Addressing the affective domain to increase effective-ness of mathematical thinking and problem solving

Conference or Workshop Item

Addressing the affective domain to increase effectiveness of mathematical thinking and problem solving

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Abstract

Mathematics anxiety, a prevalent, acquired, disabling and treatable condition, can be thought of as "an emotional handbrake" on mathematical thinking and problem solving. Thus, in order to develop mathematical thinking and problem solving as effectively as possible, it is important to enable learners to address any mathematics anxiety explicitly.

Some interventions have been found effective in reducing mathematical anxiety, and some in building mathematical resilience. Being asked to engage in problem solving can have short-term impact on learners' ability to manage the affective domain. However, the use of the Growth Zone model and other resources designed to develop mathematical resilience can have long-term positive impact on learners' ability to manage the affective domain.

The paper draws on the work of a teacher working with us on the Growth Zone model in Spain and three teachers undertaking post-graduate study at the University of Warwick to illustrate the underlying premise, that addressing anxiety will enable higher attainment in problem solving. Teachers such as Silversides (2013), Chisholm (2017) and King (2016) have found the Growth Zone model to have high efficacy in enabling learners to communicate and learn to manage their feelings when learning to problem solve.

Interviews with learners reveal that most find the Growth Zone model effective to build mathematical resilience and communicate emotions. This is important, since success in mathematics is heavily dependent on emotion. We conjecture that explicit use of a tool such as the Growth Zone model is most effective in improving learners' willingness and ability to engage in problem solving when combined with artefacts such as a 'stuck' poster.

1. Introduction

The context of this paper is one of recent changes to the mathematics curriculum which place significant emphasis on using mathematics to problem-solve (DfE, 2013). In a culture in which mathematics anxiety is prevalent but not well understood, such changes are likely to further add to students' anxiety load. In England, mathematics classes seeking to develop mathematical thinking and problem solving often contain students who experience high mathematics anxiety, and low mathematical resilience (Johnston-Wilder et al, 2013), which are known to cause low attainment (Ashcraft, 2002) and little or no progress with mathematics.

According to Schoenfeld, a problem is 'a difficult question…which exercises the mind' (Schoenfeld, 1985, p.74); Polya (1973) describes problem-solving as finding ways out of difficulty and around obstacles. The key words, in the context of this paper, are 'difficult' and 'obstacles'. Therefore success in problem solving will require the characteristics of mathematical resilience (Lee & Johnston-Wilder 2014) which are: a growth mindset, understanding of the personal value of the work that learners are engaged in, knowledge that succeeding in mathematics requires struggle and perseverance as well as a collaborative, inclusive and supportive environment in which to learn.

Working with mathematics teachers, we have found it useful to think of mathematical anxiety as "an emotional handbrake" on thinking and problem solving (for example, Johnston-Wilder et al., 2016). It has been shown that mathematics anxiety is an acquired, disabling condition but importantly it has been shown to be a treatable condition. There is a clear association between mathematics anxiety and attainment, progress and ability to think and reason clearly (Grootenboer and Marshman, 2016). Therefore, in order to develop mathematical thinking and problem solving as effectively as possible, learners will need to address any mathematics

anxiety. Typically, teachers who recognise the problem attempt to address it implicitly by encouraging and giving "this will always work" tips to remember. However the research reported in this paper shows the benefit of seeking to address the problem explicitly. The work that we have done in describing and operationalising mathematical resilience has demonstrated that anxiety does not develop if mathematics is taught in a way that develops learner resilience and that, where anxiety has already developed, learners can overcome their habitual, learned responses and develop resilience which allows for increased engagement and attainment. In a valued personal communication, Sfard (2017) states "affective factors are the first suspect in the case of lingering difficulty … your work takes the topic much further by proposing specific ways of dealing with the problem."

One tool that has been used extensively and successfully to help students to recognise and begin to deal with any negative feelings they have developed towards mathematical learning through past experiences is the Growth Zone Model, one design of which is shown below. The exact design of the model is varied by each teacher that uses it, with certain constants, the idea that in order to learn it is necessary to move beyond the comfort zone, and that learning may feel risky and uncomfortable with barriers to overcome, but that discomfort must not go too far and lead to anxiety.

The Growth Zone Model

Green – comfort, a safe but unchallenging place to be. You can build fluency but you can also get bored.

