A combinatorial approach to angiosperm pollen morphology

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Abstract
Angiosperms (flowering plants) are strikingly diverse. This is clearly expressed in the morphology of their pollen grains, which are characterised by enormous variety in their shape and patterning. In this paper, I approach angiosperm pollen morphology from the perspective of enumerative combinatorics. This involves generating angiosperm pollen morphotypes by algorithmically combining character states and enumerating the results of these combinations. I use this approach to generate 3,643,200 pollen morphotypes, which I visualise using a parallel coordinates plot. This represents a raw morphospace. In order to compare real-world and theoretical morphologies, I map the pollen of 1008 species of Neotropical angiosperms growing on Barro Colorado Island (BCI), Panama, onto this raw morphospace. This highlights that in addition to their well-documented taxonomic diversity, Neotropical rainforests also represent an enormous reservoir of morphological diversity. Angiosperm pollen morphospace
at BCI has been filled mostly by pollen morphotypes that are unique to single
plant species. Repetition of pollen morphotypes among higher taxa at BCI
reflects both constraint and convergence. This combinatorial approach to
morphology addresses the complexity that results from large numbers of
discrete character combinations, and could be employed in any situation where
organismal form can be captured by discrete morphological characters.

Keywords:

plants, morphology, morphospace, combinatorics, pollen
1. Introduction

Angiosperms (flowering plants) are an extremely diverse group of terrestrial plants and are composed of an estimated 260,000 species [1]. Angiosperms are characterised by striking morphological diversity, and the group contains an array of life forms that includes herbs, epiphytes, bulbs, aquatic plants, shrubs and trees. Research into the morphological diversity of angiosperms has involved the development of morphospaces that allow the morphology of different taxa to be quantitatively compared [2]. For example, morphospaces for angiosperm flowers have been used to investigate the frequency with which certain flower morphologies occur in nature [3] and also to examine the disparity of floral shapes [4]. Studies of fossil angiosperm pollen grains have shown that the taxonomic diversity and morphological disparity of angiosperms were decoupled during the evolutionary radiation of this clade during the Cretaceous period (~150–65 million years ago) [5], and recent work has highlighted that plant clades, including the angiosperms, typically attain high level of morphological disparity early in their evolutionary history [2].

In general, researchers recognise two approaches to the construction of a morphospace. The first approach is theoretical, which involves using a mathematical model to generate morphologies and may also include comparison with real-world forms [6,7]. In this context, simulations of plant phenotypes [8] are a botanical counterpoint to simulations of animal shell shape [9], and are a classic example of theoretical morphospace construction. The second approach is empirical, which involves constructing a morphospace from a set of empirical morphological data [2]. In an empirical morphospace, the form of an organism is typically represented by a number of discrete characters, and there is often
overlap between these characters and the set of characters that could be used for
the purpose of classification or phylogenetic analysis [10]. The dimensionality of
an empirical morphospace is usually reduced using multivariate ordination [6,7],
and the morphological disparity of a clade can be tracked through time using a
distance metric such as mean pairwise dissimilarity [e.g. 5,10]. The
morphospaces for plants constructed by Lupia [5] and Oyston et al. [2] are
empirical, and in this respect they are similar to foundational studies on the
morphological diversity of marine animals that are also based on discrete
characters [e.g. 10,11].

In addition to these theoretical and empirical approaches, however, there
are also raw morphospaces, which are formulated in terms of observed
morphological variation but prior to any multivariate ordination [7]. These
morphospaces are empirical in the sense that aspects of real form are captured
by the enumeration of discrete variables, rather than using a generative model,
but they are also theoretical in the sense that they can represent forms that do
not exist in nature [7,12]. Raw morphospaces are useful tools in the investigation
of organic form because they facilitate "the study of the evolution of the actual in
the realm of the possible" [7, p. 10]. Stebbins produced a raw morphospace for
angiosperm flowers by constructing a grid measuring 16 cells by 16 cells [3].
Each of the 256 cells in this grid represented one of 256 possible combinations of
primitive versus advanced conditions for the eight binary floral characters that
Stebbins analysed [3]. This grid allowed Stebbins to graphically represent
morphological types produced by different discrete character combinations, and
to demonstrate that the proportion of floral shapes that exist in nature is a
limited subset of the floral shapes that might be structurally possible [3]. For
Stebbins, one of the primary problems with his analysis was “the distortion produced by the recognition of only two conditions in respect to each character. This [he noted] was necessary because of the great complexity which would have resulted if more conditions had been recognized” [3, p. 302].

