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Planar millimetre-wave antenna simultaneously producing four orbital angular momentum modes and associated multi-element receiver array

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Abstract: We present the design and simulation of a planar Bull's eye antenna that can simultaneously produce four orbital angular momentum (OAM) modes with up to 2.4 GHz impedance bandwidth at 60 GHz. OAM modes $l = \pm 3, \pm 13$ are produced simultaneously by two ring resonators in the centre of the antenna. The peak gain is 13.1 (8.1) dBi at ± 10 (± 25) degrees for mode $l = \pm 3$ (± 13), with all modes present over a 30 degree angular range (15 to 45 degrees from the boresight). For reception and demultiplexing we use a discretely sampled partial aperture receiver (DSPAR), and show algebraically that such systems can achieve ideal mode orthogonality in cases where existing partial aperture receivers cannot. We present a DSPAR design to complement the Bull's eye antenna, needing only four simple receiving elements and hence more readily scaling to work over longer link distances than systems relying on horn antennas with integrated spiral phase plates.

1. Introduction

Millimetre-wave radio communications channels can gain additional capacity by encoding information using orbital angular momentum (OAM) modes [1]. Early research on OAM was conducted in the optical regime, using spiral phase plates to produce OAM [2]. Work on OAM generation and communication at millimetre-wave frequencies initially used spiral phase plates [3,4] and phased arrays [5] whose elements must be carefully arranged for good transmission and reception [6]. Compared to the size, weight and complexity of phased arrays, single antenna solutions are attractive, with elliptical patch antenna [7] and travelling wave ring-resonator [8] approaches both having been successfully demonstrated. Patch antennas have relatively low gain compared to arrays, while the travelling wave ring-resonator requires a parabolic reflector that occupies a significant volume compared to a spiral phase plate coupled with a horn antenna of equivalent gain. Given the short range of many millimetre-wave wireless communication links, array antennas are often required to counteract path loss, and therefore compact high-gain antennas are desirable. Following our earlier work on leaky-wave Bull's eye antennas [9], in this paper we propose a flat plate antenna that can similarly achieve OAM transmission but in a planar form factor that is more convenient for deployment in short-range 60 GHz communications links, and easier to manufacture because it needs only conventional computer numerical control machining of aluminium. Our planar designs can simultaneously produce either two or four OAM modes (depending how many ring

resonators are included) and may be 3D printed and coated in metallic paint for rapid production of a prototype. The structure of the paper is as follows. Section two describes the design and simulation of an antenna with a single ring resonator feed, capable of producing two OAM modes simultaneously. Section three considers the constraints on mode choice imposed by using partial aperture receivers. Section four shows the design and simulation of a dual ring resonator Bull's eye antenna, producing four OAM modes simultaneously. The transmitter is designed so as to work with known OAM receiver architectures. Section five investigates the mode orthogonality constraints imposed by discretely sampled partial aperture receivers, leading to a receiving and de-multiplexing scheme that exhibits orthogonality for a wider range of OAM modes than continuously integrated partial aperture receivers. It is also more practical to implement at link distances exceeding a few metres than those based on horn antennas with integrated spiral phase plates. We conclude the paper in Section six.

2. Bull's eye antenna producing two OAM modes

2.1. Design

We use a travelling wave ring resonator of inner radius 1.62 mm, outer radius 5.38 mm and height 1.88 mm, energised by two WR-15 waveguide ports so as to simultaneously create two independent OAM modes, of $l = \pm 3$, centred at 60 GHz. The design of the travelling wave ring resonator is similar to the one presented in [8], where further details of the physical origin of the corresponding OAM mode can be found in their Fig. 1(b). We also note that the theoretical formulation of the far-field radiation expression for a circular traveling-wave antenna with a top slit is presented in [10] and suggests that a model based on a wire bent in a circle and fed with a constant electric current, but having a consecutive phase around the circle, can provide an accurate model of an antenna fed with a circular travelling wave. It is noted in [10] that the radiation properties of OAM modes generated by such an antenna are stable even far away from the antenna, and support communications applications through multiplexing of mode numbers. For our design, the input reflection coefficient is lower than -10 dB for 2.5 GHz bandwidth. The OAM modes leak from the resonator via a continuous 0.5 mm slit in its narrow outer wall, shown on the diagram in Fig. 1a. The resonator is surrounded by a 60 mm diameter Bull's eye antenna. The Bull's eye antenna comprises a corrugated surface that is optimised for radiating surface waves in a single direction. A standard linearly polarised Bull's eye antenna of this diameter has a gain of 18 dBi [9] compared to 30 dBi for a similar sized parabolic antenna [11]. However, the parabolic antenna in [8] is approximately 50 mm high (excluding the hybrid coupler on the reverse) giving an aspect ratio of 100%, whereas our structure has an aspect ratio of less than 5%. Our structure is thicker than an eight element patch array, but needs less

precise and less expensive fabrication techniques because it does not use substrate waveguides (which are sensitive to line edge roughness). This is useful for mechanical reasons when fitting a communication system into a tight space. The dimensions of the Bull's eye antenna are given in [9], except that only rings 2 - 5 are required. The structure, including the feed, is shown in Fig. 1, with a 3D drawing of the top (radiating) side of the antenna in Fig. 1a and a plot of the resulting electric field for one of the OAM modes in Fig. 1b.