Amber (yellow) – growth – a more risky and challenging place where learning happens. You can feel pleasure and excitement but you may also feel uncomfortable.

Red – panic and anxiety - You are no longer learning and you feel unsafe – get help to return to amber or green

This paper draws on the work of four teachers to illustrate that addressing anxiety will enable higher attainment in problem solving. Teachers such as Silversides (2013), Chisholm (2017) and King (2016) have found the Growth Zone model to have high efficacy in enabling learners to communicate and learn to manage feelings when learning to problem solve. The learners themselves reveal that most find the Growth Zone model effective in building resilience and allowing them to communicate emotions. This is important, since success in mathematics is heavily dependent on emotion and on how a learner sees themselves or personal identity. According to Sfard (2005) identity is 'a collection of stories that people tell about themselves (first person identity narratives) or that others tell about them (third person identities)'. What is being done, in the research here reported, is to enable the revision of personal identity stories about mathematics from "I am not a maths person" to "I was excluded, sometimes by my choice, as I couldn't think straight because I was in my red zone. I know I can learn and feel safe in my growth zone but I need ideas, help and support to do that!" Based on the work of teachers such as these, we conjecture that explicit use of a tool such as the 'growth zone model' is most effective when combined with ways of working that are inclusive, collaborative and supportive and other artefacts such as a 'stuck' poster.

2. A changed learning environment

The first of the four projects, which followed a CPD session, focused on increasing attainment of pupils by providing a changed, more inclusive and collaborative classroom environment and some after school tutoring sessions in a school in Spain. The teaching concentrated on building the students' confidence through using a slower pace when working through some algebraic problems and through the overt use of the Growth Zone model. The environment in

the after-school sessions was designed to be much more safe and secure than the hurly burly of the general classroom which had a mix of attainment. The group also took advantage of a single session of 'inclusivity' coaching to build a safe environment.

The data, which was in the form of observations recorded in field notes, came from a selected group of six pupils who had particular difficulties in mathematics which appeared to be related to anxiety.

2.1. Findings

Verbal communication was noted as a particular difficulty exhibited by many of the children who also were noted as presenting characteristics of anxiety. It was reported about one child: “it so clear that entry into a maths classroom installs immediate 'fear' and an almost total shut down of verbal communication” and of another: “she is extremely quiet and only a simple question away from completely shutting down into a 'rabbit in the headlights' state”. Another child did not show anxiety but: “she poses quite some difficulties for all teachers as reaching and communicating with her is almost impossible.” Whereas “[when] she is with her friends she is an active and funny girl, but becomes chameleon like in maths lessons, being seemingly only visible at the start and the end of each lesson.”

There was some evidence in the data of these children not understanding the idea of ‘struggle’ in mathematics but expecting to follow a simple smooth path through any problem. “In the past he has considered learning only to be me telling him exactly what to do and him completing simple, unchallenging tasks, and his learning has been minimal.” It is also likely that the parents of these children inadvertently encouraged the idea that it is acceptable to be poor at mathematics, reinforcing a fixed mindset. “I was talking with her mom after school whilst they were together, where she used language such as 'I don't understand maths, it’s not my subject etc etc' - in front of child E.”

Following the intervention, there was a demonstrable increase in immediate attainment with all but one of the group voluntarily retaking a test on algebraic problem solving following the changed environment which resulted in increased scores from around 2/8 to on average 7/8. There have been several more changes in the attitudes of the children, “…vocal answers to simple directed questioning” and “… upon targeted questioning today, she did simply reply 'I don't know', a statement it takes quite some confidence to say, and therefore a clear indicator of her comfort in the new inclusive environment.”

The environment has resulted in several of the children being more prepared to take appropriate responsibility for their own learning “… she will now adopt the 3 before me approach and gladly works with other children, now only approaching me when she really cannot grasp a concept” and “Today watching his delight at acquiring his own learning from research and self-study was delightful, and the support offered by his peers is also very encouraging.”

2.2. Discussion

This was a small project but it demonstrates some of the disabling effects of anxiety shutting down on dialogic communication which is such a vital part of learning in the classroom. Allowing the children to develop a language of struggle using the Growth Zone model seems to have had a marked effect on the children’s learning. The children were reported as slowly changing their attitude to mathematics from one where they sit in silence and absorb very little, to one where it is alright to admit that “I don’t know” as help is readily available from other people in the classroom as well from the teacher if needed.