In this paper I address the complexity that results from large numbers of discrete character combinations by approaching angiosperm pollen morphology from the perspective of enumerative combinatorics. I begin with a schematic example that demonstrates the construction of a raw morphospace for angiosperm pollen. I use this approach to study angiosperm pollen morphology in the hyperdiverse Neotropics, and this involves comparing the morphology of 1,008 real-world pollen grains to 3,643,200 algorithmically generated pollen morphotypes. The approach I outline is not restricted to angiosperm pollen, and could be employed in any situation where organismal form can be captured by discrete morphological characters.

2. A combinatorial approach to angiosperm pollen morphology

The overall morphology of a single angiosperm pollen grain is determined by the nature and arrangement of several different individual characters. The states of these characters are typically described using specialist terminology, which allows researchers to communicate information about pollen [e.g. 13]. This specialist terminology treats each of these character states as discrete elements that contribute to the overall morphology of a pollen grain. Different pollen morphotypes reflect different combinations of these character states. This is shown in Figure 1a that contains five morphological characters, each with a
number of different states, which can be used to construct a schematic pollen grain (descriptive terminology in this paper follows [13]).

2.1. Counting the number of possible angiosperm pollen morphotypes

One possible combination of the character states shown in Figure 1a is: 3, 2, 2, 1, 2, 4. In the language of formal descriptive palynology [13], this combination would produce a tricolpate pollen grain with a tectate exine, spheroidal shape, monad dispersal, and an echinate–reticulate surface ornamentation. In more general terminology, this pollen grain has three elongated apertures in its exine, which is the outer coating of a pollen grain that is composed of sporopollenin [13] and in this case possesses a distinctive layer known the tectum that forms a roof over its internal structures [13]. This pollen grain has a spherical 3-dimensional shape, is dispersed at maturity by its parent plant as a single grain (a monad), and has spines longer than 1µm together with a network-like pattern on the surface of its exine. Generating this combination involves selecting one state from each list of aperture, exine, shape and dispersal character states. Some pollen grains posses just a single type of surface ornamentation, but others have two distinct types that form primary and secondary surface ornamentation (although the two are not hierarchically related). *Razisea spicata* (Acanthaceae), for example, has sparse echini and dense baculae [see 13 for terminology]. Accordingly, this combination involves selecting two states from the list of surface ornamentation character states.

I have thought of this process of character state selection as a combinatorial counting problem, in which the number of ways of selecting \( k \) objects from a list of \( n \) objects, if the order of selection is irrelevant, is given by
The numbers \( \binom{n}{k} \), which can be read "\( n \) choose \( k \)", are the binomial coefficients, and the value of \( \binom{n}{k} \) is 0 when \( k < 0 \) or \( k > n \) because there are no ways to select fewer than zero or more than \( n \) objects from a list of \( n \) objects [14]. In the schematic example shown in Figure 1, the values of \( \binom{n}{k} \) for each of the five characters are shown in Figure 1b.

For the aperture, exine, shape and dispersal characters, only one state is selected from each character state list, and the binomial coefficients for these characters are written \( \binom{n}{1} \) (Fig. 1b). For the aperture character, for example, there are \( \binom{3}{1} = 3 \) ways to select a single character state from this list. For surface ornamentation, two states are chosen from the list of five character states, and the binomial coefficients for this character are written \( \binom{5}{2} \) (Fig 1b). In this case there are \( \binom{5}{2} = 10 \) ways to select two states from this list. By the product rule, the number of all possible combinations of these character states is

\[
\binom{3}{1} \binom{2}{1} \binom{3}{1} \binom{2}{1} \binom{5}{2} = 360.
\]

In other words, there are 360 unique pollen morphotypes that can be generated from the five characters shown in Figure 1a using \( n \) choose \( k \) combination.

### 2.2. Enumerating and displaying angiosperm pollen morphotypes

For small values of \( \binom{n}{k} \) it is possible to count and enumerate combinations manually. For example, there are six combinations of two letters that can be produced from the four letters ABCD, and this is given by \( \binom{4}{2} \) or 4 choose 2. This
can be calculated using factorials or by using Pascal’s triangle for binomial coefficients, and these six combinations can be enumerated manually by hand: AB AC AD BC BD CD. However, manual counting and enumerating becomes rapidly intractable as the values of \( \binom{n}{k} \) increase. In order to overcome this, I have written an algorithm using the Python programming language that generates angiosperm pollen morphotypes by \( n \) choose \( k \) combination of morphological characters. An example of this algorithm, which counts and enumerates all possible combinations of the character states shown in Figure 1a, is provided in the Supplementary Material. This algorithm takes lists of morphological characters as its input, enumerates the combinations of these morphological characters using \( n \) choose \( k \) combination, and then writes these combinations to a .csv file with headers for each character. An example of a .csv file containing all possible combinations of the character states shown in Figure 1a is provided in the Supplementary Material.

