010 first quarter (mean=0.669, SD=0.210).
In the TFE model, there were more fluctuations over time but oscillates at certain time points (Figure 5.7). The high peaks are mostly on the third quarters of every year (July-September). This coincides with the beginning of financial years and assignment of budgets to the facilities for that particular financial year. This variation would probably not be seen if analysis was conducted in annually.
The efficiency measurement with the true random effects also exhibits almost same results as the TFE. The highest peak corresponds with financial year oscillation. This is shown in Figure 5.8.
5.3.7 Incorporating ownership in hospital efficiency measurement

Ownership is one of the key factors that can affect the efficiency of hospitals. Generally, hospitals that are not subsidized and depend on other sources other than from the government might have more incentive to ensure that resources are allocated and utilized efficiently. Government hospitals on the other hand might have less incentive to do so. Although, faith-based hospitals are perceived to be more efficient, there might actually be similar to the public hospitals as compared to the private hospitals. This study also examines how ownership affects efficiency of hospitals.

There are two ways of incorporating exogenous variables such as ownership as referred to Chapter 2. One, they can be incorporated as regressors and accounts for systematic differences across hospitals due to ownership structure. The limitation with this model is that it does not explicitly explain the variation in efficiency in the different hospitals.

The other alternative is to estimate the efficiency scores without the ownership variable and subsequently compares the results between the two ownership types. The disadvantage of using the two-step approach in the second method is that the estimation of the first-step might affect the results of the second stage if there are any errors. The efficiency scores might end up being either similar or different between the two ownership types depending on the bias.

However, there are some advantages of the two-stage approach. The ownership types of hospitals are assumed to affect the efficiency and not the production technology. The method also allows the use statistical tests to compare the efficiency scores. In this study,
the two-stage approach was applied. Table 5.9 shows the average efficiency scores by ownership. In the standard models (pooled, BC and PL), the public hospitals had significantly higher efficiency than the faith-based. However, when using the true effects models, there were no significant differences between the two types of hospitals in this study.

Table 5.9: Mean (SD) efficiency scores by ownership

<table>
<thead>
<tr>
<th></th>
<th>Faith-Based Mean (SD)</th>
<th>Public Mean (SD)</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pooled model</td>
<td>0.6486 (0.1774)</td>
<td>0.7432 (0.1184)</td>
<td>0.0001*</td>
</tr>
<tr>
<td>Pitt and Lee</td>
<td>0.3413 (0.1492)</td>
<td>0.6890 (0.1586)</td>
<td>&lt;0.0001†</td>
</tr>
<tr>
<td>Battese and Coelli</td>
<td>0.3434 (0.1491)</td>
<td>0.6942 (0.1577)</td>
<td>&lt;0.0001†</td>
</tr>
<tr>
<td>True Random Effects</td>
<td>0.8307 (0.0926)</td>
<td>0.8365 (0.1065)</td>
<td>0.2501*</td>
</tr>
<tr>
<td>True Fixed Effects</td>
<td>0.8431 (0.0849)</td>
<td>0.8515 (0.0799)</td>
<td>0.4873*</td>
</tr>
</tbody>
</table>

*Kruskal-Wallis test for non-normal data
†Student’s t-test for normal data

Figure 5.9 shows the efficiency scores of the public and faith based hospitals over time. Similar to the overall efficiency score results, the BC model efficiency over time by ownership was stable in both types of hospitals. The faith-based hospitals had lower efficiency constantly over time. This was different with true effects model, where the efficiency varied over time in the two hospital types. In the pooled model and the true effects models, the mean efficiency score sharply dropped in the last quarter of 2012. A possible explanation to this drop is during this period there was a major strike by health workers (doctors and nurses) in all public hospitals. This had a major effect on the hospital activities and mainly the outpatient and inpatient departments.
Figure 5.9: Mean efficiency over time by ownership - Pooled and Time-Varying models

_Solid line – Faith-Based hospitals_
_Dashed line – Public hospitals_

5.4 Conclusion

The main aim of this chapter was to estimate efficiency in selected Kenyan hospitals using a parametric approach. Stochastic frontier production function using hospital panel data between 2008 and 2012 was explored. There were strong correlations between the outputs (especially admissions) and inputs. There was a general decline in the number of outpatients and admissions over time and this similar trend was with the staffing levels and number of beds. However, expenditure increased significantly over time.
Both time-invariant and time-varying stochastic frontier panel data models were used in the Cobb-Douglas production function. The parameter estimates from the different models yielded varying results mainly driven by the underlying assumptions of the models. The model results show that the time-varying BC model and time-invariant PL model have similar results. The mean efficiency scores of these two models were lower than the true effects models. Unfortunately there is no statistical test that can be done show whether true effects models are better fit than the BC and PL models. The true effects models appear to perform better due the higher number of significant input variables (Table 5.6). There is also less variation in the mean estimates in the true effects models (Table 5.7). Choosing between the TRE and TFE depends on the assumption one wants to make about the correlation between the unobserved heterogeneity and explanatory variables. However, from the results, the similar scores in the two models indicate that a bias resulting from any correlation between the hospital specific effects and explanatory variables does not influence the estimates.

These results had similar patterns to a one study estimating efficiency in sub-Saharan Africa (sSA) where they found mean scores of 0.86, 0.91 and 0.998 for BC, TFE and TRE models respectively in Kenya (Novignon & Lawanson, 2014). The preferred model in that study was the TRE model as it can accommodate the presence of time-invariant unobserved heterogeneity in the panel data. Further analysis to explore comparisons of the SFA panel data models and identify most preferred model will be carried out in the future.

Sign and significance of the parameter estimates varied in the different models. The parameter estimate for nurses had a larger magnitude and was significant than the other inputs suggesting that nurses have a positive effect on efficiency of the hospitals. The
nurses' parameter might also indicate that nurses can also easily substitute other staff e.g. physicians, especially if there is shortage. For example, for basic care, nurses can substitute doctors if there are not available. The beds parameter was generally significant but with a negative coefficient. This indicates that beds might not be binding factors in the overall efficiency of hospitals. There is a need for further analysis to explore these aspects.

Efficiency measurement over time using data from Kenyan hospitals shows no particular trend. The pattern oscillated with high peaks during the beginning of the financial year. This was especially the case with the true effects models unlike in BC model where the mean efficiency over time was more constant.

In this study, efficiency levels were estimated by ownership. There were significant differences between public and faith-based hospitals when using the standard models (pooled, PL and BC) with public hospitals having higher estimated efficiency levels. However, there were negligible and not significant differences when using the true effects models. The faith-based hospitals still had lower levels most of the time when compared in all the time periods (Figure 5.9). This provides insight that there are no differences between the two types of hospitals. Further analysis by ownership with a larger data set might be ideal to ensure that the differences are not driven by small sample size.
6 Comparison of DEA and SFA using data from Kenyan hospitals

6.1 Introduction

Frontier analysis techniques have recently become common in measuring efficiency. As highlighted in Chapter 2, DEA and SFA have advantages and disadvantages. Estimates from the different techniques differ because of different underlying framework of the techniques, shape of the frontier and distance from the individual observations and the frontier. However, the underlying concept is the same, which is to envelop data within a frontier in order to determine relative efficiency levels compared to the frontier in individual DMU.

The SFA uses economic theory to determine the shape of the frontier. The true frontier is unobservable and this raises questions on how best to approximate it. The DEA on the other hand, requires no specification of the functional form prior to determining the frontier. The frontier is determined by data in the DEA model. SFA would be considered to be restrictive due the assumptions made on model, hence DEA may be seen as more flexible. However, since frontier calculated using DEA is dependent of data, the frontier is sensitive to observations that are unusual or combinations of different inputs and outputs.

The calculation of the distance from the frontier is also different in the two frontier models. DEA assumes that the deviation from the frontier is all due to inefficiency. In SFA, the position of the frontier is estimated by recognising both inefficiency and measurement error. DEA also generates efficiency using limited data by comparing each DMU to its peers that produce comparable outputs. If an output is unique to a particular DMU, then the DMU will have no peers to make a comparison to even though there are other outputs that might be in common. Absence of peers automatically assigns full efficiency (score of 1.0)
to the DMU. SFA on the other hand utilizes all the sample information when estimating relative efficiency indicating that efficiency estimates are more robust to any combinations of variables and presence of unusual data (e.g. outliers).

This chapter highlights the difference between the two frontier techniques using sample data from Kenyan hospitals. Estimates from a cross sectional SFA is presented and then compared to estimates obtained from Chapter 4 using DEA. Correlation of the two models is carried out to examine the relationship between the techniques.

Efficiency scores by ownership using a cross sectional data are further discussed in this chapter and a comparison between DEA and SFA by ownership determined.

6.2 Methodological framework

A cross sectional stochastic frontier analysis was used to estimate the relative efficiency with single output (output index of outpatients and admissions) and seven inputs (doctors, nurses, clinical officers, other health workers, other staff, expenditure and total beds). A Cobb-Douglas production functional form was assumed with half-normal distribution in the form:

\[
\ln y_i = \alpha + \beta' \ln x_i + \nu_i - u_i
\]  

(6.1)

where  \( i=1, ..., N; \)  \( t=1, ..., T \)

Output vector \((y) = \) output index of outpatients and admissions
Input vector ($x$) = doctors, nurses, clinical officers, other health workers, expenditure and beds

$\beta$ = Unknown parameter

$\nu$ = Noise/measurement error

$\tau$ = Inefficiency

In order to compare the two techniques the results from the cross sectional SFA estimates are compared to the DEA results obtained in Chapter 4. Since there were no significant differences between the orientations of the models and there was a strong correlation between the models (Table L.2), a DEA input oriented model was assumed in this chapter. The input-oriented model is also preferred in this setting and district hospitals as managers have control over the inputs used (refer to Chapter 4). Since not all hospitals operate at an optimal scale, the variable returns to scale (VRS) alternative over constant returns to scale (CRS) was assumed.

6.3 Results

6.3.1 Cross sectional stochastic frontier analysis

A total of 27 hospitals were included in this analysis with 20 public and 7 faith-based hospitals. Two cross sectional data sets were selected from the larger panel dataset. This included data from the years 2011 and 2012 since the quarterly data from these two years were complete and did not have missing observations. The descriptive statistics and correlation of the input and output variables are presented in Chapter 4 from Table 4.2 and Table 4.3.
Maximum likelihood estimates of the cross section Cobb-Douglas production function is shown in Table 6.1. The parameter estimates between the two years were similar with lambda of 1.6484 in the 2011 dataset and 1.9707 in the 2012 dataset. This indicates that 73% of total variation is due to inefficiency in 2011 and 80% in the 2012. The remaining 27% and 20% respectively are due to random variation.

Similar to the panel dataset, the parameters for nurses in both datasets have higher magnitudes and are significant at 1%. Potentially this means that the nurses have a significant influence on efficiency in hospitals but also indicates that if there is shortage or lack of nurses, the service delivery in hospitals can be affected negatively as they cannot be easily substituted.
Table 6.1: Maximum Likelihood estimates for parameters of stochastic Cobb-Douglas Production Function

<table>
<thead>
<tr>
<th>Output Index</th>
<th>Estimate (SE)</th>
<th>Confidence Interval</th>
<th>Estimate (SE)</th>
<th>Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-4.8867*** (1.9934)</td>
<td>-8.7938 -0.9796</td>
<td>-4.1356*** (1.0788)</td>
<td>-6.2499 -2.0210</td>
</tr>
<tr>
<td>Doctors</td>
<td>0.1149 (0.1225)</td>
<td>-0.1251 0.3550</td>
<td>0.2675** (0.1085)</td>
<td>0.0549 0.4801</td>
</tr>
<tr>
<td>Nurses</td>
<td>0.7647*** (0.2319)</td>
<td>0.3101 1.2193</td>
<td>0.6481*** (0.2092)</td>
<td>0.2381 1.0581</td>
</tr>
<tr>
<td>COs</td>
<td>0.2318 (0.2001)</td>
<td>-0.1603 0.6239</td>
<td>0.1383 (0.2186)</td>
<td>-0.2901 0.5678</td>
</tr>
<tr>
<td>Other HWs</td>
<td>-0.1631 (0.1893)</td>
<td>-0.5341 0.2078</td>
<td>-0.2608 (0.1882)</td>
<td>-0.6296 0.1080</td>
</tr>
<tr>
<td>Other staff</td>
<td>0.0043 (0.1246)</td>
<td>-0.2399 0.2486</td>
<td>0.0595 (0.1243)</td>
<td>-0.1841 0.3030</td>
</tr>
<tr>
<td>Expenditure</td>
<td>0.1768 (0.1367)</td>
<td>-0.0910 0.4447</td>
<td>0.1706** (0.0785)</td>
<td>0.0168 0.3244</td>
</tr>
<tr>
<td>Beds</td>
<td>-0.2530* (0.1327)</td>
<td>-0.5132 0.0072</td>
<td>-0.2798** (0.1308)</td>
<td>-0.5361 -0.0235</td>
</tr>
</tbody>
</table>

Noise and Inefficiency distributions

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>1.6484** (0.2067)</td>
<td>1.2432</td>
<td>2.0536</td>
<td>1.9707 (0.9478)</td>
</tr>
<tr>
<td>σ_u</td>
<td>0.3198</td>
<td></td>
<td></td>
<td>0.3596</td>
</tr>
<tr>
<td>σ_v</td>
<td>0.1940</td>
<td></td>
<td></td>
<td>0.1825</td>
</tr>
<tr>
<td>Log</td>
<td>-2.94357</td>
<td></td>
<td></td>
<td>-3.70555</td>
</tr>
</tbody>
</table>

***, **, * => Significance at 1%, 5% and 10% level

6.3.2 Efficiency score comparison between DEA and SFA

Efficiency scores were estimated using both DEA and SFA. Efficiency levels for each hospital using both data sets are presented in Table 6.2. Since DEA assigns at least hospitals that lie on the frontier 100% efficiency, there were several with a score of 1.0. SFA does not assign hospitals a score of 1.0 because of the underlying way of calculating distance of individual DMUs from the frontier. Efficiency levels for individual hospitals are different between the two methods as shown in Table 6.2.
Table 6.2: Efficiency estimates using DEA under VRS assumption and SFA Cobb Douglas production function.

