Choosing the future: A level mathematics

In this chapter I look at two questions that concern policy and research in post-compulsory mathematics. The questions are:

- How can we understand and compare different mathematical courses of study?
- How can we understand students' choices to study mathematics?

As in most long-lasting education debates, the answers are not simple: they depend on how you approach - and theorise - education, what you think is convincing evidence, and whether you are setting out to understand mathematics in schools or to change it. I discuss these ideas in the context of A-levels: the standard university preparation in England and Wales where 16-year-olds choose typically only three or four subjects to study over two years. In doing so I introduce some of my own research, notably my perspective that the ways that we understand and try to change school mathematics are not just about mathematics but also about how we view adolescence and progress.

The first occasion when UK students make a choice about studying mathematics is at age 16. And when they do get that choice, they use it. Research into students' experiences of learning mathematics suggests that many are profoundly disengaged from their mathematics lessons, at all levels of achievement, and simply do not want to continue. As one oft-quoted student puts it, "I would rather die!" (Brown, Brown, and Bibby 2008), while another less dramatically comments “You have done it every year and it just gets kind of tiring” (Murray 2011, 278).

These findings of stable and widespread disengagement suggest a downturn in A-level mathematics. In fact, examination board data over the last fifteen years has shown a dramatic fall but then rise in the number of candidates (Figure 1). It is clear that students' choices do not depend just on past experiences but also involve their future aspirations and society's expectations. Families, schools and governments understand choosing mathematics as a way to brighten individuals' life prospects and the whole country's economic future. One of the roles of mathematics education research is to examine how the curriculum could (or should) meet these economic and political goals in the light of our knowledge about students’ learning.

Think of an adult colleague who chose to stop studying mathematics at 16. What arguments might convince him or her that mathematics should have remained compulsory?

How would you argue differently to convince a current 16 year-old?

Since 2008 there have been three mathematics A-levels, although (capital-M) Mathematics is by far the most common, with 70,000 students completing in 2011. Representing 9% of all A-levels taken, this was the highest proportion since 1996. It is notoriously hard to match examination data over time, but this is probably below 1980s numbers. The second A-level, Further Mathematics, is a traditional option for students who want to extend their mathematics. It has also been viewed as an elite option available only in a minority of schools, mostly large or private ones (Matthews and Pepper, 2007). Recently numbers have risen (Figure 1) after government-funded promotion by the Further Mathematics Support Programme (FMSP). Further Mathematics is of particular interest because of its gateway function for high-status STEM fields of science, technology and engineering and mathematics. This gives it a prominent role in the processes of including and excluding students through mathematics (Smith 2010).
In contrast, Use of Mathematics (UOM), the third A-level I consider here has existed only as a restricted pilot from 2008-12, with fewer than 1500 candidates. UoM is designed as an alternative to Mathematics. Its core syllabus includes algebra, functions and calculus topics recognisable from the AS-level Mathematics syllabus. However, teaching focuses on skills of mathematical modelling and communication across context-related problems rather than fluency in the techniques needed for STEM applications. It is the least common A-level, but worth discussing as it is often mentioned as a basis for a compulsory post-GCSE pathway for students who do not intend to follow STEM courses (ACME 2010).

Potentially then, current A-levels could provide three parallel courses for pre-university study: 'parallel' in their timing but pitched at different conceptions of mathematics and its uses. These particular examples are specific to A-levels, but we find similar stratifications and purposes in other western education pre-university systems, for example Australia, or the European and International Baccalaureates. Of course, in most Baccalaureate systems mathematics is compulsory, while in the UK (and Australia) students enjoy more autonomy. However, this self-determination comes with the responsibility to make – and be seen to make - successful choices and to achieve the grades that prove them. It is a combination of freedom and self-scrutiny that follows the contemporary western model of neoliberal governance, through which individuals’ will to act is acknowledged and utilised to reinforce institutional practices (Rose 1999).