I have visualised these character combinations using a parallel coordinates plot (Fig. 1c). In this plot, each vertical axis represents a different morphological character and lines visualise associations between character states. Parallel coordinates plots can be thought of as multipartite graphs, with each vertical axis corresponding to a graph partition [15], and they have been used to visualise large volumes of network traffic data [16]. This plot represents a raw morphospace that is bounded and contains a range of possible morphologies (Fig 1c). Provided that the same characters and states are used, real-world pollen grains can be mapped onto this morphospace.

3. Hyperdiversity in the Neotropics
I have applied this combinatorial approach to the hyperdiverse angiosperm-dominated tropical rainforests of South America. This region of the world contains the highest levels of plant diversity on Earth, and supports an estimated ~16,000 tree species alone [17]. When both extant and extinct taxa are considered, the total diversity of all Neotropical plant species that have ever existed is potentially enormous. I have focussed on the morphology of angiosperm pollen grains produced by plants growing on Barro Colorado Island (BCI). Barro Colorado Island is a 1,560-ha island situated in the Panama Canal that supports hyperdiverse lowland moist tropical forest, and a 50-ha plot on BCI that has been monitored since 1980 contains just over 300 tree and shrub species [18]. This region is of particular interest in the context of this present paper because such high species diversity offers an opportunity to confront the challenge of analysing large numbers of discrete character combinations [3]. The vegetation of BCI also represents a single well-studied flora with which to examine the patterns of morphospace occupation by different plant groups, and to explore the properties of occupied and unoccupied morphospace [e.g. 12].

### 3.1. Morphospace construction

I used the methods outlined in Figure 1 to construct a raw angiosperm pollen morphospace based on 60 aperture character states, four exine character states, ten shape character states, six dispersal character states and 23 surface ornamentation character states (a full list of character states is provided in the Supplementary Material). For surface ornamentation, two states are chosen from the list of 23 character states, and there are \( \binom{23}{2} = 253 \) ways to select two states
from this list. By the product rule, the number of all possible combinations of 
these character states is 
\[
\binom{60}{1} \binom{4}{1} \binom{10}{1} \binom{6}{1} \binom{23}{2} = 3,643,200.
\]
The raw morphospace that represents these 3,643,200 algorithmically generated 
pollen morphotypes is shown in Figure 2, and this allows associations between 
character states to be visualised. These character states were selected from the 
glossary of Punt et al. [13], and each is known to exist in nature. Some 
combinations of these character states are also known to exist in nature. For 
example, the combination Aperture 3, Exine 4, Shape 9, Dispersal 1, 
Ornamentation One 2, Ornamentation Two 23, produces a pollen grain with a 
single annulate pore, a tectate exine, spheroidal shape, monad dispersal, and 
scabrate surface ornamentation. This combination represents an archetypal 
grass pollen grain [19]. However, such real-world morphologies are a limited 
subset of the total 3,643,200 possible combinations in this morphospace, and 
consequently some of the morphologies shown in Figure 2 are theoretical and 
not known to exist in nature.

3.2. Patterns of morphospace occupation by real-world Neotropical 
angiosperm pollen grains

In order to map real-world Neotropical angiosperm pollen grains onto this 
morphospace, I examined a monograph containing morphological descriptions 
and images of pollen grains produced by plants growing on BCI [20]. I scored the 
angiosperm pollen grains in this monograph for each of the characters used to 
construct Figure 2. I excluded the following taxa from this analysis: (1) those
lacking a plate, (2) those with missing characters, and (3) polyads. I followed the higher taxonomic classifications in [20], and this resulted in a dataset consisting of 115 angiosperm plant families, which contain a total of 1,008 species and produce a total of 468 pollen morphotypes (Table 1; a full list of taxa and character scorings is provided in the Supplementary Material). This is just 0.01% of the 3,643,200 pollen morphotypes that were generated algorithmically by $n$ choose $k$ combination and used to construct the raw morphospace shown in Figure 2.