<table>
<thead>
<tr>
<th></th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEA</td>
<td>SFA</td>
</tr>
<tr>
<td>1</td>
<td>0.9352</td>
<td>0.7933</td>
</tr>
<tr>
<td>2</td>
<td>1.0000</td>
<td>0.8884</td>
</tr>
<tr>
<td>3</td>
<td>1.0000</td>
<td>0.7442</td>
</tr>
<tr>
<td>4</td>
<td>0.7059</td>
<td>0.4346</td>
</tr>
<tr>
<td>5</td>
<td>1.0000</td>
<td>0.9024</td>
</tr>
<tr>
<td>6</td>
<td>1.0000</td>
<td>0.7466</td>
</tr>
<tr>
<td>7</td>
<td>1.0000</td>
<td>0.7129</td>
</tr>
<tr>
<td>8</td>
<td>1.0000</td>
<td>0.8472</td>
</tr>
<tr>
<td>9</td>
<td>1.0000</td>
<td>0.8492</td>
</tr>
<tr>
<td>10</td>
<td>0.7090</td>
<td>0.8674</td>
</tr>
<tr>
<td>11</td>
<td>1.0000</td>
<td>0.9087</td>
</tr>
<tr>
<td>12</td>
<td>1.0000</td>
<td>0.7968</td>
</tr>
<tr>
<td>13</td>
<td>0.4780</td>
<td>0.8205</td>
</tr>
<tr>
<td>14</td>
<td>1.0000</td>
<td>0.8633</td>
</tr>
<tr>
<td>15</td>
<td>0.8521</td>
<td>0.7165</td>
</tr>
<tr>
<td>16</td>
<td>0.5597</td>
<td>0.7989</td>
</tr>
<tr>
<td>17</td>
<td>0.8559</td>
<td>0.8193</td>
</tr>
<tr>
<td>18</td>
<td>0.5776</td>
<td>0.6872</td>
</tr>
<tr>
<td>19</td>
<td>0.6682</td>
<td>0.8665</td>
</tr>
<tr>
<td>20</td>
<td>1.0000</td>
<td>0.7287</td>
</tr>
<tr>
<td>21</td>
<td>0.5932</td>
<td>0.7479</td>
</tr>
<tr>
<td>22</td>
<td>0.9879</td>
<td>0.8411</td>
</tr>
<tr>
<td>23</td>
<td>0.8073</td>
<td>0.7970</td>
</tr>
<tr>
<td>24</td>
<td>1.0000</td>
<td>0.6856</td>
</tr>
<tr>
<td>25</td>
<td>1.0000</td>
<td>0.9305</td>
</tr>
<tr>
<td>26</td>
<td>1.0000</td>
<td>0.8390</td>
</tr>
<tr>
<td>27</td>
<td>0.6002</td>
<td>0.7414</td>
</tr>
<tr>
<td>Mean (SD)</td>
<td>0.8641 (0.1794)</td>
<td>0.7917 (0.0996)</td>
</tr>
</tbody>
</table>

The mean efficiency estimates using DEA was higher than the SFA method in both data sets. This difference was significant with p-value=0.0462 in the 2011 data and p-value=0.0039 in the 2012 data set. The distributions of the two analysis techniques are
shown in Figure 6.1. DEA has a lot more variation with most hospitals with high efficiency levels.

Figure 6.1: Bean plots of DEA and SFA estimated efficiency levels

- **Dotted line across**: Overall mean value
- **Solid line in individual bean plot**: Mean value for each plot
- **White lines within plots**: Individual efficiency scores, with longer lines e.g. in the DEA bean plots indicating several hospitals with the same efficiency score (e.g. score of 1.0)

The results show that the average DEA efficiency scores are higher than the SFA mean scores. This indicates that inefficiency, deviation from the frontier, is lower in DEA than in SFA. In theory, SFA efficiency estimates are usually higher than DEA because it incorporates measurement error but the disadvantage of DEA is that it is sensitive to data and tends to specify many DMUs on the frontier in this study. This factor might be driven
by relatively small sample size i.e. with fewer observations a higher number of DMUs tend to be on the frontier (Alirezaee, Howland, & van de Panne, 1998; Y. Zhang & Bartels, 1998). DEA results are highly affected by the hospitals that are on the frontier and consequently exaggerating the frontier by failing to adjust for measurement error. If the SFA efficiency scores are compared with DEA under CRS assumptions (results in Chapter 4), the SFA relative efficiency scores are higher than DEA. This is because the DEA VRS model tightly envelopes the data more than CRS and more DMUs are placed on the frontier.

6.3.3 Efficiency score comparison between DEA bootstrapped model and SFA

As earlier discussed in Chapter 4, bootstrapped DEA model allows for the correction of sample bias and statistical inference methods can be used in generating confidence intervals around the estimates. A summary of the individual hospital efficiency scores is shown in Table 6.3. Comparing SFA with the scored from a bootstrapped DEA model indicates different results from the uncorrected DEA scores. When a bootstrapped model is used, there were no significant differences between the two frontier techniques (p-value is 0.2530 and 0.8959. This is also clearly shown in the bean plot in Figure 6.2.
Table 6.3: Efficiency estimates using bootstrapped DEA model under VRS assumption and SFA Cobb Douglas production function.

<table>
<thead>
<tr>
<th></th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEA bootstrapped</td>
<td>SFA</td>
</tr>
<tr>
<td>1</td>
<td>0.8488</td>
<td>0.7933</td>
</tr>
<tr>
<td>2</td>
<td>0.8683</td>
<td>0.8884</td>
</tr>
<tr>
<td>3</td>
<td>0.8489</td>
<td>0.7442</td>
</tr>
<tr>
<td>4</td>
<td>0.6449</td>
<td>0.4346</td>
</tr>
<tr>
<td>5</td>
<td>0.8539</td>
<td>0.9024</td>
</tr>
<tr>
<td>6</td>
<td>0.8470</td>
<td>0.7466</td>
</tr>
<tr>
<td>7</td>
<td>0.8657</td>
<td>0.7129</td>
</tr>
<tr>
<td>8</td>
<td>0.8498</td>
<td>0.8472</td>
</tr>
<tr>
<td>9</td>
<td>0.8535</td>
<td>0.8492</td>
</tr>
<tr>
<td>10</td>
<td>0.6427</td>
<td>0.8674</td>
</tr>
<tr>
<td>11</td>
<td>0.8478</td>
<td>0.9087</td>
</tr>
<tr>
<td>12</td>
<td>0.8598</td>
<td>0.7968</td>
</tr>
<tr>
<td>13</td>
<td>0.4342</td>
<td>0.8205</td>
</tr>
<tr>
<td>14</td>
<td>0.8964</td>
<td>0.8633</td>
</tr>
<tr>
<td>15</td>
<td>0.7775</td>
<td>0.7165</td>
</tr>
<tr>
<td>16</td>
<td>0.5028</td>
<td>0.7989</td>
</tr>
<tr>
<td>17</td>
<td>0.7800</td>
<td>0.8193</td>
</tr>
<tr>
<td>18</td>
<td>0.5243</td>
<td>0.6872</td>
</tr>
<tr>
<td>19</td>
<td>0.6155</td>
<td>0.8665</td>
</tr>
<tr>
<td>20</td>
<td>0.8628</td>
<td>0.7287</td>
</tr>
<tr>
<td>21</td>
<td>0.5479</td>
<td>0.7479</td>
</tr>
<tr>
<td>22</td>
<td>0.9097</td>
<td>0.8411</td>
</tr>
<tr>
<td>23</td>
<td>0.7331</td>
<td>0.7970</td>
</tr>
<tr>
<td>24</td>
<td>0.8626</td>
<td>0.6856</td>
</tr>
<tr>
<td>25</td>
<td>0.8465</td>
<td>0.9305</td>
</tr>
<tr>
<td>26</td>
<td>0.8439</td>
<td>0.8390</td>
</tr>
<tr>
<td>27</td>
<td>0.5429</td>
<td>0.7414</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Mean (SD)</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DEA bootstrapped</td>
</tr>
<tr>
<td>Mean</td>
<td>0.7597</td>
<td>0.7917</td>
</tr>
<tr>
<td>(SD)</td>
<td>(0.1435)</td>
<td>(0.0996)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1601)</td>
</tr>
</tbody>
</table>
6.3.4 DEA and SFA model correlations

Correlation between corrected DEA and SFA was 0.2571 (p=0.1955) for the 2011 data and 0.4410 (p=0.0213) for the 2012 data. These are relatively weak correlations indicating that the efficiency scores in the two methods are different. The weak correlation between the two methods can be attributed to the underlying assumptions of the models. DEA results are highly affected by data and the frontier is driven by the hospitals lying on the frontier. This defines the shape and the distance of the frontier, which is different from the SFA model that is based on economic theory.
Ranking of the hospitals depending on the efficiency scores is shown in Table 6.4. The hospitals' ranking varied depending on the model and dataset but with hospital 18 generally ranking at the bottom 5 hospitals in all the models.

Table 6.4: Ranking of hospitals using bootstrapped DEA and SFA

<table>
<thead>
<tr>
<th>Hospital</th>
<th>2011 DEA</th>
<th>2011 SFA</th>
<th>2012 DEA</th>
<th>2012 SFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>17</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>20</td>
<td>11</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>27</td>
<td>22</td>
<td>27</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>14</td>
<td>19</td>
<td>17</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>24</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>9</td>
<td>14</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>8</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>21</td>
<td>5</td>
<td>23</td>
<td>9</td>
</tr>
<tr>
<td>11</td>
<td>13</td>
<td>2</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>16</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>13</td>
<td>27</td>
<td>12</td>
<td>27</td>
<td>21</td>
</tr>
<tr>
<td>14</td>
<td>2</td>
<td>7</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>15</td>
<td>18</td>
<td>23</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>16</td>
<td>26</td>
<td>14</td>
<td>25</td>
<td>15</td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td>13</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>18</td>
<td>25</td>
<td>25</td>
<td>24</td>
<td>26</td>
</tr>
<tr>
<td>19</td>
<td>22</td>
<td>6</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>22</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>21</td>
<td>23</td>
<td>18</td>
<td>26</td>
<td>22</td>
</tr>
<tr>
<td>22</td>
<td>1</td>
<td>10</td>
<td>21</td>
<td>11</td>
</tr>
<tr>
<td>23</td>
<td>19</td>
<td>15</td>
<td>1</td>
<td>17</td>
</tr>
<tr>
<td>24</td>
<td>6</td>
<td>26</td>
<td>5</td>
<td>23</td>
</tr>
<tr>
<td>25</td>
<td>15</td>
<td>1</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>16</td>
<td>11</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td>27</td>
<td>24</td>
<td>21</td>
<td>15</td>
<td>7</td>
</tr>
</tbody>
</table>

The efficiency scores and ranking results were further used to split the hospitals into three categories; 1) hospitals that were ranked at the top in both DEA and SFA, 2) hospitals...
ranked at the bottom and 3) those that were in the middle of the ranking. There were 9, 8 and 10 hospitals in each category respectively. The aim of dividing into the different categories is to explore any characteristics that are associated with a particular category. Since, there were challenges in obtaining additional data on factors that might drive efficiency, only data on inputs and outputs were used to look at the differences between the three categories. For DEA, analysis of super efficiency scores, slack positive efficient hospitals and peers are summarized in Appendix M.