THE QUESTION OF EQUIVALENCE

We are used to thinking of mathematics as a hierarchical subject, where each topic relies on knowledge lower down in the hierarchy. When we do recognise strands, we talk of organising them in a spiral curriculum, with topics building on each other and increasing in complexity and abstraction. So in describing the three courses above, it made sense to acknowledge their different starting and end points: Further Mathematics extends Mathematics, and UoM overlaps with it. Implicitly, then, we think of mathematics courses as lying along a single dimension. Unidimensionality makes them easy to compare and hard to claim as equivalent.

However, we are also used to thinking that summative assessments should be equivalent between subjects and over time. If we want to understand different routes within mathematics A-levels we have to consider how to reconcile the evident hierarchy within
mathematics with the social aim that assessments are worth the same over time and between subjects (Coe et al. 2008).

First, why do we think of A-levels as equivalent? One reason is because many of the institutional structures of schools suggest it. We can see this in how the curriculum is organised into timetable slots, in the spread of eclectic mix-and-match subject combinations, in how schools and students compare module results, and universities ask for ‘ABB or equivalent’. On top of this we have examples in other subjects. French, Spanish and German are equivalent modern languages; it is no better to be 'Late' than 'Early' when it comes to parallel Modern History A-levels. The wider practices of the A-level curriculum emphasise equivalence and ensure that students and schools are not unduly restricted (and academic disciplines do not lose out) when options narrow to only three or four subjects.

Against these institutional assumptions, we have the hierarchy of disciplines and the social judgements of employers and universities. In 2011 the elite Russell group of universities published a list of ‘facilitating’ A-levels. These included Mathematics and Further Mathematics and excluded newer A-levels such as Law. The existence of elite preferences is not surprising; it is part of the ‘grapevine’ knowledge that sustains middle-class educational privilege (Ball, Maguire, and Macrae 2000). However, it re-opened arguments that A-level subjects do have different intrinsic values, and specifically that mathematics needs more recognition and must be defended from innovation. The pilot A-level UoM was a particular focus of criticism. Educators for Reform (2009) suggested that “poorly-informed students – in particular those at the weaker schools” would be distracted by the implied equivalence, choose the wrong AS-level and end up excluded from further STEM study.

These same concerns both for and against equivalence are repeated within Mathematics A-level. The full A-level has four compulsory pure modules plus two applied options chosen from lists in Mechanics, Statistics and Decision Mathematics. This pair can be combined to give breadth (Mechanics 1 and Statistics 1) or depth (Statistics 1 and 2) or to reflect teacher expertise. Many schools allocate students in relation to other subjects so that physicists study mechanics and geographers study statistics. Universities have supported this diversity because it allows the range of applications to flourish, particularly mechanics which is difficult without physics. However a government review of A-level (Matthews and Pepper 2007) reports diversity as a problem, citing evidence that teachers and students treated Decision Mathematics as an easier option. They were particularly concerned that universities may penalise students for choosing an easier option when it is the school that did so.

We can see here how the temptingly mathematical notion of equivalence becomes enmeshed with political significance. Arguments for and against equivalence call on ideas of equity and of protecting students from unexpected consequences. For some, the problem lies in subject choice itself, and these arguments support a baccalaureate system and/or compulsory mathematics. For others individual choice is not inherently problematic but requires managing so that students are responsible only for their own decisions, not other people’s. This individualisation feels appealing, but we have seen in Andy Noyes and Peter Gates’s earlier chapter how social class inequalities are reinforced by treating students only in terms of school-based identities.

We can also see equivalence being used as a defence when students choose ‘easier’ but not ‘harder’ mathematics, so that UoM is criticised while Further Mathematics is promoted. In the next section I look at another way of unpicking the unidimensionality perspective by comparing examination questions.