I split these 115 families into monocots (monocotyledons; angiosperms whose seeds contain a single cotyledon) and dicots (dicotyledons; angiosperms whose seeds contain two cotyledons). This partitioned dataset consists of 19 monocot families, which contain a total of 184 species and produce 68 pollen morphotypes, and 96 dicot families, which contain a total of 824 species and produce 407 pollen morphotypes (Table 1). I then mapped each BCI monocot and dicot pollen morphotype onto the raw morphospace shown in Figure 2. This shows how the morphospace occupied by the pollen of these two plant groups compares to the morphospace created by 3,643,200 algorithmically generated pollen morphotypes (Fig. 3).

The number of real-world BCI morphotypes in this analysis is clearly too few to occupy the entire raw morphospace. However, visual inspection of Figure 3 highlights that despite this, a considerable amount of the available morphospace is occupied by the BCI pollen morphotypes (Fig. 3). This is particularly noticeable in the morphospace occupied by dicot pollen, and for this plant group there are some associations between character states that are completely saturated. In this context, saturation may be defined as the
proportion of possible associations between different character states that are realized in nature. For example, there is an association between the tectate exine character state and each of the shape character states, and there is also an association between the monad dispersal state and each of the shape character states (Fig. 3b). Both of these examples therefore have a saturation score of 1.

There is also a strong, but not saturated, association between these character-states for monocot pollen (Fig. 3a). For example, there are 10 possible associations between the tectate exine character state and the shape character states and six of these are realised in the BCI dataset of monocot pollen (Fig. 3a). This gives a saturation score of 0.6.

Both monocots and dicots produce pollen with a variety of surface ornamentation patterns, and for both plant groups this area of the morphospace is well explored (see the Ornamentation One axis of Fig. 3). For both of these plant groups, there is a strong association between the Ornamentation One character states and the uppermost extremity of the Ornamentation Two axis (Fig. 3). This association represents pollen grains that have just a single type of surface ornamentation. Associations between character states on the Ornamentation One axis and other character states on the Ornamentation Two axis represent pollen grains that have both primary and secondary surface ornamentation. Both monocots and dicots have evolved the means to produce pollen with both primary and secondary ornamentation, and this morphospace shows how this capability has expanded their morphological variety (Fig. 3).

This mapping also shows that the pollen grains produced by BCI monocots and dicots occupy strikingly different regions of morphospace (Fig. 3). This is highlighted by the differences in monocot and dicot aperture character
states. Apertures are a key character in the study of angiosperm pollen, and are thought to be a key difference between the pollen of the monocot and dicot clades [21]. The aperture character states of BCI monocots are situated in two groups that are positioned at opposite poles of the aperture character axis (Fig. 3a). The group at the lower extremity of the character axis contains inaperturate, monoporate, triporate, pantoporate and stephanoporate morphotypes, while the group at the upper extremity contains zonorate, trichotomosulcate, disulcate and monosulcate morphotypes (see [13] for terminology). These two groups are separated by a region of morphospace that is largely unoccupied by monocot morphotypes (Fig. 3a). This region contains tricolpate, stephanocolpate and tricolporate forms (see [13] for terminology), and is occupied by a variety of dicot pollen morphotypes (Fig. 3b). These aperture types are highly characteristic of the eudicot plant clade, but the selective pressure (if any) driving their adoption by this plant group is largely unknown [22].

4. Discussion

4.1. Pollen grains and Neotropical plant diversity

There are fewer pollen morphotypes than plant species in the dataset of BCI angiosperms that I have used in this paper. This is reflected in the number of pollen morphotypes expressed as a proportion of species for all angiosperms in the dataset as well as monocots and dicots (Table 1). Some of the plant families that constitute the BCI flora produce very few pollen morphotypes. Such families include the Poaceae (grasses), which are taxonomically very rich (53 species in this BCI dataset) but only produce six pollen morphotypes (Table 1). Other
families are also very speciose but instead produce a large number of pollen morphotypes, and an example is the Papilionoideae (a sub-family of legumes), which contains 60 species in this BCI dataset and produces 47 pollen morphotypes (Table 1).

This pattern, in which a plant group is characterised by the production of fewer pollen morphotypes than species, has been frequently observed in many geographic regions and time periods, and pollen grains are said to suffer from low taxonomic resolution [23]. This pattern holds for the BCI flora as a whole, and for certain families, (Table 1), but when plant species richness is plotted against the number of pollen morphotypes for all families at BCI there is considerable scatter in the distribution of data points (Fig. 4). This shows that the relationship between the taxonomic diversity of the BCI flora and the number of pollen morphotypes produced by its constituent plants is too variable to be encompassed by a single notion of low taxonomic resolution.