Table 6.5 and Table 6.6 summarize the mean values of the different inputs and outputs by categories created based on ranking of hospitals with both DEA and SFA. The summaries clearly show that the bottom ranking hospitals (‘inefficient hospitals’) use more resources in terms of outputs and produce less output compared to the highest-ranking hospitals. The difference between the two groups was not statistically significant in both years except for doctors in 2012. There were also both types of hospitals in all the categories of ranking. This indicates that there were no patterns / characteristics of one category of hospitals that made them particularly different or perform better (or worse) than another ranking category. It will be interesting to explore the differences between the groups with availability of more data in the future.
Table 6.5: Mean values of inputs and outputs of top, middle and bottom ranking hospitals using both DEA and SFA (2011 data set)

<table>
<thead>
<tr>
<th>2011</th>
<th>Mean value for top ranking hospitals</th>
<th>Mean value for bottom ranking hospitals</th>
<th>Mean value for hospitals ranking in the middle</th>
<th>P-value between top and bottom ranking hospitals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doctors</td>
<td>7</td>
<td>10</td>
<td>10</td>
<td>0.3210</td>
</tr>
<tr>
<td>Nurses</td>
<td>51</td>
<td>65</td>
<td>57</td>
<td>0.4440</td>
</tr>
<tr>
<td>Clinical officers</td>
<td>9</td>
<td>11</td>
<td>9</td>
<td>0.4745</td>
</tr>
<tr>
<td>Other HWs</td>
<td>20</td>
<td>26</td>
<td>21</td>
<td>0.1881</td>
</tr>
<tr>
<td>Other staff</td>
<td>37</td>
<td>39</td>
<td>24</td>
<td>0.8910</td>
</tr>
<tr>
<td>Expenditure (Kshs.)</td>
<td>15,956,095.84</td>
<td>16,304,398.00</td>
<td>22,983,772.50</td>
<td>0.9527</td>
</tr>
<tr>
<td>Total beds</td>
<td>111</td>
<td>116</td>
<td>111</td>
<td>0.8862</td>
</tr>
<tr>
<td>Outputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outpatients</td>
<td>74,897</td>
<td>58,740</td>
<td>64,917</td>
<td>0.3735</td>
</tr>
<tr>
<td>Admissions</td>
<td>5,078</td>
<td>4,575</td>
<td>5,962</td>
<td>0.7411</td>
</tr>
<tr>
<td>Ownership</td>
<td>Number of hospitals (Public, FB)</td>
<td>(7,2)</td>
<td>(6,2)</td>
<td>(7,3)</td>
</tr>
</tbody>
</table>

Table 6.6: Mean values of inputs and outputs of top, middle and bottom ranking hospitals using both DEA and SFA (2012 data set)

<table>
<thead>
<tr>
<th>2012</th>
<th>Mean value for top ranking hospitals</th>
<th>Mean value for bottom ranking hospitals</th>
<th>Mean value for hospitals ranking in the middle</th>
<th>P-value between top and bottom ranking hospitals</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doctors</td>
<td>5</td>
<td>11</td>
<td>10</td>
<td>0.0317</td>
</tr>
<tr>
<td>Nurses</td>
<td>43</td>
<td>67</td>
<td>62</td>
<td>0.1734</td>
</tr>
<tr>
<td>Clinical officers</td>
<td>8</td>
<td>12</td>
<td>9</td>
<td>0.1276</td>
</tr>
<tr>
<td>Other HWs</td>
<td>18</td>
<td>24</td>
<td>24</td>
<td>0.1947</td>
</tr>
<tr>
<td>Other staff</td>
<td>28</td>
<td>44</td>
<td>28</td>
<td>0.1553</td>
</tr>
<tr>
<td>Expenditure (Kshs.)</td>
<td>14,200,162.00</td>
<td>20,069,818.08</td>
<td>21,551,776.90</td>
<td>0.4030</td>
</tr>
<tr>
<td>Total beds</td>
<td>79</td>
<td>138</td>
<td>122</td>
<td>0.1071</td>
</tr>
<tr>
<td>Outputs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outpatients</td>
<td>67,009</td>
<td>60,346</td>
<td>70,732</td>
<td>0.6716</td>
</tr>
<tr>
<td>Admissions</td>
<td>4682</td>
<td>5286</td>
<td>5770</td>
<td>0.6829</td>
</tr>
<tr>
<td>Ownership</td>
<td>Number of hospitals (Public, FB)</td>
<td>(6,2)</td>
<td>(7,2)</td>
<td>(7,3)</td>
</tr>
</tbody>
</table>
6.3.5 Comparison of DEA and SFA by ownership

Comparing efficiency levels by ownership shows no significant differences between DEA and SFA in the public hospitals. However, in the faith-based hospitals there was evidence of difference between the two techniques. This could be driven by the small sample size and results might change if a larger dataset was used. Table 6.7 shows these differences by ownership.

In the DEA results, there were no significant differences between public hospitals and faith-based hospitals. This is also the case with SFA method indicating that overall there are no differences in efficiency level by ownership in sample data from Kenyan hospitals.

Table 6.7: Difference in efficiency levels by ownership using bootstrapped DEA and SFA

<table>
<thead>
<tr>
<th>DMU</th>
<th>2011</th>
<th>2012</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DEA</td>
<td>SFA</td>
<td>P-Value</td>
<td>DEA</td>
<td>SFA</td>
<td>P-Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public hospitals</td>
<td>0.7525</td>
<td>0.8079</td>
<td>0.1154</td>
<td>0.7607</td>
<td>0.7867</td>
<td>0.4549</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n=20)</td>
<td>(0.1560)</td>
<td>(0.0712)</td>
<td></td>
<td>(0.1786)</td>
<td>(0.0838)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Faith-based</td>
<td>0.7801</td>
<td>0.7454</td>
<td>0.5569</td>
<td>0.8161</td>
<td>0.7225</td>
<td>0.0672</td>
<td></td>
<td></td>
</tr>
<tr>
<td>hospitals</td>
<td>(0.1071)</td>
<td>(0.1536)</td>
<td></td>
<td>(0.0861)</td>
<td>(0.1799)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(n=7)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-Value</td>
<td>0.6701</td>
<td>0.1570</td>
<td>0.4415</td>
<td>0.2137</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6.4 Conclusion

In this chapter, both parametric and non-parametric methods were applied in estimating technical efficiency using data from Kenyan hospitals. The main aims of the chapter were to explore the difference in efficiency estimates by using DEA and SFA and examine the difference in ownership using these two methods. The two methods have advantages and disadvantages. As a flexible model, DEA can take multiple outputs with ease compared to
SFA and no prior assumptions are made on the functional forms. However, DEA has a major drawback in that it does not separate inefficiency and measurement error leading to biased estimates in the presence of noise in the data. The main advantage of SFA is that it takes into account random noise by separating the error term into inefficiency and measurement error. However, SFA is limited to the production of single output or extra steps in developing a single output index is done in cases with multiple outputs. Since distributional assumptions on the functional forms are made in the SFA model, the results can be affected by the presence of misspecification bias.

Results from the two techniques, indicate that there are no significant differences in the technical efficiencies in the hospitals. The slight differences can be attributed to different methodological assumptions of the models. Maximum likelihood estimates of $\lambda$ and $\sigma_a$ in the cross sectional stochastic frontier production indicate that there were inefficiencies present in the model. The estimated efficiency in the two models for the two years ranged between 0.7579 and 0.7751 when the scores were corrected for bias in DEA. The estimated SFA efficiency scores were lower than uncorrected DEA scores in this study. (Sharma, Leung, & Zaleski, 1997) highlighted SFA efficiency estimates are expected to be higher than DEA due to DEA attributing deviation from the frontier purely as inefficiency. However, the opposite can occur if DEA tightly fits the frontier to the sample data as shown in their study. This was also the case in this thesis. There were many hospitals lying on the frontier with the DEA VRS approach leading to higher mean scores compared to SFA. The results were opposite with DEA under CRS assumption with higher SFA efficiency scores. These results however changed when bootstrapped DEA was used to correct for bias. The results showed that there were no significant differences between the two techniques.
The correlation between the two techniques was positive but weak and not significant in the 2011 data but significant in the 2012 data set. This indicates that the results are sensitive to model specification and changes over time. The correlation shows that there is no perfect relationship between the two types of models in this study. This indicates that using different methods of measuring efficiency may affect interpretability of results and extra caution is needed when inferring from the results. More than one frontier technique needs to be applied before discussing policy implication in order to ascertain robustness of the results obtained.

Ownership as a driver of efficiency was assessed and the results showed that there were no significant differences between the two types of hospitals. The DEA and SFA estimates however differed in the faith-based hospitals but this cannot be conclusive due to the small sample size.

In summary, there is no clear consensus on which method would be preferable over another. The frontier methods have both advantages and disadvantages therefore if consistency estimates and comparable results are met, the methods can be used to measure efficiency depending on the data and the description of the hospitals. (Jacobs et al., 2006) discussed three groups in which to sort DMUs when comparing DEA and SFA. First group is where relative efficiency is sensitive to choice of analysis technique. Secondly are DMUs that appear efficient no matter the model specified. Finally, the third group are the ones that always appear inefficient irrespective of the model. With this categorization, there were no significant patterns to indicate why efficient hospitals were always efficient no matter the model. But, generally the efficient hospitals utilized limited resources.
(inputs) and have higher output levels. Availability of additional data will provide a platform for further analysis.

Chapter 7 on sensitivity analysis emphasizes the choices that have to be made when considering DEA and SFA in this study. Combination of input and output variables, choice of functional forms, distribution of the error term and returns to scale may have an effect on the results obtained in such studies.
7 Sensitivity analysis

7.1 Introduction

This chapter examines sensitivity analysis of the estimates obtained by using either data envelopment analysis (DEA) or stochastic frontier analysis (SFA). The two frontier techniques have advantages and disadvantages and preference over which method to use is still an on-going debate (Andor & Hesse, 2011; Jacobs et al., 2006). Comparing the two methods in this study showed differences in efficiency estimates indicating that technical efficiency is sensitive to the type of method use. Sensitivity analysis is important in determining the robustness of the estimates from the two frontier techniques.

In DEA, the frontier is formed based on data by joining the hospitals with a relative efficiency score of 1.0. DEA is highly sensitive to data and any measurement error can affect the results significantly. A DMU can be considered fully efficient in terms of only one output without considering other outputs. Therefore this particular DMU will not have peers. The non-parametric nature of DEA also makes it difficult to test for robustness of the efficiency scores. Sensitivity analysis in this case is important not only because of measurement errors but also estimate efficiency using different combinations of variables. There are several studies that examined sensitivity analysis in DEA with more emphasis on changes in input and output combinations (Boljuncic, 2006; Cooper et al., 2001; Jahanshahloo et al., 2011; Lotfi, Kharazmi, & Amin, 2009; Neralic, 1998; Wen, Qin, & Kang, n.d.).

SFA on the other hand has an advantage over DEA as the estimated frontier incorporates measurement error. However, there could be inconsistency in efficiency estimation using SFA. Jondrow, Lovell, Materov and Schmidt (JLMS) estimator is used in SFA to extract
the inefficiency component in the estimated composed error term. The JLMS estimator can be inconsistent when using a single cross sectional data. However, this can be resolved by employing a panel data with sufficient time period (Baccouche & Kouki, 2003). There could also be problems with appropriateness of the choice of the one-sided error distribution when using maximum likelihood estimation. There is still no clear consensus on the appropriate distribution of the error term. Sensitivity analysis on the different distributions is important in order to check for consistency and stability (Baccouche & Kouki, 2003).

There are several ways of carrying out sensitivity analysis of this study. Validity of the specifications of the frontier approaches and findings can be assessed in various ways and the processes and results are discussed in this chapter.

7.2 Validity of the findings

There are two ways in which validity of the results can be evaluated. One is the internal validity which examines the stability of the results using different methods and secondly, the external validity that is concerned with generalizability of the results. Internal validity can be assessed in either of the following ways:

a) Using different combinations of input and output variables

b) Assess efficiency using different specifications in DEA (input vs. output oriented)

c) Assessing efficiency using different specifications of the SFA approach (Cobb-Douglas vs. Translog or different distributional assumptions of the one-sided error term)

External validity on the other hand can be assessed by:
a) Comparing efficiency using DEA and SFA approaches with the same input and output variables

b) Estimating efficiency and testing consistency over time

7.2.1 Different combinations of input and output variables

In health care, there are multiple inputs and multiple outputs. These variables can be many in some cases or in other cases limited due to data unavailability. If the inputs and outputs are limited, all that are available can be included when estimating efficiency. However, there are cases where aggregation of variables especially inputs such staffing levels are done (Ray, 2005). This aggregation of variables might impose prior restrictions on the models (Ray, 2005). Models with aggregated data tend to hide variations in the data e.g. doctors might have a different effect on efficiency compared to administrative staff.