The new environment seemed to have made a difference to the identities of these children, for example: “Her confidence is clearly growing as she recognises her own ability, she even identified some areas in basic maths she wants to improve on”. The teacher described the changes he perceived as ‘heart-warming’.

3. A Growth Zone intervention

The research undertaken by Emma Silversides took place in a mixed, non-selective comprehensive school in a rural UK setting, with a group of 26 year 7 pupils who displayed lower to middle attainment. Silversides was already working with these pupils, and in her view 'streaming may have diminished the existence of mathematical resilience in some pupils' (Silversides, 2013).

Silversides explored the impact of one Growth Zone session on the students' capability to engage in sustained problem solving activity. She used the session to describe the attributes of the Growth Zone Model, to introduce the language of the model and to explicitly consider how to stay in the Growth Zone, "More precisely to investigate how learners, having been invited to use a variety of their own words to describe how they feel in their own green, orange and red zones and having explored strategies for keeping safe in their mathematics Growth Zone, away from panic but not cruising …, might tackle a challenge requiring mathematical resilience" (Silversides, 2013). "The planned intervention involved pupils being introduced to the Growth Zone diagram, exploring the significance and relevance of it in relation to their mathematical learning experiences to date. This mini-workshop included brainstorming activities, independent reflection and group discussion." (Silversides, 2013.) During the session, each pupil produced a personalised Growth Zone diagram. The diagrams produced by the pupils were kept in their books, accessible in subsequent lessons. In the session, the pupils were also asked to relate the Growth Zone diagram to their out-of-school activities and experience of learning new skills in a less formal setting. Following the intervention, the class frequently referred to the Growth Zone diagram in their books and used post-it notes of appropriate colour to record their feelings when problem solving.

The data were collected in the form of videos of the Growth Zone session from fixed cameras, from two questionnaires completed by the pupils, and from notes and reflections recorded by Silversides.

3.1. Findings

Prior to the intervention, 52% of the group reported that they were 'not very good at solving mathematics problems that took a while to figure out' or 'if they couldn't solve a mathematics problem in a few minutes, they probably couldn't do it at all and would stop trying'.

During the intervention, Silversides observed that "pupils could easily find words and descriptions that they personally associated with an active presence in their safe zones and their red zones [and that]recognition of feelings experienced in the red zone came easily for the majority, but many then proceeded to include several of these feelings in their learning zone; there was little difference for them between the two, with words such as 'stressed', 'angry' and 'worried' appearing in both the red zone and the learning zone. Significantly, all those displaying traits of a growth mindset and mathematical resiliency via the questionnaire generated three very distinct sets of emotions/feelings associated with each zone with no overlap whatsoever." (Silversides, 2013)

Silversides also found that: "knowledge of (and ability to identify) potential strategies for moving into the learning zone, and indeed sustaining a presence in it, were noticeably limited. [noting that] Those pupils who readily related to the initial phase of the intervention via a personal experience (sporting, musical or other) were all included in the group of pupils who created three distinct learning zones ... their ability to distinguish between being out of their learning zone completely and demonstrating resiliency within their learning zone certainly places them at an advantage when trying to personally build resiliency under instruction." (Silversides, 2013)

After the intervention, Silversides noted that: "all the pupils had a direct and accurate means of reflecting on their learning in any given lesson; they did this enthusiastically and with significant thought". In the second questionnaire, the pupils made comments such as: 'I needed my partner to help me stay in my learning zone', and 'I was in my red zone but we

got the teacher to help and then it was ok'. She felt these comments revealed that the pupils had engaged with the intervention and could apply it to their learning. She said "Not only is this a tool of reflection for pupils but it enabled me as a teacher to obtain direct feedback." (Silversides, 2013)

3.2. Discussion

Silversides found that, as the class teacher, she benefited more from descriptions relating to growth zones than from any other previously employed approaches to gain information about how her pupils were progressing. She reported: "I immediately began to develop an understanding of the emotions (frequently mentioned by pupils on the post-its) that they had experienced in that particular lesson (so how they related to that topic) and could use this to inform future planning including potential deployment of a TA. It also revealed qualities of some otherwise 'quieter' pupils; 'I could stay in my growth zone because 'x' explained this to me in a way that I could understand.'" (Silversides, 2013)

Silversides noted that intervention had several immediate effects on the pupils themselves. The duration and engagement of the pupils in collaborative work was significantly increased in comparison to that previously experienced. 50% of the pupils claimed on the second questionnaire that they had 'made more effort to be resilient in class since the intervention' and an additional 12% admitted that the intervention 'had made me think about my learning and I have thought about it since with relation to how I learn in classes'. Hence about two thirds of the class were considering how they engage with their work more and knew that learning requires some effort from them to recruit the support they might need.