Some data points are indeed situated well below the line of equality, and pollen morphotypes clearly underestimate the species diversity of these families (Fig. 4). However, there are 32 families that plot directly on the line of equality, 23 families that miss the line by just 1 (both excluding families represented by a single species on BCI), and there are several other families that plot close to this line of equality (Fig. 4). For these families, assemblages of dispersed pollen grains will provide an accurate or at least reasonable reflection of their species diversity (Fig. 4). Additionally, there are some plant species on BCI that produce dimorphic pollen, and examples include Cocos nucifera (the coconut tree), which produces pollen with either a monosulcate or a trichotomosulcate aperture.

Indeed, among monocots there are several families in which pollen aperture type
is not consistent within anthers, and these include the Agavaceae and Iridaceae (both Asparagales), as well as the Arecaceae, which contains *Cocos nucifera* [21].

It is possible, therefore, that in certain circumstances data points could lie above the line of equality in Figure 4, and that assemblages of dispersed pollen grains could overestimate the diversity of the source vegetation. The patterns of morphospace occupation by BCI angiosperm pollen grains (Fig. 3) therefore reflect a situation in which some plant families, such as the Papilionoideae, contribute a large quantity of morphological variety, whereas other taxa such as the Poaceae contribute much less morphological variety.

### 4.2. The structure and utility of a simplified palynological taxonomy

In this paper, the morphology of angiosperm pollen grains has been encoded in a system of discrete characters (see Fig. 2 and the Supplementary Material). This system contains fewer characters than would be employed by a palynologist seeking to classify pollen grains for the purpose of reconstruction vegetation history, and omits certain characters that may be phylogenetically informative such as the composition of the endexine and the nature of the intine [21].

Additionally, when observed using a microscope, the surface ornamentation of pollen grains can appear to vary continuously between individuals and species in a manner that is not well captured by the discrete approach that is taken here. As such, both the total number of character combinations that are represented by the raw morphospace shown in Figure 2, and the number of BCI pollen morphotypes mapped onto this morphospace in Figure 3, represent minimum diversity. The inclusion of just two size classes \( \left( \frac{1}{2} \right) \), for example, would double...
the total number of possible character state combinations used to construct the raw morphospace shown in Figure 2:

\[
\binom{60}{1} \binom{4}{1} \binom{10}{1} \binom{6}{1} \binom{23}{2} \binom{2}{1} = 7,286,400.
\]

In contrast, the removal of secondary ornamentation would substantially reduce the total number of possible character state combinations:

\[
\binom{60}{1} \binom{4}{1} \binom{10}{1} \binom{6}{1} \binom{22}{1} = 316,800.
\]

Secondary ornamentation is a rare character in the BCI flora (there are just 65 BCI pollen morphotypes with secondary ornamentation), and in this respect BCI is typical of angiosperm pollen morphology in other floras.

This discrete character-based system of encoding pollen morphology is comparable to formal taxonomic descriptions of pollen grains in the sense that both involve breaking pollen morphology down into individual characters, which are then described using specialist terminology (Fig. 1a) [13]. Further, in spite of its simplicity, this system contains a relationship between the number of species per pollen morphotype and the number of pollen morphotypes that is essentially of the same mathematical form as the relationship between the number of species per genus and the number of genera in other taxonomic systems (Fig. 5). These include classifications of plants, sponges, molluscs, fishes, beetles and birds [24, 25]. The correspondence between Figure 5 and similar plots from other taxonomic systems [24, 25], which are often presented on log-log plots, suggests that studying the structure of palynological taxonomy through evolutionary time could provide data on the diversification history of plant clades [see 26].
Together with data comparing plant species richness and the number of pollen morphotypes for all families at BCI (Fig. 4), Figure 5 indicates that angiosperm pollen morphospace at BCI (Fig. 3) has been filled mostly by pollen morphotypes that are unique to single species (301 instances; top-left data point in Fig. 5). There are 167 pollen morphotypes that are produced either by different species within a single family, such as the archetypal grass pollen grain, or by taxa within two or more families, and these repetitions represent 36% of the pollen morphotypes at BCI (Figure S1). This emphasizes that when studying assemblages of Neotropical angiosperm pollen grains, even a highly simplified system consisting of the five discrete characters used in this paper can capture reasonable data on the diversity of the source vegetation. The addition of further characters would likely increase the number of pollen morphotypes and move the data points in Figure 5 upwards and to the left. If these results are applicable to other regions of the world and through geological time then they provide a degree of support for studies that use pollen grains to reconstruct the diversity and composition of tropical vegetation through time.