In order to assess efficiency by using different combinations of input and output variables, the following models were specified:

- Model 1: Output index as output and doctors, nurses, clinical officers as inputs
- Model 2: Output index as output and doctors, nurses, clinical officers, other health workers as inputs
- Model 3: Output index as output and doctors, nurses, clinical officers, other health workers, expenditure, beds as inputs
- Model 4: Output index as output and all staff combined, expenditure, beds as inputs
- Model 5: Admissions as output and all seven inputs
- Model 6: Outpatients as output and all seven inputs
- Model 7: Output index as output and all seven inputs (Chapters 4 and 5)
### Table 7.1: Pooled Cobb Douglas Production function using different combinations of input and output variables (panel data 2008-2012)

<table>
<thead>
<tr>
<th>Appendix A</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6*</th>
<th>Model 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.5720***</td>
<td>-2.5179***</td>
<td>-3.1649***</td>
<td>-</td>
<td>2.5921***</td>
<td>7.0099***</td>
<td>-3.208***</td>
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<td></td>
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<td>(0.1433)</td>
<td>(0.2802)</td>
<td>-</td>
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<td>(0.4016)</td>
<td>(0.2891)</td>
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<td>0.1224***</td>
<td>0.1078***</td>
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<td>-0.0209</td>
<td>0.2266***</td>
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<td>(0.0325)</td>
<td>(0.0324)</td>
<td>-</td>
<td>(0.0408)</td>
<td>(0.0385)</td>
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<td>Cos</td>
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<td>Other staff</td>
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<td>-</td>
<td>-0.0212</td>
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<td>-</td>
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<td>All staff combined</td>
<td>-</td>
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<td>-</td>
<td>0.9567***</td>
<td>(0.0705)</td>
<td>-</td>
<td>-</td>
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<td>Expenditure</td>
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<td>(0.0465)</td>
<td>(0.0523)</td>
<td>(0.0486)</td>
<td>(0.0378)</td>
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**Noise and Inefficiency distributions**

<table>
<thead>
<tr>
<th></th>
<th>Lambda</th>
<th>Sigma_u</th>
<th>Sigma_v</th>
<th>Log Likelihood</th>
<th>Mean efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lambda</td>
<td>2.0436***</td>
<td>1.8802***</td>
<td>2.3261***</td>
<td>1.5671***</td>
<td>0.7338</td>
</tr>
<tr>
<td></td>
<td>(0.3569)</td>
<td>(0.4167)</td>
<td>(0.4232)</td>
<td>(0.5701)</td>
<td>(0.1311)</td>
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<tr>
<td>Sigma_u</td>
<td>0.4315***</td>
<td>0.4109</td>
<td>0.4394</td>
<td>0.6346</td>
<td>0.7296</td>
</tr>
<tr>
<td></td>
<td>(0.2111)</td>
<td>(0.1886)</td>
<td>(0.1889)</td>
<td>(0.2652)</td>
<td>(0.1388)</td>
</tr>
<tr>
<td>Sigma_v</td>
<td>0.2111***</td>
<td>0.2186</td>
<td>0.1889</td>
<td>0.1927</td>
<td>0.7296</td>
</tr>
<tr>
<td></td>
<td>(0.2111)</td>
<td>(0.1886)</td>
<td>(0.1889)</td>
<td>(0.2652)</td>
<td>(0.1388)</td>
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<tr>
<td>Log Likelihood</td>
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<td>(0.7307)</td>
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<tr>
<td>Mean efficiency</td>
<td>0.7338</td>
<td>0.7420</td>
<td>0.7296</td>
<td>0.7381</td>
<td>0.9943</td>
</tr>
<tr>
<td>(SD)</td>
<td>(0.1311)</td>
<td>(0.1236)</td>
<td>(0.1388)</td>
<td>(0.1142)</td>
<td>(0.1306)</td>
</tr>
</tbody>
</table>
The parameters estimates and mean efficiency scores from Table 7.1 show that having different combinations of variables does not significantly change the results in this study. The parameter estimates, noise and inefficiency distributions were consistent in 6 of the models. The mean efficiency estimates were also consistent ranging between 0.729 and 0.742. Model 6 maximum likelihood estimation did not converge causing the lambda value tend towards zero. This indicates that there was no inefficiency in the data. This means that this model specification might not have been the best fit for the data in this study.

Using the bootstrapped data envelopment analysis under the assumption of VRS with an input oriented specification, there were some similarities across the models except for models 4 and 6 as shown in Figure 7.1. The results of DEA utilised cross sectional dataset of year 2011 (Chapter 4). The mean efficiency score for models 1-7 were 0.7148, 0.7452, 0.7554, 0.5022, 0.8285, 0.5662 and 0.7589 respectively. The large variation in model 4 could be driven by the assumption of combining all staff into one input. The different cadres within a hospital setting have different responsibilities and effect on efficiency and hence lumping them together might hide any variation within one specified combined input. Variation in model 6 is also seen in the SFA pooled estimation indicating that specifying outpatients as a single output might introduce misspecification bias to the models.
Using the Cobb-Douglas stochastic production function assuming a half-normal model showed that the results vary depending on different combination of input and output variables. The mean efficiency scores of model 1-7 using the 2011 dataset were 0.7801, 0.7861, 0.7926, 0.6625, 0.9968, 0.7841 and 0.7917 respectively. There were no significant differences between models 1-3 and 6-7. Model 4 as highlighted in the pooled SFA and DEA results showed wide variation of efficiency scores probably driven by the combined definition of staffing levels as an input. Model 5 interestingly did not converge in the cross sectional dataset as compared to the pooled panel dataset. The results were also vice versa with model 6 (Figure 7.2). This shows that results vary depending on the specification of the models and choice of variables.
Distributions of Models 1–7 using cross sectional SFA

Figure 7.2: Cross sectional Cobb-Douglas Stochastic Production Efficiency Estimates (2011 dataset)

7.2.2 Input vs. Output Oriented Specification of DEA

Comparing input and output-oriented DEA approaches can also be used to check the validity of the model findings. The results from the two models using data from 2011 and 2012 are shown in section 4.3.3 and Appendix L. The distributions from the input and output oriented models were similar as shown in Appendix. There were strong correlations between the input and output models in the 2011 and 2012 datasets (0.9778 and 0.9873 respectively). This shows that DEA is stable with changes in the model assumption.

7.2.3 Cobb-Douglas vs. Translog production functions

Assessing the different functional forms of the stochastic frontier model can also be a way of testing the internal validity of the findings. The pooled estimates of the stochastic
frontier model using the panel data was described in Chapter 5 Table 5.4. The lambda value for the Cobb-Douglas functional form was 2.2914 with standard error (SE) of 0.00567 and 2.1103 for the Translog form with SE of 0.2608. The lambda values were similar with a higher standard error in the Translog form compared to the Cobb-Douglas form. The mean efficiency score using a Cobb-Douglas production function was 0.7235 with SD of 0.1381, which was statistically different from the Translog form with a mean value of 0.8145 and SD 0.0958.

The varying results from different functional forms show that the results and interpretation depends on the specification of the models. Proper procedures and detailed explanation need to be provided when choosing the different models in order for the results to be interpreted in the context of the data and analysis done.

7.2.4 Varying distributions of the error term

The results of varying distributions of the error term using pooled stochastic Cobb-Douglas production function was also discussed in Chapter 5 Table 5.4. The mean efficiency scores of the half-normal, exponential, truncated and gamma distribution were 0.7335, 0.7831, 0.7828 and 0.8089 respectively. The parameter estimates from the different distribution assumptions were stable and comparable.

7.2.5 Comparison of DEA and SFA

Comparison between the two frontier techniques has been described in detail in Chapter 6. The correlation between the DEA input oriented approach and SFA was 0.2571 with the 2011 data and 0.4410 with the 2012 data. This correlation was only significant with the 2012 data. This indicates that there is no strong consistency between the two models and
that external validity is still in question in the sample data used in this study. Choice between the two frontier approaches is important considering DEA is data-driven and highly sensitive to data. SFA has the advantage over DEA as it disentangles the error term into random noise and inefficiency.

7.2.6 Consistency of efficiency over time

Consistency of efficiency over time can either be assessed by panel data or two or more cross sectional data sets. With the panel data set as described in Chapter 5, there was consistency in efficiency levels over time with the Battese and Coelli method. The true effects models on the other hand, showed an oscillating pattern but were consistently peaking at the beginning of every financial year.

With the two cross-sectional data sets (2011 and 2012), the efficiency levels obtained by DEA model were not significantly different in the two datasets. The mean efficiency levels were also similar in the SFA model (section 6.3.2).

7.3 Conclusions

This chapter discussed various ways of testing for internal and external validity of the DEA and SFA findings. Using different combinations of input and output variables were generally consistent except for some combinations in which the model did not converge in SFA. Combining all staff into one input variable affected the results leading to inconsistency and high variation in the efficiency score in both DEA and SFA. This means that aggregating inputs might provide efficiency estimates that are not robust in the two frontier approaches. However, within the DEA model, specifying either input or output orientation produced consistent results and there was strong correlation between the two
DEA orientation specifications. This suggests that the choice of the orientation in DEA is highly stable.

In the SFA approach, choosing the functional form and distribution of the error term was also important step in checking for validity. In this study, although the mean efficiency score for the Translog function was higher than the Cobb-Douglas form, their lambda values were comparable. The parameter estimates in the two functional forms were also stable. When specifying the distributional assumptions of the error term, the parameters estimates and mean efficiency levels were stable across the different distributions.

However, when comparing the two frontier techniques, there was a weak correlation that was not significant in 2011 dataset but significant in the 2012 data set. This suggests that the choice of the frontier technique is important in analysing data and sensitivity to data and models need to be considered when measuring efficiency. Generally, efficiency levels over time were stable and only fluctuated in the SFA true effects models.
8 Conclusions

8.1 Main findings

The main aim of this thesis was to explore efficiency in Kenyan hospitals using DEA and SFA. Data were collected from level IV public and faith-based hospitals. Key input and output variables were collected from each sampled hospital but some were excluded in the analysis due to missing cases (Chapter 3). The output variables were aggregated to form an output index suggesting the models were in a single output multiple inputs nature.

The initial data collected were a panel structure in monthly format for the years 2008-2012. These data were aggregated into quarterly format forming a panel data of 20 quarters. Although the hospital workload activity data were in monthly format, the finance and human resource data were either in quarterly or annual format. Quarterly form was preferable due to availability of data and it also provides greater precision and allows measuring efficiency more immediately than annual data. Monthly data on the other hand might not provide more variations over time, as the activity data are rarely different between months. This data format was mainly used in the SF model. Since DEA model does not take in a panel structure, the data were further aggregated into annual format and only two data years (2011 and 2012) were used to explore efficiency in the DEA model. These two cross-sections were also used to estimate parameters and efficiency for the SF model that was later compared to the DEA results.

In DEA, the both the uncorrected and bootstrapped estimates were reported in the study (refer to Chapter 4). In the variable returns to scale (VRS) input-oriented model, the mean efficiency scores for the uncorrected measures were 0.8641 and 0.8721 for the 2011 and 2012 data sets consecutively. The results were different when the efficiency scores were...
corrected for bias using the bootstrapping. The mean efficiency scores were 0.7597 and 0.7751 for 2011 and 2012 data sets. Generally these results are consistent with similar studies conducted in Africa (Osei et al., 2005; Zere et al., 2001). Most of the studies (section 2.4) that used similar assumption models however had much lower efficiency scores indicating that DEA is sensitive to the data used and combination of input and output variables (Ichoku et al., 2011; Jehu-Appiah et al., 2014; Kirigia et al., 2008; Maredza, 2012; Tlotlego et al., 2010; Zere et al., 2006). This sensitivity in selection of variables was also shown in (Kibambe & Koch, 2007) with efficiency scores ranging between 0.636 and 0.903 depending on the variables in model.

In SF panel data analysis, the different time-invariant and time-varying models produced varying efficiency estimates due to the underlying assumptions. The time-invariant Pitt and Lee model estimated mean efficiency score was 0.62 and comparable with the time-varying Battese and Coelli model. This was much lower than the DEA model mean score. However, when the true effects models were specified, the average score was approximately 0.85, which was comparable to the DEA mean efficiency levels. Notably though, the main difference between DEA and SFA is the ability to differentiate the random error and inefficiency, where the DEA lumps together random errors and inefficiency. In the SF cross sectional data analysis, however the mean efficiency levels were 0.79 and 0.77 for the 2011 and 2012 datasets. These scores were comparable to the mean efficiency levels of the SF pooled panel data set (mean = 0.72).

The parameter estimate for nurses was statistically significant in all the time-invariant and time-varying models. One of the explanations for this is that nurses are considered the backbone of the health system and play a critical role in the provision of hospital services.
The other reason why the nurses' parameter was significant is possibly because nurses are not easily substituted compared to the other staff. They conduct a wide range of tasks in a hospital. For example, a nurse can carry out some of the tasks by doctors, clinical officers and health workers in specific hospital services but challenging when there is need to substitute them. This implies that they are vital in day to day running of health services. However, the results might indicate otherwise if efficiency is analysed by case-mix. In some disease areas, the doctors are more vital compared to the nurses. The results might be different in specialised care units that require specific physician skills. The different cadres of staff also have big differences in skill mix and this might indicate different effects of the various staff inputs.

