The European Large Area ISO Survey – IV. The preliminary 90-\(\mu\)m luminosity function

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ABSTRACT
We present the luminosity function of 90-\(\mu\)m-selected galaxies from the European Large Area ISO Survey (ELAIS), extending to \(z = 0.3\). Their luminosities are in the range \(10^9 < h_{65}^2 L/L_\odot < 10^{12}\), i.e. non-ultraluminous. From our sample of 37 reliably detected galaxies in the ELAIS S1 region from the Efstathiou et al. \(S_{90} > 100\) mJy data base, we have found optical, 15-\(\mu\)m or 1.4-GHz identifications for 24 (65 per cent). We have obtained 2dF and UK Schmidt FLAIR spectroscopy of 89 per cent of identifications to rigid multivariate flux limits. We construct a luminosity function assuming that (i) our spectroscopic subset is an unbiased sparse sample, and (ii) there are no galaxies that would not be represented in our spectroscopic sample at any redshift. We argue that we can be confident of both assumptions. We find that the luminosity function is well described by the local 100-\(\mu\)m luminosity function of Rowan-Robinson, Helou & Walker. Assuming this local normalization, we derive luminosity evolution of \(1 + z)^{2.45\pm0.85}\) (95 per cent confidence). We argue that star formation dominates the bolometric luminosities of these galaxies, and we derive comoving star formation rates in broad agreement with the Flores et al. and Rowan-Robinson et al. mid-infrared-based estimates.

Key words: surveys – galaxies: evolution – galaxies: formation – galaxies: starburst – cosmology: observations – infrared: galaxies.

1 INTRODUCTION

The study of the star formation history of the Universe is an extremely active field, in which much of the current debate is centred on the uncertainties in dust obscuration. Selecting star-forming galaxies in the ultraviolet is relatively cheap in observing time, but such selection makes the samples extremely sensitive to dust obscuration. Detecting the reprocessed starlight requires sub-millimetre surveys (e.g. Smail, Ivison & Blain 1997; Hughes et al. 1998; Barger et al. 1998, 1999; Eales et al. 1999; Peacock et al. 2000; Ivison et al. 2000a,b; Dunne et al. 2000) and/or space-based mid–far-infrared surveys (e.g. Rowan-Robinson et al. 1997; Taniguchi et al. 1997; Kawara et al. 1998; Flores et al. 1999; Puget et al. 1999; Oliver et al. 2000; see e.g. Oliver 2001 for a review). As such, the European Large Area ISO Survey (ELAIS) is well-placed for the study of the evolution and obscuration of the star formation in the Universe, and strong constraints at \(z \leq 1\) are possible from ELAIS (Oliver et al. 2000).

The scientific aims and strategy of ELAIS were presented in detail in Paper I (Oliver et al. 2000). In Papers II and III (Serjeant et al. 2000; Efstathiou et al. 2000) we presented respectively the ELAIS preliminary analysis source counts from the CAM (6.7 and 15 \(\mu\)m) and PHOT (90 \(\mu\)m) instruments on ISO. The 90-\(\mu\)m sample covered 11.6 square degrees, and the source counts were found to agree well at the bright end with the IRAS 100 \(\mu\)m counts. Excellent agreement was also achieved with a parallel independent pipeline (Surace et al., in preparation).

In this paper we present the first 90-\(\mu\)m luminosity function from our initial spectroscopic campaigns in the ELAIS S1 field (Gruppioni et al., in preparation; La Franca et al., in preparation;
Section 2 defines our sample. In Section 3 we use this catalogue to derive a 90-μm luminosity function. We discuss the implications of our results in Section 4. We assume $H_0 = 65 \text{ km s}^{-1} \text{Mpc}^{-1}$, $\Omega_0 = 1$ and $\Lambda = 0$ throughout.

## 2 Sample Selection and Data Acquisition

The parent sample for this study is the preliminary analysis catalogue of Paper III, with 90-μm fluxes satisfying $S_{90} > 100 \text{ mJy}$. The completeness of this sample falls approaching this limit, but has been well-quantified with simulations (fig. 6 of Paper III). We will use the calibration adopted in Paper III. The 90-μm flux calibration is still uncertain to within $\pm 30$ per cent, as discussed in Paper III; however, in order to be consistent with the $100 \text{ mJy}$ flux calibration as being accurately known. Nevertheless, in predicting the redshift distributions of other 90-μm surveys, we will not neglect the flux calibration systematics.