2.2. Simulation results

The structure was simulated using CST Microwave Studio's time-domain solver. Open boundaries were spaced 10 wavelengths from the structure, and the assumption of lossy aluminium metal with conductivity $3.56 \times 10^7 \text{ S.m}^{-1}$. A 3D view of the far-field radiation pattern is shown in Fig. 2a, with a plot of the far-field cut for $\varphi = 0^\circ$ in Fig. 2b. There is a characteristic null along the boresight, with a peak gain of 13.5 dBi at $\pm 11^\circ$ from the boresight. There is an offset of 2.3 dB as a consequence of the cut we have taken, and the cause is the asymmetric feeding network. The gain is reduced compared to a standard Bull's eye antenna because there is no direct emission from the feed. Compared to an eight-element array, our far-field pattern is smoother since it does not exhibit the characteristic sidelobes, as expected from a Bull's eye antenna [9].

For comparison, we modelled an eight-element array of WR-15 waveguide apertures on the circumference of a circle of radius 3λ . Typically, radii of previously reported multi-element arrays range from $0.5-2\lambda$, and use isotropic sources or patch antennas [1,5]. For the sake of a tougher comparison, we used a larger aperture array with higher gain elements. The eight element array had lower peak gain

Fig. 1. Flat-plate antenna for OAM ± 3

a 3D view of radiating top side with cut-away view of the central resonator (inset)

b Electric field distribution for the $l = +3$ mode, on a plane 10λ above the antenna

(10.8 dBi) and a larger null, with peak gain at $\pm 16^\circ$ from the boresight for $l = \pm 3$.

The phase of the far-field radiation from our Bull's eye antenna is shown in Fig. 2c, and clearly exhibits a spiral phase pattern that accumulates 6π of phase in each clockwise rotation about the boresight, indicating an OAM mode with $|l| = 3$, with phase increasing in a right handed manner indicating the positive mode $l = +3$. In the following, for conciseness, we do not show the corresponding data for the other mode because it is nearly identical (except reversed in its handedness). We analyse the mode purity by calculating the phase difference at a notional two-element array in the far-field as described in [12]. The results are shown in Fig. 2d for three values of theta ($\theta = 4, 11, 26^\circ$) where theta is the angle from the boresight as shown in

Fig. 2. Simulations of the single ring resonator antenna producing $l = +3$ OAM mode ($l = -3$ results are similar, not shown)
a 3D view of far-field radiated pattern
b Gain in the xz -plane
c Phase plot for the radial component of the electric field (E_ϕ) 10λ above the antenna surface.
d Mode purity for three different angles of theta, confirming the mode is $l = +3$.

the inset. The chosen values correspond to peak gain, and -5 dB on either side of that. For angular separations of the receive elements of $\alpha > 60^\circ$, the estimated OAM mode value is correct to ± 0.3 for all values of theta within the range plotted.

Eight element arrays perform better for small values of α , and so can use smaller separations between the receive elements [12]. For a fixed size receiving array at $l = \pm 3$, these effects approximately offset each other, giving our design similar performance but with reduced complexity. Assuming a propagation distance of 10 m, $\alpha = 60^\circ$, and $\theta = 4^\circ$, the receive elements would be separated by 73 cm. For an eight element array, the same gain is achieved at $\theta = 10^\circ$, and the receive separation can be reduced to $\alpha = 20^\circ$, giving 61 cm separation between the receive elements. However, our antenna has a smaller null, can simultaneously produce two modes, and permits a much larger range of receiver element separations if desired. This larger parameter space may offer an advantage to the design of multi-element receive arrays [12], particular when multiple modes are transmitted simultaneously.

3. Partial aperture receivers for multiplexed OAM modes

Recent work on multiplexing OAM modes in the optical regime has taken advantage of the relatively limited spread of the transmitted mode in optical fibre-based transmission schemes, for example multiplexing with two OAM modes $l = \pm 1$ and two linear polarisations in conjunction with 4x4 MIMO-DSP processing to recover the data from the four channels after transmission over 5 km of fibre [13]. In this way, the whole spatial extent of the mode is used in the de-multiplexing process, allowing all OAM modes to retain their mutual orthogonality [14-16]. A similar approach has been applied to millimetre wave communications links over a 2.5 m distance using four antennas and spiral phase plates operating with modes $l \in \{\pm 1, \pm 3\}$ and cross-talk between of -12 to -14 dB [4]. However, whole-aperture receivers are unattractive for longer free-space link distances because the receiving aperture must increase with the link distance in order to receive sufficient power. Partial aperture receiving methods are possible, and have been used for free space optics [17], and in the infra-red [18], but they can introduce cross talk between modes [19]. However, a sub-set of the modes can retain ideal orthogonality [18] for a given angular aperture of $2\pi/n$ as follows:

$$U = \frac{n}{2\pi} \int_0^{2\pi/n} e^{j(l_1 - l_2)\varphi} d\varphi = \begin{cases} 1, & l_1 = l_2 \\ 0, & l_1 - l_2 = mn \\ > 0, & \text{otherwise,} \end{cases} \quad (1)$$

where m is an integer. Table 1 presents possible pairs of mode numbers for values of $n \in \{2, \dots, 6\}$ and $m \in \{1, \dots, 5\}$ that are obtained using the rule $l_2 = l_1 + mn$ [18]. The location of the plus-minus symbol is

used to indicate the number of orthogonal modes that are supported. For example, for $m = 1$, and $n = 6$, the table entry $\pm(1,7)$ means that modes $l_1 = 1$ and $l_2 = 7$ are orthogonal to each other, and so too are $l_1 = -1$ and $l_2 = -7$, but not $l_1 = 1$ and $l_2 = -7$, nor $l_1 = -1$ and $l_2 = 7$, i.e. only two-way orthogonality is achieved. The notation $(\pm 3, \pm 13)$ means that four-way orthogonality exists, e.g. $l_1 = -3$, $l_2 = 3$, $l_3 = -13$, and $l_4 = 13$ are mutually orthogonal for the given values of m and n (here $m = 5$ and $n = 2$). From Table 1, at $n = 6$, simultaneous transmission of four OAM modes is possible if the first mode is $l = 3$, but not if it is $l = 1$. For $n = 2$, simultaneous transmission of four OAM modes is possible for all five values of m shown. This gives design flexibility that suits the Bull's eye antenna, because it has mode purity for $\alpha > 60^\circ$. This motivates the search for Bull's eye design that can produce four modes simultaneously, the first mode pair being $l = \pm 3$, and the second mode pair being $l \in \{\pm 5, \pm 6, \pm 7, \pm 9, \pm 11, \pm 12, \pm 13, \pm 15, \pm 18\}$ which are values taken from rows $n = 2, 3$ in Table 1. This ensures that the receiver design is not unduly constrained, and can therefore be a whole aperture receiver, partial aperture receiver with $n = 2$, or a discretely sampled receiver. We note in passing that, according to these limits, a multi-element array *transmitter* would require a minimum of 14 elements in order to transmit two different mode numbers simultaneously. Typically, mode purity is only obtained over apertures approximately $\alpha < 60^\circ$ [12], corresponding to $n \geq 6$, limiting the available mode choices compared to an antenna that can produce mode purity over a larger aperture.

Table 1 Orthogonal mode pairs for partial apertures of $2\pi/n$, for selection of starting mode numbers

n	m	1	2	3	4	5
6		$\pm(1,7)$ $(\pm 3, \pm 9)$	$\pm(1,13)$ $(\pm 3, \pm 15)$	$\pm(1,19)$ $(\pm 3, \pm 21)$	$\pm(1,25)$ $(\pm 3, \pm 27)$	$\pm(1,31)$ $(\pm 3, \pm 33)$
5		$\pm(3,8)$	$\pm(3,13)$	$\pm(3,18)$	$\pm(3,23)$	$\pm(3,28)$
4		$\pm(3,7)$	$\pm(3,11)$	$\pm(3,15)$	$\pm(3,19)$	$\pm(3,23)$
3		$(\pm 3, \pm 6)$	$(\pm 3, \pm 9)$	$(\pm 3, \pm 12)$	$(\pm 3, \pm 15)$	$(\pm 3, \pm 18)$
2		$(\pm 3, \pm 5)$	$(\pm 3, \pm 7)$	$(\pm 3, \pm 9)$	$(\pm 3, \pm 11)$	$(\pm 3, \pm 13)^*$