Measuring efficiency over time was done using a panel stochastic frontier model. Using the panel data of 20 quarters, there was no evidence of either an increasing or declining efficiency over time. The trend in efficiency over time varied in the different models. Battese and Coelli model showed a constant trend in all the quarters. There were no significant changes in this model and similar with the true effect models that had more of oscillating changes over time. There were specific high peaks of relative efficiency scores at the beginning of the financial years. When using the two cross-sections data (2011 and 2012) using DEA, there were no significant differences between the two years. In summary, this indicates that there were no significant trends of efficiency over time.

Examining factors that drive efficiency in this study, ownership of the hospitals was explored. In DEA, ownership does not significantly influence efficiency in Kenyan hospitals. Using a truncated regression model, the results showed that being a public hospital had a negative impact on efficiency but this was not statistically significant. In
SFA, the difference in efficiency between public and faith-based varied depending on the model. There were significant differences in the two types of hospitals when using standard models (PL and BC models) with public hospitals having higher efficiency levels. However, the results were different when using the true effect models with no difference between the two types of hospitals. The results on the effect of ownership on efficiency might be driven by the small sample size in this study.

Considering possible measurement errors in DEA and inconsistency in model specification for the SFA, sensitivity analysis was carried out to check for robustness of the efficiency estimates. When the combination of input and output variables was varied, the estimates were different. This means that the models selected in measuring efficiency were sensitive to different combinations of input and output variables. However, the choice of orientation model in DEA and functional form or distribution of the one-sided error term in the SFA model was consistent and stable in the different specifications. Overall the choice of the frontier technique is important and need to be considered when measuring efficiency, as there were different efficiency estimates in the two models.

From this research study, the following can be concluded:

i. There is evidence of technical and scale inefficiency in the Kenyan hospitals sampled in this study.

ii. Applying frontier analysis techniques, DEA and SFA, yields varying efficiency estimates. This indicates that careful considerations have to be taken when choosing the analysis technique for measuring efficiency. With the challenges in hospital data in developing countries, model sensitivity to data has to be assessed and considered before choosing the appropriate technique.
iii. Within each frontier technique, the efficiency estimates were stable in the different model assumptions. The choice of functional form, distribution assumptions, orientation models and returns to scale assumptions did not significantly affect the efficiency estimates obtained.

iv. Nurses’ parameter was significant indicating their importance in running of key hospital services. The results might be different in specialized care that requires doctors’ skills.

v. There were no significant changes in efficiency over time. There was no particular upward or downward trend but had more of an oscillating feature in this data set.

vi. Ownership was not a significant factor driving efficiency in Kenyan hospitals. There were no significant differences between public and faith based hospitals except for specific SFA standard models (pooled, PL and BC), where public hospitals had significantly higher efficiency levels than the faith-based hospitals.

vii. Sensitivity analysis carried out in the study showed that estimates vary when different combinations of input and output variables were used.

### 8.1.1 Generalizability of the findings

Generalizability of efficiency measurement is a challenge because the results are mainly based on relative efficiency in a specific sample. In this study, data analysed was sampled from level IV hospitals in Kenya. These hospitals are the cornerstone of the health system as they are the main providers of primary care. The efficiency scores estimated in the study are a reflection of relative distance from the frontier that was formed by fully efficient hospitals only within the sample. The small sample size can also be a limiting factor in generalizing the results in similar levels. A larger sample size might be ideal for discussing the effect in all levels of care. The panel structure provides a platform to analyse the data.
with a larger sample size as done in this study. This helps in showing that the findings will be similar to other hospitals with similar incentives. Each hospital also has its own needs with more emphasis on particular input or output compared to another hospital.

The overall results in this study give an insight into the state of efficiency in Kenyan level IV hospitals. Frontier modelling techniques were used in this study, providing more information regarding efficiency measurement methods not only in Kenya but also in Africa. The specific problems with inefficiency can be addressed and solutions developed to tackle individual constraints.

Using the sample data from Kenyan hospital, there was little evidence of differences between public and faith-based hospitals. Ownership has been perceived to influence efficiency in Kenyan hospitals but according to the results of this study there were no indications of significant effect of ownership on hospital efficiency. Ownership might not be a driving factor of efficiency but stronger conclusions might be possible with a larger sample size.

### 8.2 Contributions of the study

This study has some contributions to the measurement of hospital efficiency in Kenyan hospitals and also in developing country context. Most of the studies highlighted in section 2.4 that were conducted in Africa have used mainly DEA to measure efficiency. This study estimated relative efficiency using both frontier analysis techniques, DEA and SFA. Although DEA is more flexible because it has no prior assumptions on the functional form, it assumes that the distance from the frontier is solely due to inefficiency. This does not reflect the true nature of estimates, as there is always random noise/statistical error when
measuring efficiency. This is particularly the case if there are any measurement errors in data due to poor data recording and keeping in most developing countries. SFA disentangles the distance from the frontier into inefficiency and measurement error. These techniques might be more applicable especially if choice of technique is sensitive to data. SFA, however, requires prior assumption on the distribution of the functional form therefore introducing chances of misspecification.

Previous studies in Kenya hospitals utilized secondary data sets obtained from DHIS at the Ministry of Health (MoH) (Kirigia et al., 2002). Data compiled and sent to MoH might not be a reflection of quality data (Kihuba et al., 2014). The data in this study were sampled from Kenyan hospitals and collected from the individual hospitals. Collecting data from the individual facilities although challenging has an advantage in that any gaps or inconsistent data can be clarified directly from the hospitals records staff. Although the data from individual facilities might not have been completely accurate, any errors seen were only those that occur at this level and not overlaid by additional errors. This was better than obtaining secondary data (normally sent by hospitals to the national level through the former district office) that had gone through further steps of record entry and editing by different individuals.

Specifically to the Kenyan setting, this study used panel data from 2008 to 2012 and analysed in quarterly format. Therefore the study had 20 panels to assess efficiency over time. This contributes to knowledge in measuring efficiency over time in Kenyan hospitals. This study gave insight in how efficiency levels change over time not annually but quarterly providing additional detail to efficiency measurement over time.
The other major contribution of this study in efficiency measurement in Kenyan hospitals is assessing ownership as a factor of efficiency. This study explored how ownership affects efficiency and differences between public and faith-based hospitals. Studies conducted in other countries in Africa have shown varying results with some indicating non-public hospitals performing better than public hospitals (Maredza, 2012; Masiye, 2007; Masiye et al., 2006) and other studies showing the vice versa (Jehu-Appiah et al., 2014). In this study, there were no significant difference between the public and faith based hospitals except for the standard SFA panel models that showed that public hospitals had higher efficiency levels. These results provide a platform for further analysis and discussion on the effect of ownership in efficiency measurement in health care.

Sensitivity analysis of estimated efficiency scores was conducted in this study adding to the hospital efficiency measurement knowledge in developing countries especially in Africa. Considering most of the studies conducted in hospitals in African countries used DEA, sensitivity analysis is important in order to check for robustness of the results. DEA is sensitive to data and the results might be inconsistent to changes in data or model specification. Since sensitivity analysis had not been done before in detail, this study examined some of the aspects of ascertaining the stability of the efficiency scores.

8.3 Policy implications

This study used a relatively small sample size from level IV hospitals, nonetheless the analysis and the findings are important contributions to the knowledge of measuring efficiency in Kenyan hospitals and have potential implication to our health policy. For
example, the results from this study suggest that there are technical inefficiencies (see chapters 4 and 5) within the health system which if addressed can lead to potential savings.

The estimated efficiency levels using both DEA and SFA show existence of overall technical inefficiencies. Hospital managers in developing countries such as Kenya use flexible methods such as simple ratio analysis and DEA to measure efficiency. Simple ratio analysis is a traditional measure of efficiency that does incorporate multiple inputs and outputs and it also does not estimate efficiency relative to the ‘best’ practice. As highlighted in the thesis, DEA is more common frontier analysis approach used in developing countries compared to SFA. Although DEA is a more flexible method, it is prone to error since it is sensitive to data. The advantage of SFA is that it estimates the frontier by incorporating both inefficiency and measurement error. The choice of analysis approach should be discussed carefully considering the key research questions, type and availability of data and possible biases that can affect the results.

This study also provides a platform for further exploration and discussion in regards to the effect of ownership on hospital efficiency. Generally, there were no significant differences between public and faith-based hospital despite the perception that faith-based hospitals deliver health care services more efficiently. This is new knowledge based on Kenyan data, indicates that not only should there be emphasis that public hospitals use resources efficiently but also the same should be conducted by the faith-based hospitals.

Emphasis has been placed in strengthening the health information systems in Kenya. However, there are still some major challenges in accessing good quality and up to date data from the individual hospitals. Activity data were mostly available but other data such
as finance, human resource and administrative statistics were not available centrally. The quality of the data is also poor and strengthening of the information systems by the government needs to be emphasized.

In summary, efficiency analysis is important in contributing to information and ways in improving health care delivery. Although there were limitations with data, frontier efficiency measurement provides insight into hospital performance and government should make this process a part of regular assessment.

8.4 Limitations and areas of future research

8.4.1 Limitations of the study

There were a few limitations in this study. In section 3.9.1 some of the challenges regarding missing data were highlighted. Obtaining data that was mostly accurate and of best quality was a challenge in this study. Although the data was collected from the individual facilities unlike from the central database (has more measurement errors), there were still some missing data and some inaccurate calculations. Some of the data were completely missing for a hospital or within observations and variables. Ways that these issues were handled in this study are highlighted in section 3.9. A more complete and comparable data set would be preferable for future research.

The types of input and output variables used in this study were also limited. Inclusion of other variables and especially data on factors that might affect efficiency would be ideal. The preferable outcome measure in efficiency measurement is the final outcome on quality of care. This is complex to measure and due to limiting factor the study used only the intermediate outcomes on hospital services.
The study used a relatively small sample size due to lack of complete data sets from the hospitals, time and budget constraints. Larger sample size would provide more stable results and lack of any biased results due to small sample size. Data compiled centrally from the hospitals would be better in obtaining data from more hospitals and observations but the current state of data at the district health information system (DHIS) is a constraint. As highlighted in Figure 3.2, the initial sample size of the study was intended to be 52 hospitals. Due to constraints highlighted in Figure 3.2, 27 hospitals were only included in the final analysis. This can potentially introduce selection bias from sample attrition and therefore a key limitation to the study.

In this study, data were not adjusted for quality or case-mix. The data were collected from level IV hospitals and hence and assumption made that they have the same case-mix. If case-mix differences are not captured, the results might be biased either upwards or downwards and hence critical in adjusting for quality and case-mix (Björkgren, Fries, & Hakkinen, 2004; Grosskopf & Valdmanis, 1993).

8.4.2 Areas of future research

There are several areas and suggestions in which future research can be done. Considering more/alternative variables in measuring efficiency is one of the areas. This includes incorporating additional health services such as deliveries, surgeries, immunizations, laboratory tests and radiology services as outputs.

Larger panel data sets can also be assessed in measuring efficiency. If resources are available for obtaining larger datasets, further analysis will be ideal in estimating
efficiency and less biased results due to small sample size will be obtained. Malmquist index can be used in the future to measure productivity change in DEA using panel data.

This study only measured efficiency in level IV (district) hospitals. Future research can be conducted in other levels of health care facilities to assess at each level the efficiency estimates. Other types of hospitals such as private hospitals and specialized facilities might be interesting to assess and this can be compared with the public and faith based hospitals.

This study estimated only technical and scale efficiency in Kenyan hospitals. Due to the lack input prices, allocative efficiency was not estimated in this study.

Other determinants of efficiency can also be assessed in future research. In this study, only ownership as a factor of efficiency was explored. There are additional determinants that can drive inefficiencies in health facilities such as average length of stay, bed occupancy rate, hospital size, population level, urbanization level, specialization number and distance of other hospitals in the same locality. Further analysis on factors that influence efficiency will help guide strategies in improving efficiency by the policy makers.

Hospital outputs such number of patients seen and number of procedures done are easier to measure than the outcomes related to patients such as life expectancy and quality of care received. Since the same level of hospitals was used in this study, the mix of cases was assumed to be similar across the hospitals. Insignificant effect of incorporating case mix adjustments has also been shown in other studies (Grosskopf & Valdmanis, 1993; Ozcan, 1992). If different levels and types of hospitals are included in future research, then accounting for difference in quality might be critical (Björkgren et al., 2004). In specific
health services such as immunization, the quality of care data can be easily defined because quality does not vary much. However, the measurement of quality might be more challenging in general treatment of patients. There are however, cases in which estimation of efficiency can incorporate quality-adjusted inputs and outputs (O'Donnell & Nguyen, 2011). Examples of variables that can be included in future research are average length of stay, waiting times, patients' satisfaction, patients’ characteristics (e.g. gender, age, ethnicity etc.) and socio-economic status.