COMPARING A LEVEL QUESTIONS
The variables \( x \) and \( y \) are related by an equation of the form
\[
y = ax + \frac{b}{x + 2}
\]
where \( a \) and \( b \) are constants.

(a) The variables \( X \) and \( Y \) are defined by \( X = x(x+2) \), \( Y = y(x+2) \).
Show that \( Y = aX + b \). (2 marks)

(b) The following approximate values of \( x \) and \( y \) have been found:

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<td>3.35</td>
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(i) Complete the table in Figure 1, showing values of \( X \) and \( Y \). (2 marks)

(ii) Draw on Figure 2 a linear graph relating \( X \) and \( Y \). (2 marks)

(iii) Estimate the values of \( a \) and \( b \). (3 marks)
Maria has completed a university course and has a student loan of £10 000 to repay. Each year, Maria has to pay back 9% of anything she earns over £15 000 during that year.

Throughout this question, you may assume that interest is not charged on a student loan.

Maria earns £25 000 in the first year and gets an increase in her earnings of £2000 per year in each subsequent year.

The following recurrence relations are used to model this situation until the loan is repaid:

- \( E_n = E_{n-1} + 2000 \) gives Maria’s earnings each year \((E_1 = 25 000)\);
- \( R_n = 0.09(E_n - 15 000) \) gives the amount Maria repays each year;
- \( L_n = L_{n-1} - R_n \) gives the amount of Maria’s loan that remains at the end of each year \((L_0 = 10 000)\).

(a) Given that \( E_1 = 25 000 \) and \( L_0 = 10 000 \), show that:
   (i) \( R_1 = 900 \);  
   (ii) \( L_1 = 9100 \).  
   (4 marks)

(b) Use the recurrence relations to complete the table on the answer sheet.  
(6 marks)

(c) How long will it take Maria to pay back her student loan?  
(1 mark)

(d) Complete the graph of loan remaining, \( L_n \), plotted against year, \( n \), on the grid on the answer sheet.  
(2 marks)

(e) Explain why the points on your graph do not lie on a straight line.  
(2 marks)

Question B (AQA 2008b)

I selected these two questions because they have a similar structure. Both ask the student to complete a table of values with pre-labelled columns for relevant algebraic variables, and both provide a set of labelled axes for plotting a graph. As well as these similarities, you will have identified a range of differences between them. Let's discuss those that are characteristic.

Question A is from Further Mathematics, and this is evident in the abstraction of the mathematical components and the links between them. Here the variables \( a, b, x \) and \( y \) are all introduced together and operated on together. In two sub-questions the students need to devise a strategy: part (a) algebraic manipulation to show \( ax + b \), and part (ciii) reading off graphs to estimate gradient and intercept. Each of these requires technical comprehension. Apart from these parts, the necessary information is linked closely to the question, and responses need little organisation.

Question B is from UoM. This is evident from the foregrounding of the finance context, and the way that variables are connected through calculation rather than algebraic substitution. Initially, after introducing \( E_n \) variables are defined one at a time. In (b) they require evaluating in a different order, by year, and rounding off. Students also need to devise a strategy in part (c) to move from table to context, and in (e), to comment on the graph. The suggested answer is 'Maria does not pay off equal amounts each year', which requires some
technical comprehension of gradient, and organising a response. Information is given early in the text, where it is distant from where it is needed, but is often repeated close by.

Comparing these two questions shows that they do have distinctive features that reflect the courses' purposes and different classroom practices. There is a hierarchy of complexity: Further Mathematics is concerned with abstract relationships and algebraic manipulation, while UoM focuses on producing and interpreting numerical and graphical representations. But there is also evidence here to challenge a unidimensional model: UoM has weaker links between the question and the information needed to answer it. It demands that students read and write about mathematics in a way that is absent in the pure questions of Further Mathematics or indeed in Mathematics.