We restrict our study to the ELAIS S1 field with 90-μm, 1.4-GHz and 15-μm survey coverage, which covers 3.96 square degrees and has the most extensive available optical spectroscopy.

### Table 1. ELAIS S1 90-μm sample.

<table>
<thead>
<tr>
<th>Name</th>
<th>RA (J2000)</th>
<th>Dec. (J2000)</th>
<th>$S_{90}$/Jy</th>
<th>$S_{15}$/mJy</th>
<th>$S_{1.4}$/mJy</th>
<th>$R$</th>
<th>$\hat{z}$</th>
<th>$z_{\text{max}}$</th>
<th>$L/L_\odot$</th>
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<td>-44 16 33.6</td>
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<td>0.1274</td>
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<td>18.4</td>
<td>0.211</td>
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</table>
3 RESULTS: THE 90-μM LUMINOSITY FUNCTION

Our selection function is as follows. In order to be in the spectroscopic target list, a 90-μm galaxy must have either (i) a reliable optical identification brighter than $R = 17$, or (ii) a reliable 15-μm or 1.4-GHz identification, together with an optical identification brighter than $R = 20$. The 15-μm identifications were made to $S_{15} \geq 2$ mJy, and the radio identifications to 0.2–0.4 mJy depending on position (Gruppioni et al. 1999). Only 90-μm galaxies in the area with radio and 15-μm survey coverage were considered. We used the M82 starburst model from Rowan-Robinson et al. (1997) to estimate the $K$-corrections, with $S_{90\mu m} = 120S_{1.4GHz}$ and a radio spectral index of $\partial \log_{10} S / \partial \log_{10} \nu = -0.8$. For optical $K$-corrections we assumed an optical spectral index of $-3$. The redshift–luminosity plane is shown in Fig. 1, with bolometric luminosities calculated at 90 μm assuming constant $L_{ν}$ (for ease of conversion to other luminosity scales). Our method for calculating the luminosity function in the face of this complicated selection function is dealt with in the appendix. Note that our method relies on the underlying assumption that no galaxies are missing at all redshifts owing to the multivariate flux limits. However, we can be confident of this in our case, as any sufficiently low-redshift galaxy will have an optical identification that will pass criterion (i) of our selection function. A hypothetical local population of very optically faint but far-infrared-bright galaxies can already be excluded from IRAS samples. We must also assume that our 89 per cent complete optical spectroscopy is a random sparse sample of the total identifications (although not necessarily of the total 90-μm sample). Although with our current spectroscopic sample this is an a posteriori selection rather than an a priori one, there were no selection biases in the spectroscopic data acquisition that could skew the sample selection function. We therefore corrected our effective areal coverage by a factor of 0.89 to account for the small optical spectroscopic incompleteness.

The integral luminosity function is given by

$$\Phi(L) = \sum_{L>L_{max,i}} V^{-1} \, \left[ \frac{1}{2\pi\sigma} \right] \exp \left[ -\frac{1}{2\sigma^2} \log^2 \left( 1 + \frac{L}{L^*} \right) \right],$$

where the sum is performed over all objects having luminosities greater than $L$. In Fig. 2 we show this integral luminosity function, and compare it with the local 100-μm luminosity function derived by Rowan-Robinson, Helou & Walker (1987) (binned data). The latter was derived from a sample in the North Galactic Pole, and is subject to possible large-scale structure variations. To estimate this, we compared the normalization of their 60-μm luminosity function with that of Saunders et al. (1990), and derived a correction factor of 0.77 to the normalization of their 100-μm luminosity function. The best-fitting function to these local data is plotted as a full line. The adopted functional form is identical to that of Saunders et al. (1990):

$$\Phi(L) = \phi_0 \left( \frac{L}{L^*} \right)^{-\alpha} \exp \left[ -\frac{1}{2\sigma^2} \log^2 \left( 1 + \frac{L}{L^*} \right) \right],$$

and the best-fitting parameters are given in Table 2. Also plotted in this figure are the models of Rowan-Robinson (2000).