Considering the presence and different levels of technical inefficiencies, there might be significant implications on economic evaluation results. If technical efficiencies exist, it means that a cost effectiveness (CE) ratio does not reflect the minimum efficient point of production at a given level (Walker, 2006). This will have an impact on decisions made based on cost effectiveness ratios. Exploring this area in the future using data from hospital services will give insight on whether the CE ratios will be affected by presence of technical inefficiencies.

8.5 Final thought

Efficiency measurement is important in health care as it contributes to improving use of limited resources. In developing countries emphasis on efficiency measurement should continue to be the key objective in the strategic plans for the governments. Identifying areas to improve efficiency and tracking how the health systems are performing should be an exercise done by the governments in order to ensure proper use of resources is practiced.
Analysis techniques for measuring efficiency should be carried out with caution depending on the objective and data used in the analysis. It is easy to pick a more flexible method but proper choice of the method should be assessed before in order to ensure less biased efficiency estimates are obtained.

This study contributed to the knowledge on efficiency measurement in Kenya and also in Africa, but future research needs to be conducted to add to the information obtained in this study.
References


Press, USA.
Analysis. Population.


StataCorp. (2011). Stata Statistical Software: Release 11. College Station, TX: StataCorp LP.
in Primary Care, 21(6), 345–357.


Appendix A  : Pilot study summary

A pilot study was conducted before the actual data collection for main study. The main aim of the pilot study was to assess the availability and format of data in hospitals. It was conducted in two hospitals: Malindi and Msambweni district hospitals.

Table A.1: Summary of the pilot study

<table>
<thead>
<tr>
<th>Input Variables</th>
<th>Malindi D. Hospital</th>
<th>Msambweni D. Hospital</th>
</tr>
</thead>
<tbody>
<tr>
<td>Staffing levels</td>
<td>Data on patient-care staff (doctors, dentists, pharmacists, COs, nurses, etc.) and non patient-care staff (administration, transport, maintenance etc.) were all available. The only categories that were not available are residents and nursing aids (currently known as nursing officers). Data available from 2000 but better from 2006/2007. 2005 – available 2006 – available for May, June, July &amp; last quarter 2007 – available 2008 – available only for last quarter 2009 – available except last quarter 2010 – available 2011 – available 2012 – available Data is in paper-based format. Information available in the HR data are names, p/no, designation, gender, marital status, date of birth, qualification, date of 1st appointment, date of current promotion, job group, years in the station, terms of service, area of deployment, position held and paying ministry. However, there are some quarters that have a summarized form and not all of the above details.</td>
<td>Data on patient-care staff (doctors, dentists, pharmacists, COs, nurses, etc.) and non patient-care staff (administration, transport, maintenance etc.) were all available. The only categories that were not available are residents and nursing aids (currently known as nursing officers). Data available only as from 2006 with some of the years having only available data for specific quarters. 2005 - None 2006 – available for last quarter only 2007 – available for 1st quarter only 2008 – available for 1st and last quarters 2009 – available for 1st and 3rd quarters 2010 – available for 1st quarter only 2011 – doctors and COs, the rest similar to 2012 2012 – available Data is in paper-based format. There were two forms; staff returns and integrated payroll and personnel database (IPPD). The staff returns form had information on name of employee, p/no, designation and station. This data was only available for only 8 quarter in the period of 2006 – 2012 hence very few. The IPPD data was available for most of the</td>
</tr>
<tr>
<td></td>
<td>Malindi D. Hospital</td>
<td>Msambweni D. Hospital</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>--------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Period however payrolls might underestimate the number of employees in the hospital as this only reflects those paid by government. It also not divided into different job groups.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Wages and salary payment</strong></td>
<td>Within the facility level the only data that is available will be payment to casuals staff as they are paid directly by the hospital. This available as general recurrent expenditure. Government pays the rest of the staff and hence data can only be obtained from national level. The only information that can be obtained from facility would be job groups and this can be linked with level for payment to that job group. Data available from 2005.</td>
<td>Within the facility level the only data that is available will be payment to casuals staff as they are paid directly by the hospital. This available as general recurrent expenditure. There is a muster roll with each casual employee and their salary as from 2005. Data available from 2005.</td>
</tr>
<tr>
<td><strong>Number of hospital beds</strong></td>
<td>Information on number of beds is mostly available but also indicated in some cases are the number of cots. These are categorized into authorized and actual physical. Data is available from 2005.</td>
<td>Information on number of beds is mostly available but also indicated in some cases are the number of cots. These are categorized into authorized and actual physical. Data is available from 2006. Data for some months in the years 2007-2009 were missing.</td>
</tr>
<tr>
<td><strong>Equipment</strong></td>
<td>There was no recorded information on the number of equipment. Data was however available as recurrent expenditure both for purchase and maintenance. Recurrent expenditure data available from 2005. Cost sharing data available from 2009.</td>
<td>There was no recorded information on the number of equipment. Data was however available as recurrent expenditure both for purchase and maintenance. Recurrent expenditure data available from 2008. 2008 data was available for the months of Jul, Sept and Oct only.</td>
</tr>
<tr>
<td><strong>Drugs and supplies</strong></td>
<td>There was no recorded information on the number of drugs and supplies (both pharmaceutical and non-pharmaceutical). Data was available from 2008.</td>
<td>There was no recorded information on the number of drugs and supplies (both pharmaceutical and non-pharmaceutical). Data was available from 2008.</td>
</tr>
<tr>
<td>Malindi D. Hospital</td>
<td>Msambweni D. Hospital</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------------------</td>
<td></td>
</tr>
<tr>
<td>however available as recurrent expenditure.</td>
<td>however available as recurrent expenditure.</td>
<td></td>
</tr>
<tr>
<td>Recurrent expenditure data available from 2005.</td>
<td>Recurrent expenditure data available from 2008. 2008 data was available for the months of Jul, Sept and Oct only.</td>
<td></td>
</tr>
<tr>
<td>Data is in paper-based format.</td>
<td>Data is in paper-based format</td>
<td></td>
</tr>
<tr>
<td>Capital costs</td>
<td>This information was difficult to obtain. The number of key buildings in 2012 was possible to know.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data for previous years more difficult to know as was never recorded.</td>
<td></td>
</tr>
<tr>
<td>Output variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Admissions</td>
<td>Data available in the workload sheet. It has been categorized into general adults, paediatrics, maternity (mothers only) and amenity.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data available from 1994 but data better from 2005 with the year 2006 data missing for the months of Jul, Sept-Dec.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data is in paper-based format. E-form is available for data from Jul 2010</td>
<td></td>
</tr>
<tr>
<td>Outpatient visits</td>
<td>Data available in the workload sheet. Several sub-categories of general outpatient, special clinics, MCHI/FP Clients and dental clinic.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data available from 1994 but better from 2005.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data is in paper-based format. E-form is available for data from Jul 2010</td>
<td></td>
</tr>
<tr>
<td>Length of hospital stay</td>
<td>This is in a separate form from the workload sheet. ALOS has been</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Malindi D. Hospital</td>
<td>Msambweni D. Hospital</td>
</tr>
<tr>
<td>-------------------------</td>
<td>---------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td></td>
<td>calculated as Occupied Bed Days / Total no. of deaths and discharges. Data available from 2008 with the year 2009 missing the month of June</td>
<td>calculated as Occupied Bed Days / Total no. of deaths and discharges. Data available from 2008 with 2009 missing the months of Sept – Dec.</td>
</tr>
<tr>
<td></td>
<td>The data is in <strong>paper-based</strong> format</td>
<td>The data is in <strong>paper-based</strong> format</td>
</tr>
<tr>
<td><strong>Diagnosis/health outcome</strong></td>
<td>This data has not been recorded well. It was a challenge to obtain information, as it has not been summarized. Only information available will individual patient case or probably top ten causes of morbidity.</td>
<td>This data has not been recorded well. It was a challenge to obtain information, as it has not been summarized. Only information available will individual patient case or probably top ten causes of morbidity.</td>
</tr>
<tr>
<td><strong>Mortality</strong></td>
<td>Data available for inpatient cases. This is in the workload sheet. Data available from 1994 but better from 2005. Data missing for the year 2006 in the months of Jul, Sept – Dec.</td>
<td>Data available for inpatient cases. This is in the workload sheet. Data available from 2006. Missing data were: 2007 – available except Aug and September 2008 – available except May 2009 – available for last 2 quarters only 2010 – available for first 2 quarters only</td>
</tr>
<tr>
<td></td>
<td>Data is in <strong>paper-based</strong> format. <strong>E-form</strong> is available for data from Jul 2010</td>
<td>Data is in <strong>paper-based</strong> format. <strong>E-form</strong> is available for data from Jul 2010</td>
</tr>
<tr>
<td></td>
<td>Data is in <strong>paper-based</strong> format. <strong>E-form</strong> is available for data from Jul 2010</td>
<td>Data is in <strong>paper-based</strong> format</td>
</tr>
<tr>
<td><strong>Referrals</strong></td>
<td>Informed that referrals were indicated under Turn Over Interval (TOI). Available from 2008 with 2009 missing the month of June.</td>
<td>This data was not recorded in any of the years.</td>
</tr>
<tr>
<td></td>
<td>Data is in <strong>paper-based</strong> format.</td>
<td></td>
</tr>
<tr>
<td>Service Type</td>
<td>Malindi D. Hospital</td>
<td>Msambweni D. Hospital</td>
</tr>
<tr>
<td>---------------------</td>
<td>--------------------------------------------------------------------------------------</td>
<td>---------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Radiology services</td>
<td>Data in workload sheet. Categorized into simple and special. Later in years 2009, this was further categorized into plain without enhancement, enhancement with contrast media, ultrasound, and special magnetic process (MRI, CT Scan). Available from 1994 but better from 2005. Data is in paper-based format. E-form is available for data from Jul 2010</td>
<td>Data in workload sheet. Categorized into simple and special. Later in years 2009, this was further categorized into plain without enhancement, enhancement with contrast media, ultrasound, and special magnetic process (MRI, CT Scan). Data available from 2006. Missing data were: 2007 – available except Aug and September 2008 – available except May 2009 – available for last 2 quarters only 2010 – available for first 2 quarters only Data is in paper-based format</td>
</tr>
<tr>
<td>Maternity services</td>
<td>Data available in workload sheet and list of various categories available e.g. deliveries, referrals, deaths etc.. Available from 1994 but better from 2005. Data is in paper-based format. E-form is available for data from Jul 2010</td>
<td>Data available in workload sheet and list of various categories available e.g. deliveries, referrals, deaths etc.. Data available from 2006. Missing data were: 2007 – available except Aug and September 2008 – available except May 2009 – available for last 2 quarters only 2010 – available for first 2 quarters only Data is in paper-based format</td>
</tr>
<tr>
<td>Surgical theatre</td>
<td>Data available in workload sheet</td>
<td>Data available in workload sheet</td>
</tr>
<tr>
<td>Malindi D. Hospital</td>
<td>Msambweni D. Hospital</td>
<td></td>
</tr>
<tr>
<td>---------------------</td>
<td>-----------------------</td>
<td></td>
</tr>
<tr>
<td>Data is in paper-based format. E-form is available for data from Jul 2010</td>
<td>Data is in paper-based format</td>
<td></td>
</tr>
<tr>
<td>Casualty</td>
<td>This data was not available. It has always been lumped together with OPD data until 2011 when it was split.</td>
<td></td>
</tr>
<tr>
<td>Emergency</td>
<td>Data has never been recorded. Lumped together with OPD Data has never been recorded. Lumped together with OPD</td>
<td></td>
</tr>
<tr>
<td>Pharmacy (no. of prescriptions)</td>
<td>Data available and categorized into common, drugs, antibiotics, special drug, and drugs for children. Data available from 2005.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data is in paper-based format. E-form is available for data from Jul 2010</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Data is in paper-based format</td>
<td></td>
</tr>
</tbody>
</table>
Appendix B : Hospitals sampled in the study

The following are hospitals that were sampled in the study:

**Coast Province**

**Public District Hospitals**
1. Port-Reiz District Hospital
2. Mariakani District Hospital
3. Malindi District Hospital
4. Likoni District Hospital
5. Kwale District Hospital
6. Msambweni District Hospital

**Faith-based hospitals**
2. Sayyida Fatima, Kisauni
3. Tawfiq Muslim Hospital, Malindi

**Nairobi Province**

**Public District Hospitals**
1. Mbagathi District Hospital

**Faith-based hospitals**
1. St. Mary’s Mission hospital, Langata
2. Jamaa Mission hospital
3. Coptic Hospital, Ngong road

**Central Province**

**Public District Hospitals**
1. Gatundu District Hospital
2. Kiambu District Hospital
3. Murang’a District Hospital
4. Kerugoya District Hospital
5. Tigoni District Hospital
6. Karatina District Hospital
7. Maragua District Hospital
8. Mukurweini District Hospital