In the end, what strikes me most about analysing these questions is how similar they are despite their different purposes. Both use a structure and wording that restrict opportunities for students to select strategies, to move back and forth between representations and to organise responses. They signal a very clear procedure for candidates and mark-schemes to follow. That is not to say that candidates would find the questions easy: many would not. However, this perception that examinations systematically reduce variation lies behind calls for mathematics curricula to move away from a hierarchy of technical fluency and towards problems that prepare students to think mathematically (London Mathematical Society 2010).

The analytic framework I introduced here is drawn from studies that compare the demands of examination questions through their structure (Hughes, Pollitt, and Ahmed 1998), their mathematical steps, complexity and familiarity (Kathotia 2012) and grammatical complexity (Morgan, Tang, and Sfard 2011). You might wonder why recent research does not try the intuitive experiment of assessing demand by comparing the performance of similar groups of students. A little thought shows us that it is not so easy. Firstly, we meet problems in defining the ways in which the groups should be similar. Secondly, these are not questions that can be accessed without teaching. Although it would be politically popular, it is impossible to design an experiment that can distinguish whether differences in performance are due to easier examinations or the fact that students have better learning experiences (Coe et al. 2008).

**STUDYING MATHEMATICS**

Research into student experiences gives another perspective through which to understand different mathematics courses. Studies agree that the students most likely to study A-level Mathematics are students who have already achieved well at GCSE, and are thus more likely to have higher socioeconomic status. Exploring the interactions of gender, ethnicity, class and prior attainment leads to intriguing variations, with one being that students from certain ethnic groups (Chinese, African, Indian and Pakistani) are more likely to choose mathematics regardless of attainment (Noyes 2009; Reiss et al. 2011).

This gives us a background for the Transmaths project (Wake, Williams, and Drake 2011), that tracked non-traditional participants in AS-level mathematics, i.e. students with B or C grades at GCSE mathematics. They found UoM students were quite different from the students who studied Mathematics: unlikely to choose STEM careers and more diverse in ethnicity, class and prior attainment. Those with the lowest GCSE grades made more progress than similar Mathematics students, and were less likely to drop out or get an ungraded result. We are back to unpicking whether UoM is easier or more engaging.

Transmaths points to the impact of lesson activities preparing for UOM coursework tasks. Students found it refreshing to research what mathematics to use, to make and connect meanings for themselves, and to explain mathematics to others. They talked about the challenge but also how these lessons increased their understanding. From this research we
get another response to the criticisms above: different mathematics courses should not be viewed as competing for students. UoM has proved engaging for a particular group of learners and successfully made it easier for them. There are issues in generalising from these experiences to a wider school population, not least because coursework has finished. Nevertheless there are many students with Bs or Cs in GCSE mathematics who currently study no more mathematics, and UoM may prove an instructive model in how to engage them.

Comparing Further Mathematics and Mathematics has been much less controversial. STEM employers, universities and schools largely agree that further mathematics should be similar to mathematics just more extreme: deeper in its abstraction, broader in its applications and sharper in its eventual function of selection. Further mathematics is even more closely linked with the “clever core” of high-attaining students, and is popular amongst students from professional backgrounds and Asian ethnicities (Matthews and Pepper 2007). My own research has followed the promotion of Further Mathematics in state schools, investigating the accounts of non-traditional students who took up Further Mathematics as a twilight-hours course. Unlike the research above, I take a poststructural approach that considers what identities are offered and formed in these experiences. Students’ accounts of doing mathematics have to make sense of the practices they engage in and also the fact that they chose/choose to participate, and because of this reflexivity I consider experience and choice together.

Think back to when you chose to continue mathematics, perhaps to A level or degree standard. What kind of decision was this? Who influenced you?

Would you describe your choice as rational - weighing up the costs and benefits? Or as expressive - reflecting who you are, setting out who you want to be?