4. Bull's eye antenna producing four OAM modes

We introduce into the Bull's eye antenna a second ring resonator at mode number 13, so as to achieve four-way orthogonality of the OAM modes with a partial aperture receiver of arc 180° ($n = 2$). Fig. 3 shows the two ring resonators in different colours for ease of identification (red for $l = \pm 3$, green for $l = \pm 13$). Each ring resonator is fed as described in Section two, giving four modes in total. The top

structure has a circular resonator of outside radius of 5.38 mm, surrounded by two indented rings, for a total outside radius of 16.4 mm. This structure is designed to produce OAM mode $l = \pm 3$. The bottom structure produces OAM mode $l = \pm 13$ and has a circular resonator of outside radius of 16.2 mm, with four indented rings. The overall structure is $73 \times 73 \times 5 \text{ mm}^3$. In order to preserve energy efficiency and permit longer transmission distances, low OAM mode numbers are preferred. For the inner feed, OAM modes $l = \pm 3$ are the lowest mode numbers for which we can build a travelling wave ring resonator. For the outer feed, the radius of the ring resonator must be large enough to surround the inner portion of the antenna as well as providing modes that are orthogonal to $l = \pm 3$; the smallest mode numbers that satisfy these two criteria are $l = \pm 13$. We note that Bull's eye antennas typically produce smaller nulls than multi-element arrays, reducing the penalty of choosing slightly larger mode numbers. Fig. 4a shows the magnitude of the electric field and Fig. 4b the phase distribution on a $173 \times 173 \text{ mm}^2$ plane 10λ (50 mm) above the antenna, when two OAM modes $l = +3$ and $+13$ are produced simultaneously. Fig. 4a shows the characteristic null

Fig. 3. Stacked OAM Bull's eye antenna for $l = \pm 3$ and $l = \pm 13$

a 3D view

b Cut view

of OAM modes $l \neq 0$, and Fig. 4b, the phase singularity at boresight ($x = y = 0$ mm). Fig. 4c shows the mode purity analysis for $\theta = 15, 30$ and 45° , for mode $l = +3$ and $l = +13$ (modes $l = -3$, and $l = -13$ are similar).

Both modes are present across a wide range of angles of theta, beyond the angles shown. This is confirmed by plotting a cut of the far-field for each mode separately, and then combined, in Fig. 4d. The

Fig. 4. Simulations of the double ring resonator antenna for modes $l = +3, +13$ (results for $-3, -13$ are similar, not shown)
a Absolute value of the electric field distribution on a 173×173 mm² plane 10λ above the antenna
b Phase distribution on the same plane as a
c Mode purity analysis for modes +3, and mode +13, conducted separately but plotted on same graph
d xz -plane far-field cut for OAM stacked Bull's eye $l = +3, +13$ and $(+3, +13)$. The Front/Back ratio is 32 dB for $l = +3$, 23 dB for $l = +13$, and 26 dB for the combined mode with $l = +3, +13$.

impedance bandwidth is 2.4 GHz at mode $l = \pm 3$, and 0.6 GHz at mode $l = \pm 13$, which is consistent with the increase in ring resonator quality factor with mode number. The peak gain is 13.1 dBi at 10° from boresight for mode $l = \pm 3$, and 8.1 dBi at 25° from boresight for $l = \pm 13$.

Our results show the characteristic increase in null width with mode number. The ratio of the divergence angle of two different mode numbers is related to the absolute value of the mode numbers and the standard deviation of the spatial distribution of the beams, which is given by the square root of the radial variance of the intensity distribution [20]:

$$\frac{\alpha_{l_4}}{\alpha_{l_2}} = \frac{(|l_4| + 1) r_{rms}^{l_2}(z)}{(|l_2| + 1) r_{rms}^{l_4}(z)} \quad (2a)$$

$$r_{rms}^{l_n}(z) = \sqrt{2\pi \int_0^\infty r^2 I_{l_n}(r, z) r dr} \quad (2b)$$

For our antenna, the divergence angle for mode number $l_2 = 3$ is $\alpha_{l_2} = 10^\circ$ and for $l_4 = 13$ is $\alpha_{l_4} = 25^\circ$. The divergence angle has scaled according to $\alpha_{l_4}/\alpha_{l_2} = (|l_4|/|l_2|)^{0.625}$, which is in between the expected limiting values of 0.5 – 1.0 in the exponent [20], with 0.5 representing a constant beam waist, and 1.0 representing a constant standard deviation of the spatial distribution. The intermediate value is obtained because we use different size apertures in the generation of the two modes. Going further, this ratio of $\alpha_{l_4}/\alpha_{l_2} = 2.5$ implies that the ratio of the standard deviations of the spatial distributions at a z value in the far-field should be $r_{rms}^{l_2}/r_{rms}^{l_4} = 0.714$, according to Eq. (2a). Applying Eq. (2b) to the intensity data from which Fig. 4d was plotted, yields a ratio of $r_{rms}^{l_2}/r_{rms}^{l_4} = 0.733$. This confirms that the divergence of the two different modes is behaving in accordance with expectation. We note that the Bull's eye antenna's divergence angles are lower, and scale better with mode number, than for multi-element arrays as we found in Section two.

Despite the different peak gain positions for the two mode numbers, there is enough overlap in the two modes to allow both to be detected over an angular range in excess of 30° ($15^\circ \leq \theta \leq 45^\circ$). However, a two-element receive array implementing the phase gradient method is unable to be used in this case, because it cannot discriminate between modes of different numbers when both are present (except at the extreme edges of the angular range where one mode is substantially stronger than the other). A multi-mode receiver of some type is preferred, so that it can be placed at a position where the power of both modes is high, and hence maximise the link distance.