4.3. The production and distribution of biological form

In this paper I have not generated morphologies using a growth model that incorporates morphogenetic processes [cf. 6,27,28]. Instead, I have represented morphogenesis in angiosperm pollen by the algorithmic combination and enumeration of discrete morphological characters (e.g. Fig. 1), and this combinatorial approach allows for the explicit comparison of occupied and unoccupied morphospace. Each of the 3,643,200 morphotypes that can be produced from the characters I have used in this study (Fig. 2; Table 1;
Supplementary Material) is biologically plausible and structurally possible. There are no morphotypes with both tricolpate and monocolpate apertures, for example, and there are no morphotypes that contain a character state that is not known to exist in nature. However, the number of these plausible and possible angiosperm pollen morphotypes is far greater than the number of angiosperm species on Earth as a whole (~260,000; [1]) and in the BCI dataset I have used here (1008; Table 1). This means that when real-world forms are mapped onto the raw morphospace that contains these 3,643,200 morphotypes there are large areas of unoccupied morphospace (Fig. 3). Although the BCI dataset used in this study represents just a small fraction of angiosperm life, I suspect that a similar analysis incorporating all known angiosperm species would produce a similar pattern.

One way of interpreting patterns of morphospace occupation is to think of the entire morphospace as a fitness landscape, with occupied areas representing local optima and unoccupied areas representing less fit alternatives [12]. This view of morphospace occupation is informed by Wright's studies of allele combinations and fitness landscapes, in which he viewed a possible $10^{1000}$ allele combinations as a rugged field with "an enormous number of widely separated harmonious combinations" [29, p. 358]. Wright viewed these harmonious combinations as adaptive peaks separated by maladaptive valleys, and Stebbins, without directly referencing Wright's work, seems to interpret the patterns in his floral morphospace in a similar fashion: "there are three types of combinations which, although structurally possible, are found so rarely that they are apparently poor adaptations, usually of low survival value" [3, p. 314].
Many attempts have been made to establish the function of the morphological characters that together make up a pollen grain [e.g. 30], and discussion has ranged from the role of surface ornamentation in pollination [22], to the role of apertures in the desiccation, hydration and germination of pollen grains [31]. Consequently, it is currently unclear which morphological features of pollen grains are adaptive and which are not, and this means that the relative roles of chance versus necessity in generating the morphological diversity of pollen grains are largely unknown. Indeed, the wide range of plausible functional roles for the morphological features of pollen grains, suggests that they are "not 'optimally' designed for a specific function, but merely structures that work with varying efficiency in a specific ecological and evolutionary context" [30, p. 182].

With this in mind, I do not interpret the areas of unoccupied morphospace in Figure 3 as regions containing pollen morphotypes that are maladaptive in the context of the BCI flora. It is possible, however, that further experimental work on the function of specific morphological features may reveal regions of morphospace that are characterised by forms with specific functional properties. Such work could also include analyses of the tendency of certain morphological features to arise by self-assembly processes, which may produce forms that reflect convergence upon a minimum free energy state rather than the effects of natural selection [32].

Alternative interpretations of morphospace occupation include the suggestion that the present-day distribution of morphology (e.g. Fig. 3) reflects an earlier more continuous distribution that has been subsequently fragmented by extinction [12], as well as Gould's classic idea that chance played a major role in distributing form, with the contingent elimination of forms early in their
evolutionary history producing a clumpy distribution of morphology among living organisms [12,33]. The forms of such living organisms may represent relatively small historically defined groups that are drawn from more-or-less unlimited possibilities [e.g. 3], or could instead be fundamentally limited by structural and/or phylogenetic constraints (see [5] for angiosperm pollen and [34] for animal skeletons). Results of this analysis suggest that areas in morphospace that are unoccupied by modern organisms (e.g. Fig. 3) could reflect the fact that many potentially viable morphologies have not yet been produced by the evolutionary process [35].