Silversides felt that one intervention lesson was insufficient to cover all the aspects that her classes needed. She reported that she felt that further sessions addressing strategies for movement into the learning zone, from the safe zone and from the red zone, as well as how to sustain an active presence in the learning zone, would have been of benefit to many of her pupils.

A clear outcome from Silversides research was that some pupils take to the ideas and language of the Growth Zone model easily, using it to their benefit. Others find it harder to identify the emotions they might experience in their learning zone, often equating those feelings with those of the red zone. She felt that focusing on such pupils in order to help them to experience and identify the positive feelings of struggling and overcoming barriers was imperative if they were to become effective learners and problem solvers.

4. The "Stuck Poster"

Chris Chisholm conducted a longitudinal study on a year 9/10 class that started shortly after the students had joined the high school where he was Head of the Mathematics Department. The learners in his research were grouped together as low attaining, and Chisholm studied the effects on their attitude to mathematics of acting to increase their mathematical resilience when asked to problem solve by using various ideas inspired by Lee & Johnston-Wilder (e.g. 2013, 2016).

Chisholm designed many "interventions" over the twelve month data collection period. Each intervention lasted from one to two weeks; he asked the students to engage in problem solving activities such as planning and costing a trip to London for two people, which included a visit to a theatre. He collected data in a research journal which contained his planning for the lessons during the intervention, data collected during lessons such as quotes from the students, his reflections on the lessons afterwards and some overarching reflections of the whole intervention. He also asked his students to complete learning journals.

The complete report of his findings is available at wrap.warwick.ac.uk/88601/1/WRAP_Theses_Chisholm_2017.pdf

4.1. Discussion

Chisholm found that his most significant focus was on helping the learners cope with being 'stuck'. He found that when the students felt "stuck", this was one of the biggest barriers to their mathematical learning, especially when it came to problem solving. Hence the main focus of the interventions became developing strategies to help the students understand that problem solving in mathematics involved perseverance and struggle. He felt that when they experienced barriers rather than having ways to overcome the barrier they equated this to being "stuck". He introduced the Growth Zone model to help these students understand the emotions they felt when they were in their "growth zone". However he reported that "that learners were hesitant to work in the Growth Zone as a consequence of the fear of 'getting stuck' and not knowing what to do when stuck. Many were seen to give up on a task if the teacher did not tell them exactly what they had to do next."

As with Silverside, Chisholm discovered that he needed resilience as a teacher in order to help his pupils change their identities from someone who is unable to do mathematics and someone who has no idea how to proceed once they feel "stuck" to someone who can engage in problem solving, whose ideas are worthwhile and worth pursuing and who can persevere. He continued to use the Growth Zone model and found "As the research progressed, I observed learners spending longer periods of time working in the Growth Zone." He saw that "learners were not giving up as easily when the learning got more challenging".

He directly tackled the "stuck" problem by developing a "Stuck Poster" using ideas about how to become "unstuck" contributed by the class, which became more sophisticated over time. This poster was one large sheet of paper and contained many ideas such as "look in a book" and "ask someone else". It was created by the learners, it was visible at all times and it was referred to on a regular basis. Whenever anyone in the class had a good idea about becoming "unstuck", it was recorded on the poster, which was rolled up after each lesson and displayed again in the next lesson, which was often in a different room.

The 'Stuck Poster' seemed to help the students become more independent learners and grow their resilience. Chisholm reports: "The majority of learners within the group referred to the usefulness of the 'Stuck Poster' in allowing them to proceed when stuck, either in discussion or in journal entries." (Chisholm, 2017.) The mean mark gained by the research group at the end of the year was higher than a comparable group taught by the same teacher the previous year, indicating that the interventions seem to have had a positive effect on the learning of the group.