**Faith-based hospitals**
1. Kikuyu PCEA hospital
2. Gaichanjiru hospital
3. Githumu hospital
4. Kalimoni hospital
5. Kijabe AIC hospital
6. Kiriaini-Mathiyoya hospital
7. Mwea Mission hospital
8. Nazareth hospital
9. Tumutumu PCEA
10. Mathari Mission hospital
11. Mary Immaculate hospital, Nyeri
12. St. Mulumba hospital

Nyanza province
Public District Hospitals
1. Bondo District Hospital
2. Gucha District Hospital
3. Homa Bay District Hospital
4. Iyabe District Hospital
5. Kisumu District Hospital
6. Kombewa District Hospital
7. Manga District Hospital
8. Masaba District Hospital
9. Migori District Hospital
10. Nyando District Hospital
11. Nyamira District Hospital
12. Rachuonyo District Hospital
13. Rongo District Hospital
14. Siaya District Hospital

Faith-based hospitals
1. St. Monica hospital, Kisumu
2. Christa-Marianne Hospital
3. Kendu-Adventist hospital
4. Maseno mission hospital
5. Nyabondo mission hospital

Map of Kenya – Location of hospitals sampled in the study

Figure B-1: Geographical location of hospitals sampled in the study
## Appendix C: Workload MoH 717 Form - Outpatient services

<table>
<thead>
<tr>
<th>District:</th>
<th>Month:</th>
</tr>
</thead>
</table>

### EUR EPUBLIC OF KENYA

Ministry of Health

Monthly Workload Report for Health Facilities

<table>
<thead>
<tr>
<th>EXPECTED REPORT</th>
<th>TOTAL REPORTS AT DISTRICT LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** Complete every line - leave no blanks. If the health institution does not provide a specific service, write "NS" ("No Service"). If the institution provides the service, but workload data are unavailable, write "NR" ("Not Recorded"). At the end of each month, this form should be completed in 2 copies and delivered by the 5th day of the following month. (January statistics should be posted by 5th February, February statistics by 5th March, and so forth). The copies should be distributed as follows:

### A. OUTPATIENT SERVICES

#### A.1 GENERAL OUTPATIENTS (FILTER CLINICS)

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Over 5 - Male</th>
</tr>
</thead>
<tbody>
<tr>
<td>Over 5 - Female</td>
</tr>
<tr>
<td>Children Under 5 - Male</td>
</tr>
<tr>
<td>Children Under 5 - Female</td>
</tr>
<tr>
<td>TOTAL GENERAL OUTPATIENTS</td>
</tr>
</tbody>
</table>

#### A.2 CASUALTY

#### A.3 SPECIAL CLINICS (if recorded separately from General Filter Clinics)

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| E.N.T. Clinic |
| Eye Clinic |
| TB and Leprosy |
| Sexually Transmitted Infections |
| Psychiatry |
| Orthopaedic Clinic |
| All other Special Clinics |
| (Medicine, Paediatrics, Surgery etc.) |
| TOTAL SPECIAL CLINICS |

#### A.4 MCH/FPP CLIENTS

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| CWC Attendances |
| ANC Attendances |
| PNC Attendances |
| FP Attendances |
| TOTAL MCH/FPP |

#### A.5 DENTAL CLINIC

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Attendances (Excluding fillings and extractions) |
| Fillings |
| Extractions |
| TOTAL DENTAL SERVICES |

#### A.6 TOTAL OUTPATIENT SERVICES

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
</tbody>
</table>

| (= A.1.5 + A.2 + A.3.7 + A.4.5 + A.5.4) |

#### A.7 MEDICAL EXAMINATIONS (except p3)

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| A.10 INJECTIONS |

#### A.8 MEDICAL REPORTS (incl. P3, compensation, insurance, etc)

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| A.11 STITCHING |

#### A.9 DRESSINGS

<table>
<thead>
<tr>
<th>NEW</th>
<th>RE-ATT</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| A.12 P.O.P |
Appendix D: Workload MoH 717 Form - Inpatient, maternity, operations, pharmacy and special services

### B. INPATIENT SERVICES

<table>
<thead>
<tr>
<th>B.1 INPATIENTS</th>
<th>GENERAL ADULTS</th>
<th>GENERAL PEDIATRICS</th>
<th>MATERNITY Mothers Only</th>
<th>AMENITY</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1.1 Discharges</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.2 Deaths</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.3 Absconeees</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.4 TOTAL DISCHARGES, DEATHS, etc.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.9 Admissions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.10 Paroles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.11 Occupied Bed Days- NHI Members</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.11a Occupied Bed Days- Non-NHI Members</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.12 Well Persons Days</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.13 Beds- Authorized</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.14 Beds- Actual Physical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.15 Cots- Authorized</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B.1.16 Cots- Actual Physical</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### B.2 MATERNITY SERVICES

| B.2.1 Vaginal delivery (includes Normal and assisted delivery) | | | | | |
| B.2.2 Caesarian Sections | | | | | |
| B.2.3 Fresh Still Birth | | | | | |
| B.2.4 Macerated Still Birth | | | | | |

### B.3 OPERATIONS

<table>
<thead>
<tr>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.3.1 Minor Surgeries (excluding circumcisions)</td>
</tr>
<tr>
<td>9.3.2 Circumcision</td>
</tr>
<tr>
<td>9.3.3 Major Surgeries</td>
</tr>
</tbody>
</table>

### D. PHARMACY - No. of prescriptions

<table>
<thead>
<tr>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.1 Common Drugs</td>
</tr>
<tr>
<td>D.2 Antibiotics</td>
</tr>
<tr>
<td>D.3 Special Drugs</td>
</tr>
<tr>
<td>D.4 For Children</td>
</tr>
</tbody>
</table>

### E. MORTUARY

<table>
<thead>
<tr>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.1 Body days</td>
</tr>
<tr>
<td>E.2 Embalment</td>
</tr>
<tr>
<td>E.3 Post-mortem</td>
</tr>
<tr>
<td>E.4 Unclaimed body days</td>
</tr>
</tbody>
</table>

### F. MEDICAL RECORDS ISSUED

| F.1 New Files |
| F.2 Outpatient records |

### C. SPECIAL SERVICES (includes both inpatients and outpatients)

<table>
<thead>
<tr>
<th>C.1 Laboratory- Number of Tests</th>
<th>Routine</th>
<th>Special</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>C.2 X-Ray- Number of Examinations</td>
<td>Plain without enhancement</td>
<td>Enhancement with contrast media</td>
<td>Ultrasound</td>
</tr>
<tr>
<td>C.3 Physiotherapy - Number of Treatments</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.4 OCCUPATIONAL THERAPY</td>
<td>Private</td>
<td>Non-private</td>
<td>Total</td>
</tr>
<tr>
<td>C.5 Orthopaedic Technology - Orthopaedic Technology - No of ITEMS e.g a pair of crutches, Prostheses etc count as one item</td>
<td>Private</td>
<td>Non-private</td>
<td>Total</td>
</tr>
</tbody>
</table>

Name  | Signature  | Date  | Designation |
--- | --- | --- | --- |
Prepared by: | | | |
Checked by: | | | |
Entered by: | | | |
<table>
<thead>
<tr>
<th>PROVINCE</th>
<th>COAST</th>
<th>DISTRICT</th>
<th>MAUNDI</th>
<th>FACILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAME</td>
<td>DAYS</td>
<td>SEDS</td>
<td>COTS</td>
<td>ADM</td>
</tr>
<tr>
<td></td>
<td>DISCH</td>
<td>T/M</td>
<td>T/OUT</td>
<td>DEATHS</td>
</tr>
<tr>
<td></td>
<td>ABC</td>
<td>REFER</td>
<td>D&amp;D</td>
<td>WELL</td>
</tr>
<tr>
<td>PEOPLE</td>
<td>VBD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ORB</td>
<td>ABD</td>
<td>% OCC</td>
<td>ALOS</td>
</tr>
<tr>
<td></td>
<td>AV. OCC</td>
<td>TOE</td>
<td>TOI</td>
<td></td>
</tr>
</tbody>
</table>

**Male** (1)

**Female** (2)

**Paediatric**

**Maternity**

**Total**

<table>
<thead>
<tr>
<th>INDICATOR</th>
<th>VARIABLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Average length of stay</td>
</tr>
<tr>
<td>2</td>
<td>Percentage age occupancy</td>
</tr>
<tr>
<td>3</td>
<td>Average occupancy</td>
</tr>
<tr>
<td>4</td>
<td>Turn over interval</td>
</tr>
<tr>
<td>5</td>
<td>Percentage of well people to patients</td>
</tr>
</tbody>
</table>
Appendix F: Monthly Payment and Commitment Summary

<table>
<thead>
<tr>
<th>Expense Item Code</th>
<th>Expense Item Description</th>
<th>New A.I.E.s Received This Month</th>
<th>Cumulative A.I.E.s This Financial Year</th>
<th>Payments Made This Month</th>
<th>Cumulative Payments This F.Y.</th>
<th>New Commitments Made This Month</th>
<th>Outstanding Commitments This F.Y.</th>
<th>Available Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>501</td>
<td>Transfers to MOH</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>502</td>
<td>Transfers to PMO</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Date of Reconciliation of Vote Book with District Accountant's Vote Book

For Facility, MOH or PMO: ____________________________
For District Accountant: ____________________________
Entered By: ____________________________
Signature: ____________________________
Date: ____________________________
Print (or Handwritten) Name: ____________________________
### Staff Returns Form

<table>
<thead>
<tr>
<th>D/NO</th>
<th>ADMINISTRATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NAME(S)</td>
</tr>
<tr>
<td></td>
<td>D/NO</td>
</tr>
<tr>
<td></td>
<td>DESIGN/JR. GROUP</td>
</tr>
<tr>
<td></td>
<td>GENDER</td>
</tr>
<tr>
<td></td>
<td>MARRITAL STATUS</td>
</tr>
<tr>
<td></td>
<td>DATE OF BIRTH</td>
</tr>
<tr>
<td></td>
<td>QUALIFICATION/CADE/HIGHEST</td>
</tr>
<tr>
<td></td>
<td>DATE OF EST. APP.</td>
</tr>
<tr>
<td></td>
<td>DATE OF CURRENT PROMOTION</td>
</tr>
<tr>
<td></td>
<td>JOB GROUP</td>
</tr>
<tr>
<td></td>
<td>YEARS IN THE STATION</td>
</tr>
<tr>
<td></td>
<td>TERMS OF SERVICE</td>
</tr>
<tr>
<td></td>
<td>AREA OF DEPLOYMENT</td>
</tr>
<tr>
<td></td>
<td>POSITION HELD &amp; INCREASE</td>
</tr>
<tr>
<td></td>
<td>MVC/PROGRAMME/DEPARTMENT</td>
</tr>
<tr>
<td></td>
<td>REMARKS/FAYING MINISTRY</td>
</tr>
</tbody>
</table>

**Appendix G**
Appendix H : Second Stage Analysis – Tobit Model

The first stage in DEA is estimating the frontier and the efficiency scores of the hospitals. Further analysis in exploring whether the efficiency scores are affected by environmental factors is done in the second stage analysis. In this study, ownership was assessed to check if there is any effect. One way of checking this is by applying non-parametric tests to assess whether there are any differences between the two types of ownership. The other common method is the Tobit regression. This section describes the theoretical model of the Tobit regression.

Tobit regression is similar to OLS except that the noise term is truncated. A simple regression model takes the form:

$$ y = \beta_0 + \beta_i x_i + \epsilon $$

( H.1 )

where $y$ is the dependent variable regressed against independent variables $x$. $\beta$'s are unknown parameters and $\epsilon$ is a random error term that does not reflect efficiency levels. This is the main disadvantage of OLS. The Tobit model for censored regression model can help solve this problem. The dependent variable in the Tobit model is either zero or positive. The Tobit regression model is as follows:

$$ tobit (y_i) = \alpha_0 + \alpha_1 x_{i1} + \alpha_2 x_{i2} + \cdots + \epsilon_i $$

( H.2 )
where $Y_i$ is the efficiency score for the $j^{th}$ hospital, $x$ is the explanatory variable (for example ownership), $\alpha$s are the unknown parameters and $\xi_i$ are the disturbance error term assumed to be normally distributed with mean $\mu$ and standard deviation $\sigma$. 
Appendix I: Robust Estimators and Heteroscedasticity of the Cobb-Douglas Production Function

In order to check for the assumption of homoscedasticity, OLS estimators are compared to White’s robust estimates. This is the heteroscedasticity corrected covariance matrix for the least square estimates. The standard errors are comparable between standard OLS and robust estimates (Table 1.1). The Breusch and Pagan Lagrange multiplier (LM) chi-square test statistic was 113.74 with 7 degrees of freedom. The p-value was 0.0000 indicating that the assumption homoscedasticity was rejected.