**WHY CHOOSE AN A-LEVEL IN MATHEMATICS OR FURTHER MATHEMATICS?**

Students who describe their reasons for choosing Mathematics A-level usually give one or more of the following:

1. mathematics leads to good careers,
2. mathematics is useful,
3. I have always been good at maths,
4. I enjoy mathematics,
5. mathematics is hard.

Which of these reasons entered into your decisions?

Calling these 'reasons' suggests that choosing is a practice of weighing up contributory factors, some concerned with mathematics, some with the social world, and some the individual chooser. In this approach to theorising choice; the next step is to identify such factors, and to investigate how changing the image of mathematics, the practices of society and students' self-perceptions, would make a difference to choice 'calculations'. Debate in this area is often quasi-statistical in tone, centred on identifying which factors are significant and how they interact. For example, the political think-tank Reform has suggested that mathematics should have more university-entry points than other subjects so students understand its higher value (Kounine, Marks and Truss 2008). This would change social practices (university entrance procedures) in order to affect the image of mathematics; it is a 'policy lever' that influences people without constraining them. Another example is often
suggested in response to girls' continued lower participation in A-level Mathematics (stuck around 40% since 2001): adopting classroom teaching methods that promote confidence and enjoyment for girls would encourage them to stay on (Brown, Brown, and Bibby 2008).

Nevertheless it seems likely that choice is more complex, because sociological research tells us that choices are both rational and expressive (Rose, 1999). Choosing articulates what we think is important, and that gives a message about who we are. In the list above, the first two reasons – 'mathematics leads to good careers', and 'mathematics is useful' - are about the role of mathematics in a western technological society. When students choose mathematics for these reasons they align themselves with high-status, powerful positions in society and with modernist views of progress and utility. We can see here how easily the qualities of mathematics in society can be read onto the choosing student as showing a self-entrepreneurial spirit.

What about the next two reasons in the list? 'I have always been good at mathematics' and 'I enjoy mathematics' are both qualities of the student. Using knowledge about yourself is a widely accepted rationale for making life-choices. Still, for these to be valid factors in choosing mathematics, we need to understand them as stable characteristics, unlikely to change or to become irrelevant as learning shifts to A-level. The first in particular suggests that mathematical ability resides in a person, and makes that person apt for any formulation of school mathematics. The earlier chapter by Mark Boylan and Hilary Povey interrogates this fixed conception of mathematics ability, whereas sociocultural research such as Solomon (2007) examines what teachers and students do to give value to some mathematical efforts and not others, constructing rather than uncovering ability.

The 'personal enjoyment' rationale also suggests taken-for-granted understandings about there being kinds of people who like 'mathematics', rather than perhaps enjoying solving equations but not sketching asymptotic graphs. It can come as a surprise to students that A-level topics such as statistics and mechanics have very different feels. Although enjoyment may feel quite a trivial reason to continue or discontinue mathematics, it is intricately bound up in how we present ourselves as successful in making choices and in managing the work we need to do. I take an example drawn from an interview project considering how students account for themselves as studying mathematics and simultaneously on themselves as becoming mature, autonomous, disciplined adults (Smith 2010).

Hayley and Esther have just explained that they chose AS-level because they enjoyed mathematics. Esther contrasts it with biology where she once cried because she couldn't understand her homework:

Cathy  And you wouldn't see maths making you cry?
Esther  Oh it probably could! Stats probably could make me cry.
Hayley  Frustration. I don't cry like with things like that but ... but frustration could make me like ugh! I can't work it out, I can't work it out. I don't know how she has done it!
Esther  Oh, I get like that when I don't understand something. But I still enjoy the subject.
Hayley  Yes. I love numbers.
Esther  I will still come back and try to learn more.
Hayley  I love the fact that if I can't do it I will just do more and more to see how I can get to the answer. Like try different methods and you always know that there
is a way that you can do it. And if you haven't done it then you need to work out what it is.