5. Reducing Anxiety and building Mathematical Resilience

This fourth study investigated the outcome of two interventions aimed at impacting positively on students' attitudes towards problem solving in mathematics. These interventions focussed on both reducing mathematical anxiety and building mathematical resilience. The first intervention used the 'Blond Hair' problem (https://www.youtube.com/watch?v=mt8ba2Z5s1w). The second was a 'growth mindset' workshop, which contained various activities including Lee & Johnston-Wilder's (2016) Growth Zone model and a 'stuck' poster.

A mixed-methods case study was carried out with fifty participants aged 11-12 years. The participants were in two middle attaining mathematics sets. The students' levels of mathematical anxiety and mathematical resilience were measured pre- and post-intervention. The measures used were modified versions of the 10-item Betz Mathematics Anxiety Scale and the Mathematical Resilience Scale (Kooken et al., 2016). Interviews with each of the participants were also held.

5.1. Findings

Both interventions led to a significant decrease in levels of students' mathematical anxiety. However, neither intervention led to a significant increase in resilience in the short-term, although the growth mindset workshop did lead to a significant increase with respect to

growth mindset traits as measured on the resilience scale. There were some students in each group who displayed low mathematical resilience and there was fairly low average mathematics anxiety pre-intervention in the first group (although 18% were reported as highly anxious). The second group of students had higher anxiety levels overall and 26% were highly anxious, which was a concerning result, particularly given the young age of students and the finding that anxiety levels increase with age.

5.2. Discussion

The interventions were successful at lowering anxiety levels, although results of some other studies (e.g. Donohoe, 2012) have concluded that short-term interventions do not impact on attainment levels in the long-term. Both interventions were successful in the short-term, in positively impacting some aspect of students' attitudes towards problem-solving. The interviews with students revealed that they found the Growth Zone model a useful way to communicate their feelings. Students also gave positive feedback on the 'stuck' poster.

This was a very small scale study but it does again demonstrate some benefits from using the Growth Zone model, combined with the 'stuck' poster as a way to help students become more proactive and resilient problem solvers. There is some evidence that such interventions can impact positively on students' attitudes towards problem solving, and their ability to manage their emotional responses when learning.

6. Reflections

The four research studies reported here demonstrate something that is well known (see OECD, 2010 for example) that mathematics anxiety is real and is widespread. However they do more than that, they illustrate that when teachers take steps to "treat" mathematics anxiety, using ideas from the extant and growing literature on mathematical resilience, their students can and do overcome anxiety and this has a positive impact on their attainment. This is important when considering problem solving, as problem solving is by nature difficult and requires persistence and perseverance to keep going when barriers are encountered.

The four teachers' reports show how easily exclusion from learning mathematics can start in schools. In many cases in these reports it presents as self-exclusion, staying quiet in lessons or simply declaring "I'm stuck". The more this need for self-exclusion is challenged by ideas such as those used in the inclusive and collaborative environment developed in the Spanish school, the more empowered students will be to learn mathematics.

These studies have shown the teachers just how narrow some students' growth zone can appear to be when they start to try and sustain themselves within that emotional space. It is clear from both Silverside's (2013) report and Chisholm's (2017) that for many students it is hard to distinguish between learning and panic. They can find anything beyond the utterly familiar very difficult to deal with, in the case of Chisholm's students moving directly to stating firmly "I'm stuck" in order to avoid facing the difficult emotions encountered in their "red" zone. However, in all these cases there is hope; certainly not all students overnight became efficient and effective problem solvers but with some persistence and perseverance on the part of their teachers, every student in these studies found ways to get into their growth zone and to stay there longer.

The difficulty that getting into their Growth Zone presents for some students may well mean that they need to build a set of personal resources to sustain them in their Growth Zone. Chisholm developed the "Stuck Poster" to give his students ideas to sustain them, the Spanish school suggested collaboration with peers in the "3 before me" approach and Silversides suggested that her students look at what they do in less formal learning settings to give them clues about how they can learn without anxiety.

The required use of problem solving may well give both teachers and students the opportunity to experience that mathematics is not all about remembering the right way to apply an algorithm. Problem solving is difficult; there is no one way that will always get to the correct answer. Calm thought will be needed, perseverance will be rewarded by the pleasurable feelings of successfully overcoming barriers and developing resilience and new learning in the process. However unless students learn that they do not need to struggle alone, help is available from peers, texts, adults, the internet, they may become anxious. Calm, reflective thinking does not easily occur, if at all, when someone is anxious.

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