5. Multi-mode receiver design

Our goal in this section is to establish whether multi-mode receivers made of multiple receive elements (discretely sampled partial aperture receivers, or DSPAR) have the same constraints on orthogonality of modes as continuously integrated partial aperture receivers. In this way we extend on previous work that looked at the bit error rate of 2-OAM and 4-OAM systems using multiple-antenna receivers but that did not explicitly make the connection between mode number choice and system performance [6]. We also highlight the benefit of the non-uniform spacing of the receive elements implicit in the DSPAR concept, by comparing performance to uniformly sampled partial aperture receivers (USPAR). Assuming the DSPAR or USPAR has N elements, we can establish a discrete form of Eq. (1) using point summation (which can be seen as representing a portion of a uniform circular array):

$$\begin{aligned} U(l_1, l_2) &= \frac{1}{\varphi_{N-1} - \varphi_0} \int_{\varphi_0}^{\varphi_{N-1}} e^{j(l_1 - l_2)\varphi} d\varphi \\ &\approx \frac{1}{N} \sum_{k=0}^{N-1} e^{j(l_1 - l_2)\varphi_k} \end{aligned} \quad (3)$$

The lower bound on N is related to the number of OAM modes. Here we target four OAM modes (4-OAM), so choose $N = 4$, because it was not significantly outperformed by DSPAR with more elements in the test cases we explored, while DSPAR and USPAR with $N < 4$ were not satisfactory. We note that DSPAR with $N = 4$ is intended for use with 4-OAM transmitters such as the Bull's eye antenna from Section four, whereas a two-element phase gradient method detector will suffice for 2-OAM, such as when using a Bull's eye antenna from Section two. Since the number of elements $N = 4$ is small, there is limited performance benefit in considering uneven amplitude weightings, especially given the cost implication of adding 16 additional amplifiers to adjust the relative amplitudes of the signals.

For each DSPAR design, we calculated the value of U according to Eq. (3) for each of the 16 possible combinations of the four mode numbers we are notionally transmitting, with the restrictions that $l_1 = -l_2$ and $l_3 = -l_4$ for $l \in \{l_1, l_2, l_3, l_4\}$. For the twelve of these calculations that measure unwanted leakage between modes, we refer to the result as the side mode rejection ratio (SMRR). The SMRR between the given modes l_a and l_b (for an ideal noise-free, fading-free) channel is

$$\text{SMRR}(l_a, l_b) = 20 \log_{10} \left(\frac{U(l_a, l_b)}{U(l_a, l_a)} \right), \quad (4)$$

where $a, b \in \{1, 2, 3, 4\}$ and $a \neq b$. Table 2 summarises the results for a subset of mode combinations given in Table 1.

Table 2 Side mode rejection ratio (SMRR, dB) for DSPAR (non-uniform spacing) and USPAR (uniform spacing) with $N = 4$ receivers, using point summation, for selected mode pairs from Table 1.

Mode pair ($\pm l_1, \pm l_2$)	n	DSPAR positions ($^\circ$) *	SMRR (dB): DSPAR / USPAR			
			($-l_1, +l_1$) ($+l_1, -l_1$)	($-l_1, -l_2$) ($+l_1, +l_2$) ($-l_2, -l_1$) ($+l_2, +l_1$)	($-l_1, +l_2$) ($+l_1, -l_2$) ($-l_2, +l_1$) ($+l_2, -l_1$)	($-l_2, +l_2$) ($+l_2, -l_2$)
(3,5)	2	[0,62,80,158]	-13.2 / 0.0	-17.6 / -12.0	-15.8 / -12.0	-15.3 / -12.0
(3,9)	6	[0,18,31,45]	-21.4 / -12.0	-21.4 / -12.0	-19.7 / -12.0	-13.3 / 0.0
(3,11)	4	[0,24,37,62]	-19.1 / -324	-18.3 / -12.0	-17.8 / -7.27	-22.4 / -7.27
(3,12)	3	[0,22,34,56]	-21.5 / -12.0	-17.1 / 0.0	-12.1 / -12.0	-23.1 / -12.0
(3,15)	6	[0,17,31,47]	-31.6 / -12.0	-16.1 / -12.0	-22.1 / 0.0	-17.8 / -12.0
(3,18)	5	[0,27,35,62]	-27.9 / -12.0	-18.4 / 0.0	-12.7 / -12.0	-312 / -12.0
(3,19)	4	[0,32,40,73]	-22.9 / -324	-16.7 / -12.0	-22.1 / -7.27	-14.7 / -7.27
(3,21)	6	[0,21,29,51]	-31.6 / -12.0	-22.1 / 0.0	-13.7 / -12.0	-14.9 / -12.0
(3,23) ⁺	5	[0,20,27,46]	-20.2 / -12.0	-21.2 / -12.0	-11.3 / -23.1	-14.6 / -0.98
(3,27)	6	[0,23,30,53]	-317 / -12.0	-19.6 / -12.0	-309 / -12.0	-301 / 0.0
(3,28)	5	[0,22,30,51]	-31.6 / -12.0	-31.4 / -12.0	-17.9 / -0.98	-12.4 / -23.1
(3,33)	6	[0,15,30,45]	-323 / -12.0	-303 / -12.0	-315 / 0.0	-297.5 / -12.0