However, if a morphospace is defined using character states that are sufficiently broad in scope [e.g. 34] then the possibility of unrealized morphologies can be effectively eliminated, whereas if large numbers of character states are used to define a morphospace (as in Fig. 2), then the number of unrealized morphologies can be increased at will. Despite this scaling issue, each of the character states used to construct the raw morphospace shown in Figure 2 is known to exist in nature and is very general compared to the complexity of pollen morphology as viewed through a microscope [13]. Additionally, none of the 3,643,200 pollen morphotypes that these general and simplified character states produce seems biologically implausible. In this light, the possibility of large numbers of unrealized angiosperm pollen morphologies seems real, and the number of such potential forms that have not yet evolved appears to be vast. This is in contrast to analyses of animal skeletons, for which "the number of potential skeletal designs, defined in very general terms, is not inordinately large" [34, p. 350]. However, the difference in the outcomes of these two analyses (angiosperm pollen versus animal skeletons) is primarily a
consequence of the different scope of the characters that are defined in each case.

In the dataset of BCI angiosperm pollen I have analysed in this paper there are 167 morphotypes that are repeated among higher taxa (Fig S1, Fig. 6), and these repetitions highlight the numbers and kinds of morphotypes that reflect phylogenetic constraint as well as evolutionary convergence. The greatest number of these repetitions occurs among different species or genera within single families \((n=68; \text{Fig. 6})\). Examples of this include the pollen of grasses (single annulate pore, tectate exine, spheroidal shape, monad dispersal, scabrate surface ornamentation; 31 repetitions in one family), laurels (inaperturate, intectate exine, spheroidal shape, monad dispersal, echinate surface ornamentation; 11 repetitions in one family) and figs (diporate, tectate exine, oblate shape, monad dispersal, psilate surface ornamentation; 7 repetitions in one family). These morphologies exhibit strong phylogenetic constraint because they are common within these families but are restricted to them.

In contrast, there are relatively few morphotypes that are repeated in three–nine families (Fig. 6). These 35 morphotypes are notably similar in terms of their aperture character state: 23 of these morphotypes share the same aperture character state (tricolporate; 65.7%), and all the morphotypes that are repeated among six–nine families share this character state (Fig. 6). With respect to aperture character states, this highlights a degree of nestedness in the distribution of angiosperm pollen morphology at BCI. Seven different morphotypes are repeated in both the monocot and dicot clades (Fig. 6). These morphotypes also have a very uniform set of aperture characters: two are inaperturate and five have multiple pores (pantoporate) (Fig. 6). As quoted in
The systematic distribution of periporate [pantoporate of many authors] pollen in extant angiosperms indicates that this aperture configuration evolved independently in many different groups" [36, p. 406]. The repetition of these aperture configurations across a fundamental divide in plant phylogeny is an example of evolutionary convergence, and in this respect, the BCI flora reflects a broader pattern in angiosperm evolution.

5. Concluding remarks

The combinatorial approach to angiosperm pollen morphology that I have outlined in this paper provides an example of how the general challenge of analysing large numbers of discrete character combinations [3] can be overcome. This approach generates biological forms by algorithmically combining character states and enumerating the results of these combinations (Fig. 1). I have used this approach to generate 3,643,200 angiosperm pollen morphotypes and produce a raw morphospace (Fig. 2). Some of these morphotypes correspond to real-world forms whereas others do not.

I have mapped the pollen of 1008 species of Neotropical angiosperms growing on BCI, Panama, onto this raw morphospace (Fig. 3). This allows realized and unrealized forms to be compared, and shows that despite the relatively small size of the BCI dataset, these real-world pollen grains occupy a considerable amount of the raw morphospace (Fig. 3). This highlights that in addition to their well-documented taxonomic diversity, Neotropical rainforests also represent an enormous reservoir of morphological diversity. Some plant families, such as the Papilionoideae, contribute a large amount of morphological
diversity, whereas other taxa such as the Poaceae contribute much less morphological diversity (Fig. 4).

In this paper, the morphology of angiosperm pollen grains has been encoded in a highly simplified system of five discrete characters. Despite this simplicity, this system contains a relationship between the number of species per pollen morphotype and the number of pollen morphotypes that has essentially the same mathematical form as the relationship between the number of species per genus and the number of genera in other taxonomic systems (Fig. 5). This highlights that angiosperm pollen morphospace at BCI (Fig. 3) has been filled mostly by pollen morphotypes that are unique to single species (Fig. 5). Angiosperm pollen morphotypes that are repeated among higher taxa at BCI reflect both phylogenetic constraint and evolutionary convergence (Fig. 6).

Data accessibility. The datasets and code supporting this article have been uploaded as part of the Supplementary Material.