Both girls emphatically describe unhappy feelings in mathematics but still claim to "enjoy the subject". They make sense of this tension by separating Statistics and things they don't understand from their relationship with mathematics as a whole, which Hayley describes as loving "numbers". By returning to primary school mathematics (and emotions), she carries the successes of the past into the present, and positions enjoyment as an expression of an 'authentic' identity. Such repositioning that looks both backwards towards childhood and forward towards accomplishment is part of the self-work of contemporary adolescence (Lesko, 2001). Hayley's last sentences repeat her love of mathematics as something established through practice and over time, giving her knowledge about mathematics and about herself-in-mathematics. She reworks frustration into something more enjoyable - the certainty of having an answer - and uses it to position herself as persistent and confident. This is an example of what I mean by working-on-yourself through mathematics. There is an interesting context to this mathematical friendship: Hayley comes from a lower GCSE set than Esther, and struggles throughout year 12. Yet in year 13, Hayley continues mathematics and Esther gives up. I suggest that we see here how Hayley can justify carrying on (to herself and others) despite some discouraging results and painful times. She uses the promise of eventual success as a means of feeling happy in her mathematics and in her self-work.

I chose this excerpt because it also links to the last of the common reasons: 'mathematics is hard'. At first sight this looks like a reason not to study mathematics. If maths is too hard, you are unlikely to enjoy it or do well. However it does feature significantly in students' reasoning. Hayley can work towards happiness by working on a mathematics problem, and the acknowledgement that mathematics is hard renders her own self-management even more impressive. Continuing with mathematics helps her demonstrate something special about herself. This notion of proving yourself with mathematics has been analysed by Heather Mendick (2006). In 'proving themselves' with mathematics, students align themselves with a powerful authority and reconcile two contemporary discourses of adolescence: knowing who you 'really' are, and ensuring that you will make progress.

CLOSING THOUGHTS
In the discussion above I have highlighted approaches to choice that are relevant to thinking how mathematics courses differ and how students understand mathematics. The approach that is dominant in policy-making has been to consider mathematics and identity as separable, so that any patterns in who chooses mathematics are side-effects, treatable as mis-functions of our educational system. Or we can think about reasons for choosing mathematics (or not) as being closely connected with constructing identities of entrepreneurial progress, ability, enjoyment, challenge and masculinity. This is not deterministic; on the contrary, contemporary understandings of selfhood and adolescence place a high premium on managing oneself. Nor is it equitable since we cannot all take up different identities freely. But from this perspective we have another way of understanding and researching different mathematics courses: as contexts in which mathematics, success and difficulty are differently positioned in relation to adolescence and self-work.

FURTHER READING
Matthews, A. & Pepper, D. (2007) Evaluation of participation in GCE mathematics. This QCA report reviews national survey data on who studies mathematics A-level and the different routes. It has been influential in critiquing mathematics as the perceived home of a “clever core” of students.

A readable introduction to post-structural perspectives on identity and how they play out in/through mathematics. The main focus is on gender, but the approach is equally influential in thinking about mathematics and social class.

**Wake, G., Williams, J. & Drake, P. (2011) Special Issue: Research in Mathematics Education.**

This collection of Transmaths project papers analyses how students negotiate their identities as they move into ‘advanced’ mathematics. It illustrates how quantitative and qualitative analytic methods can be mutually supportive.

[http://www.nuffieldfoundation.org/fsmqs](http://www.nuffieldfoundation.org/fsmqs)

The website of the free-standing mathematics qualifications that support UoM. There are interesting level 3 activities for teaching calculus, decision mathematics and dynamics.

[http://www.furthermaths.org.uk](http://www.furthermaths.org.uk)

The website of the Further Mathematics Support Programme showing the teaching resources and events it offers. The student area page titled ‘Why study mathematics?’ exemplifies the FMSP discourse that choosing is both rational and expressive.

**REFERENCES**


Reiss, Michael, Celia Hoyles, Tamjid Mujtaba, Bijan Riazi-Farzad, Melissa Rodd, Shirley