* The position of the receivers for USPAR are: [0,60,120,180] for $n = 2$, [0,40,80,120] for $n = 3$, [0,30,60,90] for $n = 4$, [0,24,48,72] for $n = 5$, and [0,20,40,60] for $n = 6$.

⁺ For mode pair (3,23), the DSPAR SMRR slightly exceeds -12dB when l_1 and l_2 have opposite signs, and requires a weighting to be applied in summation, or additional receivers, to overcome (nonetheless it is still better than USPAR).

5.1. Finding orthogonality not predicted in continuous method

We note that in Table 2, the apertures defined by the discrete points are typically smaller than the $2\pi/n$ constraint implied by Eq. (1), implying that the constraints on two-way and four-way orthogonality are weaker for the discrete case, even if some DSPAR designs require more than the absolute minimum number of $N = 4$ elements. The relaxation of the constraint is not surprising because introducing discrete sampling is equivalent to coding the aperture, which is known to enhance signals [21].

We now give a procedure for algebraically finding receiver positions that give ideal OAM mode orthogonality with DSPAR that would otherwise appear to be prohibited according to Eq. (1). We minimise Eq. (3) through appropriate choice of the receiver positions φ_k , for DPSAR with even numbers of elements N , by placing the receivers such that the individual terms in the summation fall, in pairs, on opposite sides of the circumference of the unit circle in the complex plane representing the possible values of the exponential. The relative positioning of the DSPAR receive elements is shown on the complex

plane of the exponential in the sum terms of Eq. (3) in Fig. 5a, with the actual position in real space in Fig. 5b for an example case where $l_1 = 2$, $l_2 = 8$, with a partial aperture corresponding to $n = 4$, i.e. 90° or $\pi/2$ radians. Note that the ordering around the circle in the complex plane deliberately does not match the ordering around the arc in real space. It is only for convenience in the explanation that the pairs are aligned along the real and imaginary axes, but each member of a pair of points must be on opposite sides of the circle, and ideally there is the maximum practical spacing between pairs around the complex circle (note this does imply even spacing in the physical arrangement). The general procedure for finding the points (in order of $\varphi_0, \varphi_3, \varphi_1, \varphi_2$) is as follows:

1. Locate φ_0 at an arbitrary point within the aperture (e.g. $\varphi_0 = 0$).
2. Find φ_3 according to

$$\varphi_3 = \varphi_0 + \frac{(1 \pm 2q)\pi}{l_1 - l_2}, \quad 0 \leq \varphi_3 \leq \frac{2\pi}{n}, \quad (5a)$$

where $q \in \{0, 1, 2, 3, \dots\}$ and need not be kept the same value in the remaining Eqs. (5b,c).

3. Then locate φ_1 at an arbitrary point within the aperture, but preferably using the rule

$$\varphi_1 = \varphi_0 + \frac{(\frac{1}{2} \pm 2q)\pi}{l_1 - l_2}, \quad 0 \leq \varphi_1 \leq \frac{2\pi}{n}, \quad (5b)$$

4. Such that

$$\varphi_2 = \varphi_1 + \frac{(1 \pm 2q)\pi}{l_1 - l_2}, \quad 0 \leq \varphi_2 \leq \frac{2\pi}{n}. \quad (5c)$$

Care must be taken in choosing the signs of the mode numbers. The results of Eqs. (5) hold for modes that meet the same polarity criteria, which we define as either positive polarity ($l_1 l_2 > 0$) or negative polarity ($l_1 l_2 < 0$). We take as an example $l_1 = 2$, $l_2 = 8$, with a partial aperture corresponding to $n = 4$, i.e. 90° or $\pi/2$ radians. For this mode pair, $l_2 - l_1 \neq mn$ for any integer m , hence according to Eq. (1) the modes cannot be orthogonal to a partial aperture receiver of arc $\pi/2$. Following Eqs. (5) yields one possibility as $\varphi_k \in \{0, 3\pi/20, \pi/4, \pi/2\}$. This gives a DSPAR with ideal two-way orthogonality over an aperture of

exactly the same extent as for which Eq. (1) shows it cannot be obtained using a continuously integrated partial aperture receiver.