Competing interests. I declare I have no competing interests.

Author contributions. LM conceived, designed and undertook the study. LM wrote the paper.

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**Figure and table captions**

**Figure 1.** (a) Schematic illustration of characters that make up an angiosperm pollen grain. Aperture character states: inaperturate (1), triporate (2), tricolpate (3). Exine character states: atectate (1), tectate (2). Shape character states: oblate (1), spheroidal (2), prolate (3). Dispersal character states: monad (1), tetrad (2). Surface ornamentation character states: baculate (1), echinate (2), clavate (3), reticulate (4), no ornamentation (5). Different pollen morphotypes reflect different combinations of these character states. (b) Binomial coefficients
for each character. (c) Showing how these different combinations can be
represented using a parallel coordinates plot.

Figure 2. Parallel coordinates plot of 3,643,200 angiosperm pollen morphotypes
generated algorithmically by n choose k combination. Numbers enclosed within
white circles represent each of the character states used to produce this
morphospace. Only the first and last aperture and surface ornamentation
character states are labelled to prevent overcrowding of these axes. A full list of
caracter states corresponding to these numbers is provided in the
Supplementary Material.

Figure 3. Parallel coordinates plots showing how pollen grains produced by
angiosperms growing on Barro Colorado Island map onto the morphospace
shown in Figure 2. In both plots this morphospace is shown in light grey, and is
overlain by (a) monocot pollen morphotypes in dark blue and (b) dicot pollen
morphotypes in orange. A full list of taxa and character scorings is provided in
the Supplementary Material.

Figure 4. Scatterplot comparing the species richness of angiosperm families
from Barro Colorado Island with the number of pollen morphotypes produced by
each of these families. Datapoints have been plotted for each of the 19 monocot
and 96 dicot families analysed in this paper but datapoints for certain families
overlap. Both axes are plotted on a logarithmic scale. Diagonal dashed line
represents a line of equality.
Figure 5. Scatterplot showing the hollow-curve relationship between the number of species per angiosperm pollen morphotype and the number of angiosperm pollen morphotypes on Barro Colorado Island.

Figure 6. Proportions of angiosperm pollen morphotypes that are repeated among higher taxa at BCI (green vertical bars). Numbers enclosed within grey vertical boxes show the distribution of aperture character states that are repeated among higher taxa at BCI (see dataset S1 for a full list of aperture character states). Key aperture character states repeated among three–nine families as follows: Inaperturate (1), Pantoporate (11), Tricolpate (25), Tricolporate (44), Tricolporate Marginate (45), Pantocolporate (48), Monosulcate (54). Cartoons to the right are schematic pollen grains that graphically display each of these aperture character states.

Table 1. Summary of the data and results discussed in this paper.

Supplementary material

Dataset S1. List of characters used to generate the raw morphospace shown in Figure 2.

Dataset S2. Taxa and character scorings of BCI pollen grains used to generate Figure 3. Refer to Dataset S1 for list of characters.

Supplementary File S1. An example Python algorithm that counts and enumerates all possible combinations of the character states shown in Figure 1a.
Supplementary File S2. An example of a .csv file containing all possible combinations of the character states (in numerical form) shown in Figure 1a.

Figure S1. Barchart of the frequency of angiosperm pollen morphotypes at BCI.
Figure 1. (a) Schematic illustration of characters that make up an angiosperm pollen grain. Aperture character states: inaperturate (1), triporate (2), tricolpate (3). Exine character states: atectate (1), tectate (2). Shape character states: oblate (1), spheroidal (2), prolate (3). Dispersal character states: monad (1), tetrad (2). Surface ornamentation character states: baculate (1), echinate (2), clavate (3), reticulate (4), no ornamentation (5). Different pollen morphotypes reflect different combinations of these character states. (b) Binomial coefficients for each character. (c) Showing how these different combinations can be represented using a parallel coordinates plot.
Figure 2. Parallel coordinates plot of 3,643,200 angiosperm pollen morphotypes generated algorithmically by \( n \) choose \( k \) combination. Numbers enclosed within white circles represent each of the character states used to produce this morphospace. Only the first and last aperture and surface ornamentation character states are labelled to prevent overcrowding of these axes. A full list of character states corresponding to these numbers is provided in the Supplementary Material.

187x75mm (300 x 300 DPI)
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177x177mm (300 x 300 DPI)
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204x153mm (300 x 300 DPI)
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<th>Morphotypes (n)</th>
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