Re-estimating the linear regression with a cluster correction when panel data aspect is included produced slightly lower standard error estimates than OLS and White’s robust estimates.

Carrying out a Wald test to check for joint significance of the variables, showed a chi-square test of 22.35 with 7 degrees of freedom and p-value of 0.0000. This means that the null hypothesis that the variables were not significant jointly is rejected implying that all variables are important and should be maintained in the model.
Table I.1: OLS, White’s robust and cluster Cobb-Douglas production function estimates

<table>
<thead>
<tr>
<th>Output index</th>
<th>OLS Estimates</th>
<th>White’s Robust estimates</th>
<th>Cluster estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Doctors</td>
<td>0.092</td>
<td>0.092</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>(0.033)**</td>
<td>(0.033)**</td>
<td>(0.023)**</td>
</tr>
<tr>
<td>Nurses</td>
<td>0.766</td>
<td>0.766</td>
<td>0.766</td>
</tr>
<tr>
<td></td>
<td>(0.057)**</td>
<td>(0.074)**</td>
<td>(0.042)**</td>
</tr>
<tr>
<td>Clinical Officers</td>
<td>0.234</td>
<td>0.234</td>
<td>0.234</td>
</tr>
<tr>
<td></td>
<td>(0.061)**</td>
<td>(0.057)**</td>
<td>(0.038)**</td>
</tr>
<tr>
<td>Other health workers</td>
<td>-0.218</td>
<td>-0.218</td>
<td>-0.218</td>
</tr>
<tr>
<td></td>
<td>(0.047)**</td>
<td>(0.051)**</td>
<td>(0.022)**</td>
</tr>
<tr>
<td>Other staff</td>
<td>-0.070</td>
<td>-0.070</td>
<td>-0.070</td>
</tr>
<tr>
<td></td>
<td>(0.032)*</td>
<td>(0.033)*</td>
<td>(0.025)*</td>
</tr>
<tr>
<td>Expenditure</td>
<td>0.085</td>
<td>0.085</td>
<td>0.085</td>
</tr>
<tr>
<td></td>
<td>(0.024)**</td>
<td>(0.024)**</td>
<td>(0.020)**</td>
</tr>
<tr>
<td>Beds</td>
<td>-0.076</td>
<td>-0.076</td>
<td>-0.076</td>
</tr>
<tr>
<td></td>
<td>(0.042)</td>
<td>(0.050)</td>
<td>(0.035)*</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.909</td>
<td>-3.909</td>
<td>-3.909</td>
</tr>
<tr>
<td></td>
<td>(0.298)**</td>
<td>(0.302)**</td>
<td>(0.241)**</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.77</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td>$N$</td>
<td>432</td>
<td>432</td>
<td>432</td>
</tr>
</tbody>
</table>

* $p<0.05$; ** $p<0.01$
Appendix J: Normality of the least square residuals

This section shows results on the (non) normality of the least squares. Initially, the OLS residuals were computed and normality checked. Figure J-1 shows the density plot of a normal curve and the OLS residuals. The distribution of the OLS residuals is bell-shaped but there is a noticeable skew. In order to check for non-normality Chi square test was used. The results showed a chi-square of 4.71 with p-value 0.0957 indicating that there is limited evidence to reject the null hypothesis that distribution is normal.

Figure J-1: Density curve of OLS residuals and Normal curve
Appendix K: Correlation different distributional assumptions

Vuong statistic is a non-nested test used to compare two models. A large positive test statistic provides evidence of superiority of the first model over the second one and a large negative test statistic is evidence of superiority of the second model over first. If the test statistic is small, then the models are indistinguishable as showed in Table K.1. This supports the hypothesis that the choice of the one-sided error distribution might not affect the efficiency results.

Table K.1: Correlation between distributions of the one-sided error term

<table>
<thead>
<tr>
<th>Error Distributions</th>
<th>Spearman rank Correlation</th>
<th>Vuong Statistics</th>
<th>Preferred Error distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half-normal Exponential</td>
<td>0.9725</td>
<td>-1.246</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>Half-normal Truncated</td>
<td>0.9729</td>
<td>-1.256</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>Half-normal Gamma</td>
<td>0.9557</td>
<td>-1.093</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>Exponential Truncated</td>
<td>0.9999</td>
<td>0.305</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>Exponential Gamma</td>
<td>0.9952</td>
<td>-0.547</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>Truncated Gamma</td>
<td>0.9951</td>
<td>-0.549</td>
<td>Inconclusive</td>
</tr>
</tbody>
</table>
Appendix L: DEA output-oriented efficiency scores

In the output-oriented DEA model, the efficiency scores under the CRS assumption are similar to input-oriented model. In the 2011 data, the average efficiency under the VRS assumption on the other hand had mean score of 0.8736 with a SD of 0.1818. There were 14 hospitals that were on the frontier in the output-oriented model and the same hospitals were also on the frontier in the input-oriented model. A total of 6 hospitals were scale efficient in the output-oriented model. Similar to the input-oriented results, hospitals 2, 5, 6, 9, 11 and 24 were on the frontier under both the CRS and VRS assumption and they were also scale efficient in the output-oriented model.

The 2012 data also exhibit similar pattern and results with average scores with a mean value of 0.7624 under CRS assumption and 0.8719 in the VRS assumption. Twelve hospitals were lying on the frontier under the VRS assumption. The mean scale efficiency score was 0.8696 with SD of 0.1262.

The results from the two years show similar patterns. Hospitals 2, 5, 6 and 11 were considered to be on the frontier (fully efficient) and had a scale efficiency of 1.0 in both input and output orientation assuming either CRS or VRS. The number of hospitals in the different efficiency score ranges is shown in Figure L-1. The output-oriented assumption efficiency score results for each hospital are summarized in Table L.1.
Number of hospitals in different efficiency score ranges in an output-oriented model

<table>
<thead>
<tr>
<th>Efficiency Score Range</th>
<th>2011</th>
<th>2012</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 ≤ E &lt; 0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3 ≤ E &lt; 0.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4 ≤ E &lt; 0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5 ≤ E &lt; 0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6 ≤ E &lt; 0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.7 ≤ E &lt; 0.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8 ≤ E &lt; 0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9 ≤ E &lt; 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E = 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure L-1: Efficiency score ranges in an output-oriented model
Figure L-2 shows the distribution of relative efficiency scores under the VRS assumption.

The distributions indicate similarities in the input and output oriented models with both 2011 and 2012 datasets.
Generally there was a strong and significant correlation between the ranking of the hospitals in the two data sets in the input and output oriented models (Table L.2). This should be expected as both orientation models estimate the same frontier and identify similar set of efficient DMUs (Coelli et al., 2005). The efficiency scores of the inefficient DMUs might differ but not significantly as shown in some studies (Rajasekar & Deo, 2014; Varabyova & Schreyögg, 2013).
Table L.2: Correlation of the hospital ranks under the VRS assumption

<table>
<thead>
<tr>
<th>Year</th>
<th>Input Oriented</th>
<th>Output Oriented</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2011</td>
<td>2012</td>
</tr>
<tr>
<td><strong>Input Oriented</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2012</td>
<td>0.6766</td>
<td>1</td>
</tr>
<tr>
<td><strong>Output oriented</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2011</td>
<td>0.9778</td>
<td>0.7014</td>
</tr>
<tr>
<td>2012</td>
<td>0.6619</td>
<td>0.9873</td>
</tr>
</tbody>
</table>
Appendix M: Assessment of efficient hospitals

Super efficiency scores

Analysis of super efficiency scores is a way of differentiating among frontier hospitals. Super efficiency scores examine the change in which the hospitals lying on the frontier remain relatively efficient. The higher the value of the super efficiency score, the higher the ranking of the hospital among the relative efficient ones. Super efficiency score can be obtained for both inefficient and efficient hospitals but the scores remain the same for the inefficient ones and higher values are obtained for the efficient units. The values for super efficiency scores are therefore not restricted to a maximum of 1.0 (for efficient hospitals) but can take a value ≥ 1.

Table M.1 outlines the individual hospital super efficiency scores across the input and output oriented models under both CRS and VRS assumptions. The hospitals that had the traditional efficiency of 1 can be differentiated with different values above 1 indicating that they could have reduced their inputs or outputs and still not dominated by the feasible reference hospital.
Table M.1: Super Efficiency Scores for hospitals on the frontier

<table>
<thead>
<tr>
<th></th>
<th>2011 Input Oriented</th>
<th>2011 Output Oriented</th>
<th>2012 Input Oriented</th>
<th>2012 Output Oriented</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CRS</td>
<td>VRS</td>
<td>CRS</td>
<td>VRS</td>
</tr>
<tr>
<td>DMU</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.1438</td>
<td>1.2401</td>
<td>1.1438</td>
<td>1.4039</td>
</tr>
<tr>
<td>3</td>
<td>N/A</td>
<td>1.5083</td>
<td>N/A</td>
<td>Inf</td>
</tr>
<tr>
<td>5</td>
<td>1.4395</td>
<td>0.7059</td>
<td>1.4395</td>
<td>1.4420</td>
</tr>
<tr>
<td>6</td>
<td>1.1904</td>
<td>1.5407</td>
<td>1.1904</td>
<td>Inf</td>
</tr>
<tr>
<td>7</td>
<td>N/A</td>
<td>1.2684</td>
<td>N/A</td>
<td>1.1851</td>
</tr>
<tr>
<td>8</td>
<td>N/A</td>
<td>Inf</td>
<td>N/A</td>
<td>Inf</td>
</tr>
<tr>
<td>9</td>
<td>1.0012</td>
<td>1.3275</td>
<td>1.0012</td>
<td>1.2578</td>
</tr>
<tr>
<td>11</td>
<td>4.4919</td>
<td>4.5240</td>
<td>4.4919</td>
<td>Inf</td>
</tr>
<tr>
<td>12</td>
<td>N/A</td>
<td>1.2150</td>
<td>N/A</td>
<td>1.1373</td>
</tr>
<tr>
<td>14</td>
<td>N/A</td>
<td>0.4780</td>
<td>N/A</td>
<td>1.0320</td>
</tr>
<tr>
<td>20</td>
<td>N/A</td>
<td>1.2438</td>
<td>N/A</td>
<td>1.2578</td>
</tr>
<tr>
<td>26</td>
<td>N/A</td>
<td>2.0752</td>
<td>N/A</td>
<td>Inf</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hospitals on the frontier</td>
<td>6 14</td>
<td>6 14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N/A means that the hospitals were not on the frontier under the particular assumption Inf – infeasibility
However, this method has a drawback where the super efficiency results might not have a feasible solution. Under the CRS assumption, both models had feasible solutions for data from both 2011 and 2012. However, under the VRS assumption, hospital 8 had infeasible solution in the input oriented model for both 2011 and 2012. This result was also the same in the output-oriented model with hospitals 3, 6, 11 and 26 using 2011 and 2012 data and additional hospital 27 in the 2012 data. Infeasibility means that there are no other hospitals within which to assess the particular hospital with an infeasible solution.

**Slack positive efficient hospitals**

There are instances where reduction of inputs or augmentation of outputs is not sufficient. A gap might still exist even when inputs are reduced or outputs are increased. In data envelopment analysis, this is referred to as a slack, which is the excess input or less output even after proportional change in the inputs and outputs. The existence of slack in the hospitals that lie on the frontier can be assessed. After modifying the efficiency scores with the slack-based measure, none of the hospitals on the frontier had a positive slack. Meaning that there was no excess reduction in inputs or increase in outputs required for these hospitals.

**Peers**

If the input and output oriented models and assuming a variable returns to scale, one can identify hospitals that can be compared to each other (Table M.2 shows the different potential peers for the hospitals. Hospitals can be compared with other potential peers but for cases of hospitals that lie on the frontier (fully efficient), they are compared to themselves e.g. hospital 8.
Table M.2: Peers for individual hospitals assuming VRS in input and output oriented models

<table>
<thead>
<tr>
<th>Hospital</th>
<th>Peer 1</th>
<th>Peer 2</th>
<th>Peer 3</th>
<th>Peer 4</th>
<th>Peer 5</th>
<th>Peers</th>
<th>Peer 1</th>
<th>Peer 2</th>
<th>Peer 3</th>
<th>Peer 4</th>
<th>Peers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>6</td>
<td>11</td>
<td>NA</td>
<td>NA</td>
<td>Same</td>
<td>6</td>
<td>11</td>
<td>24</td>
<td>NA</td>
<td>6,11,23</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>Same</td>
<td>2</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>Same</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
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<td>NA</td>
<td>NA</td>
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<td>Same</td>
<td>3</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>Same</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>6,11</td>
<td>6</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
<td>6,11</td>
</tr>
<tr>
<td>5</td>
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*O-O: Outputs oriented model column compared to the input oriented model. Same means that the peers are the same in both the input and output oriented models.