DSPAR element positions φ_k for four-way orthogonality can be calculated by solving each of Eqs. (5) simultaneously for all possible values of (l_1-l_2) in Eq. (3), which for the present example is 4, 6, 10, 16. Since this is a system of congruencies, it has strict conditions on the existence of solutions, which do not represent an accurate indication of the degree of orthogonality that can be obtained in practice (because we can accept our free parameter q deviating slightly from integer values). We calculated some example receiver designs using a Monte Carlo approach with ensemble size of 100,000, and these are listed in Table 3. Note that we found a design for four-way orthogonality with the mode set $l = (\pm 2, \pm 8)$, with $\varphi_k \in \{0, 33, 53, 87^\circ\}$, confirming that the algebraic limits in this case do not represent a limit on the available receiver designs. This section confirms via an algebraic and a numerical approach that DSPAR can obtain ideal orthogonality between a wide number of modes, including those that cannot be achieved in continuously integrated partial aperture receivers, and thus provides a sound basis for using DSPAR receivers in multiplexed OAM links. We note that DSPAR is inherently lower in energy efficiency than a whole aperture receiver, but has the advantage of producing a smaller, lighter physical structure and it is also consistent with the traditional ability of radio to serve more than one user simultaneously (a single transmitter can communicate to more than one DSPAR receiver simultaneously, which is not possible with whole aperture receivers).

Fig. 5. Selection of receive antenna positions

a Position on the complex plane of the exponential from Eq. (3), dashed arrow shows order of point selection process
b Position in real space

Table 3 DSPAR element positions φ_k in degrees, for selected cases where DSPAR obtains orthogonality (SMRR < -12 dB) unavailable to a continuous receiver for $n = 4$, or USPAR with receiver positions at [0,30,60,90].

Mode pair ($\pm l_1, \pm l_2$)	DSPAR positions ($^\circ$)	SMRR (dB): DSPAR / USPAR			
		($-l_1, +l_1$) ($+l_1, -l_1$)	($-l_1, -l_2$) ($+l_1, +l_2$) ($-l_2, -l_1$) ($+l_2, +l_1$)	($-l_1, +l_2$) ($+l_1, -l_2$) ($-l_2, +l_1$) ($+l_2, -l_1$)	($-l_2, +l_2$) ($+l_2, -l_2$)
(2,8)	[0,33,53,87]	-18.8 / -12.0	-15.3 / -324	-27.2 / -7.27	-23.6 / -12.0
(2,12)	[0,38,52,90]	-24.7 / -12.0	-15.3 / -7.27	-23.2 / -7.27	-39.2 / 0.0
(2,13)	[0,34,48,83]	-26.6 / -12.0	-25.0 / -1.55	-14.2 / -303	-18.9 / -7.27
(3,8)	[0,33,47,79]	-19.9 / -324	-23.1 / -13.0	-12.2 / -1.55	-15.4 / -12.0
(3,10)	[0,26,40,67]	-20.2 / -324	-27.5 / -13.0	-18.7 / -1.55	-21.2 / -12.0
(3,12)	[0,22,34,56]	-21.5 / -324	-17.1 / -319	-12.0 / -303	-23.1 / 0.0
(3,14)	[0,31,47,78]	-27.8 / -324	-14.1 / -1.55	-21.2 / -13.0	-17.8 / -12.0

5.2. DSPAR for dual resonator Bull's eye

We design a DSPAR with $N = 4$ elements for four-way orthogonality for modes $l = (\pm 3, \pm 13)$ in a partial aperture of 180° using the same Monte Carlo approach as before, with ensemble size of 100,000. We set an 180° partial aperture ($n = 2$) as the upper bound for our search space. It is not necessary to constrain the DSPAR aperture by Eq. (1), because we have shown it does not directly apply. There were 172 candidate arrays with the worst SMRR of -12 dB or better, as shown in Fig. 6a. We chose a design with receivers at positions of $\varphi = 10, 43, 61$, and 94° , giving us an angular dimension of 84° for the full array, so that the antennas at $43, 61$, and 94° are effectively referenced to the antenna at 10° , rather than each other. The theoretical SMRR, calculated for OAM modes $(\pm 3, \pm 13)$ with Eq. (3), is shown with in Fig. 6b and clearly illustrates the four-way orthogonality.

Note that this design is different to the ‘two-pairs’ scheme proposed in [6]. In order to test the sensitivity to rotational fading, so as to confirm that no special alignment between the mode and the receiver must be made (at least rotationally), the SMRR were analysed using far-field data for mode 3 and mode 13 from the dual resonator antenna at $\theta = 30^\circ$, where both modes have approximately equal power, with the receive antenna offset progressively at 1° rotational increments. Those results were plotted in Fig. 7a. The SMRR was less than -8 dB for all positions and -15 dB in average for orthogonal mode 3 and mode 13. Results are only shown for modes +3 and +13 for clarity, because the opposite handed modes were similar.