The hatched area shows the ±1σ errors from our sample. There is a marginally significant excess at the highest luminosities. The $(V/V_{max})$ statistic for this sample is 0.54 ± 0.07; a Kolmogorov–Smirnov test on the $V/V_{max}$ distribution shows only a 34 per cent probability of inconsistency with the top-hat distribution $U[0,1]$. Assuming $(1+z)^3$ luminosity evolution (Fig. 2) gives $(V/V_{max}) = 0.51 ± 0.07$, and, in both the evolving and non-evolving models, a Kolmogorov–Smirnov test on the observed luminosity distribution also gives acceptable confidence levels. Our spectroscopic sub-sample on its own is therefore not sufficiently large to detect evolution reliably.

A much stronger constraint on the strength of the evolution comes from the source counts slope. The 90-μm counts in the entire ELAIS areas are significantly non-Euclidean (Paper III), but the counts do show a surprising upturn at the faintest end (Fig. 3). The cause of this upturn is not clear, but it is too large to be entirely attributable to uncertainties in the completeness correction. One possibility is that some of the faintest sources are spurious glitch events, but all the sources have been eyeballed and
accepted by at least two observers. Such a glitch population would have to be somewhat pathological. Evidence for comparably strong evolution at 200 mJy in the Lockman Hole is also reported by Matsuhara et al. (2000) based on model fits to the fluctuations in their 90-μm maps, consistent with the faintest ELAIS points. Nevertheless, in the absence of a clear model-independent interpretation, we confine our source count fits to the 200 mJy region. We note that the sources used in the luminosity function are all reliably cross-identified at other wavelengths. The source counts at 200 mJy are in good agreement with the Lockman Hole counts from Linden-Vørnle et al. (2000) (Fig. 3), who use a very different source extraction and flux calibration procedure from ELAIS. The ELAIS and Lockman counts for the faintest bin in Fig. 3 (165–294 mJy) are 27 ± 4 and 22 ± 9 deg⁻² dex⁻¹ respectively, and in the brightest bin (294–522 mJy) are 19 ± 4 and 4.3 ± 3 deg⁻² dex⁻¹ respectively. Note that this represents only a single source in the Lockman Hole, so Poissonian errors would underestimate the error in this case. The ±1σ bounds in the case of one observed source are 0.18–3.3, and all the remaining errors quoted in the counts are ±1σ Poissonian errors.

If we assume that the local luminosity function derived above

Figure 2. Luminosity function assuming no evolution (left) and (1 + z)³ luminosity evolution (right). The upper panels show the integral luminosity function; the lower panels show the differential form. The hatched area is the ±1σ error region from this study. For the differential counts the hatched area indicates the error on the luminosity function in a ±0.5-dex bin; the differential counts are therefore effectively smoothed by a 1-dex boxcar. The data points show the local 100-μm luminosity function of Rowan-Robinson et al. (1987), and the full line is a fit to these local data. The broken lines show the populations in the Rowan-Robinson (2000) model. The long-dashed line denotes the 'cirrus-like' population, i.e. galaxies with bolometric luminosities dominated by new stars heating previously created dust. Note that this population is not necessarily identified with NGC 6090-like cirrus galaxies. The short-dashed line denotes the starburst population; the dash–dotted indicates Arp 220-like galaxies; the dash–double-dotted line shows the AGN population.
undergoes $(1 + z)^\alpha$ luminosity evolution, we can obtain a constraint on the evolution parameter $\alpha$. Note that this is relatively insensitive to the flux calibration uncertainty (Paper III). The correct relative normalization between 90 and 100 $\mu$m for these purposes is one that achieves continuity in the source counts. The current 90-$\mu$m calibration satisfies this almost exactly (Paper III) in that the bright end of the 90-$\mu$m PHOT counts dovetails with the faint end of the 100-$\mu$m $\text{IRAS}$ counts, with little room for error in the relative calibration. The Kolmogorov–Smirnov test on the shape of the source counts yields a slightly shallower slope than its Euclidean equivalent. Also shown as open symbols are the Lockman Hole counts from Linden-Vørnle et al. (2000). All errors are Poissonian, except in the case of a single object in a bin for which the $\pm 1 \sigma$ bounds on the number of objects in the bin are 0.18–3.3.