Fig. 6. Results of the Monte Carlo process and illustration of the four-way orthogonality for the receivers at position $\varphi = 10, 43, 61, \text{ and } 94^\circ$

a Top 172 results from Monte Carlo design process with ensemble size 100,000, graded according to best case side mode rejection ratio (SMRR), and inset, the angular position in degrees for the first 20 antennas

b SMRR calculation illustrating the four way orthogonality for modes $l = (\pm 3, \pm 13)$ calculated with the point summation (Eq. 3) for one of the optimised DSPAR. For comparison we show the results for USPAR with an equivalent partial aperture of $n = 4$, and one with $n = 2$, neither of which achieves orthogonality.

We propose an installation concept for the dual resonator Bull's eye transmitter (Tx) and DSPAR receivers (Rx1-Rx4) in Fig. 7b that facilitates OAM links over longer distances than implementations using spiral phase plates. We envisage the use of masts if a free standing system is required, or for the antennas to be mounted on the opposing faces of buildings or other infrastructure. Antennas are exaggerated in size for clarity, but the overall scheme is to scale. The receive array is distributed over an area, but unlike conventional large aperture antennas, it is sparsely populated and hence requires no interstitial structures, overall requiring less weight, giving lower windage and costing less to install. We analysed the sensitivity to the masts moving, finding that acceptable performance is obtained even with up to 3 cm of lateral movement over 3 m link distance (at larger link distances the sensitivity to movement is reduced).

The implementation of the DSPAR receiver is straightforward. We show four antennas in Fig. 7b. These are connected by a phase-preserving system such as waveguides of equal length, or radio-over-fibre [22], to the receiving electronics. Each of the antenna's signals (R_1, R_2, R_3, R_4) are split into four to create sixteen signals ($R_1|_{l_1}, \dots, R_1|_{l_4}, R_2|_{l_1}, \dots, R_4|_{l_4}$). Each signal is then phase shifted according to the scheme outlined above. Groups of four signals are then summed to give the four OAM modes. The four resulting signals are then mixed down to baseband. It may be necessary to adjust each phase shift so as to

compensate for small relative phase shifts arising from manufacturing tolerances or non-idealities in the installation.

Fig. 7. Four element receiver array design optimised to suit transmissions from dual ring resonator Bull's eye antenna
a SMRR calculation for modes +3 and +13 when all four modes $\pm 3, \pm 13$ are transmitted from the Bull's eye antenna, for the optimal antenna array from *a* at each possible ϕ position around the mode (i.e. examining the sensitivity to rotational fading, which is relatively weak)
b 3D view of the installation concept for the $l = (\pm 3, \pm 13)$ OAM with four receivers at $\phi = 10, 43, 61$, and 94° with $\theta = 30^\circ$, showing an architecture over a longer link distance with a transmitting (Tx) OAM antenna and 4 receivers (Rx1, Rx2, Rx3 and Rx4) mounted on masts

Our work is expected to find application wireless communications systems that require enhanced spectral efficiency over line of sight links, including wireless back haul, and short range communications within data centres [4]. These line of sight links offer no diversity gain, eliminating the advantages of MIMO, and the line of sight requirement precludes the use of multiple links via beamforming techniques.

6. Conclusion

We present the design and simulation of a planar millimeter-wave antenna for simultaneously producing four orbital angular momentum modes with up to 2.4 GHz impedance bandwidth centred about 60 GHz, and an associated receiving and de-multiplexing scheme based on a sparse four-element array of simple antennas such as dipoles or patches. The planar transmit antenna is of the Bull's eye type, incorporating two ring resonators each producing two modes, for four modes in total. We chose modes $l \in \{\pm 3, \pm 13\}$ which are fully mutually orthogonal even for receivers with particular partial apertures. The Bull's eye antenna has less complexity than the equivalent multi-element transmit array, particularly when mode number constraints imposed by the receiver are considered. It is straightforward to fabricate in

aluminium using conventional computer numerical controlled machines, but can also be three dimensionally printed. For receiving and de-multiplexing, we do not use the same antenna because at large link distances there is little power radiated along the boresight (for any OAM mode $l \neq 0$, regardless of transmitter antenna type). We showed that multi-element receive arrays, our so-called discretely sampled partial aperture receivers (DSPAR), can achieve ideal orthogonality between modes in cases for which it is denied to continuously-integrated partial aperture receivers. In addition to compactness, and freedom of mode choice, the main practical benefit of using DSPAR as receivers in multiplexed OAM links is that they do not require a large solid antenna structure, but instead only a small number of compact, low-cost single-element antennas, joined by waveguide or radio-over-fibre links so as to maintain phase coherence. Hence the DSPAR receiver can be readily scaled to receive transmissions from the Bull's eye antenna at much larger distances than is practical for existing millimetre-wave OAM links that use horn antennas incorporating spiral phase plates.

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8. References

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