We can use these results to predict the redshift ranges of current and future 90-$\mu$m surveys. We assume that the evolution extends to between $z = 1$ and 2 and is then non-evolving. Fig. 4 shows the 95 per cent confidence limit for surveys of various depths, incorporating the uncertainty in the $\text{ELAIS}$ 90-$\mu$m absolute flux calibration. One corollary is that at least 10 per cent of the currently unidentified 90-$\mu$m galaxies in $\text{ELAIS}$ extend to $z > 0.5$, possibly even $z > 2$.

Is it reasonable to assume that star formation dominates the bolometric luminosities of our 90-$\mu$m sample? Genzel et al. (1998), Lutz et al. (1998) and Rigopoulou et al. (1999) have presented mid-infrared spectroscopy of ultraluminous galaxies ($L > 10^{12} L_{\odot}$) in the same redshift interval as our sample, and using the polycyclic aromatic hydrocarbon (PAH) features as a star formation indicator find that star formation dominates the bolometric power outputs in 70–80 per cent of ultraluminous infrared galaxies (ULIRGs). The active galactic nucleus (AGN) bolometric fraction derived in this way decreases monotonically at lower luminosities (e.g. Lutz et al. 1998). In local infrared galaxies between $10^{11}$ and $10^{12} L_{\odot}$, $\sim 15$ per cent have Seyfert 2 spectra (Telesco 1988), although care should be taken in interpreting AGN dominance from optical spectra (e.g. Lutz, Veilleux & Genzel 1999; Taniguchi et al. 1999). At $L < 10^{11} L_{\odot}$ most local infrared galaxies are single, gas-rich spirals (e.g. Sanders & Mirabel 1996) in which many lines of argument point to starburst dominance in the infrared (e.g. Telesco 1988); for example, the similarity of the spectral energy distributions to those of Galactic star-forming regions; the linear $L_{\text{IR}}-L_{\text{CO}}$ correlation over many orders of magnitude in luminosity; and optical/infrared spectroscopic confirmation of star formation activity. It therefore seems likely that our sub-ultraluminous population should not be powered by active nuclei, as is also suggested by the low fraction

\begin{table}
\centering
\caption{Best-fitting parameters of the local luminosity function, with $\sigma = 0.724$ assumed. The reduced $\chi^2$ of the best fit is 0.66.}
\begin{tabular}{ccc}
$\Phi_{\text{IR}} L^{-3} \odot$ & $\log_{10} L_{\odot}/L_{\text{CO}}$ & $\alpha$ \\
\hline
5.4 ± 1.8 & 9.67 ± 1.47 & 1.73 ± 0.04
\end{tabular}
\end{table}
of spectra in our sample (10 per cent) with Seyfert 2 features. (This does not imply that AGN activity cannot be present at a weak or bolometrically negligible level: e.g. Ho, Filippenko & Sargent 1997.) At the lowest luminosities there may be a significant contribution from cirrus which is at least partly illuminated by the old stellar population in the galaxies (Telesco 1988; Morel et al. 2001), although such galaxies nevertheless also still obey the starburst radio–far-infrared relation (e.g. Condon 1992). We conclude that the galaxies in our sample have their bolometric luminosities dominated by star formation.

The Saunders et al. (1990) local luminosity density implies a local star formation rate of 0.023 ± 0.002 M⊙ yr⁻¹ Mpc⁻³ (Oliver, Gruppioni & Serjeant 2001) assuming a Salpeter initial mass function from 0.1 to 125 M⊙. Our results imply that this rises to 0.036 ± 0.009 M⊙ yr⁻¹ Mpc⁻³ by z = 0.2 (95 per cent confidence), significantly higher than the Tresse & Maddox (1998) Hα-based estimate of 0.022 ± 0.007 M⊙ yr⁻¹ Mpc⁻³. (Note that the two Seyfert 2 spectra in our sample are at the brightest end of the luminosity function where the contribution to the luminosity density is the smallest. The luminosity density is dominated by the galaxies around or below the break in Table 2.) We believe that this is due to two factors: first, the Hα-based estimates are not immune to extinction effects, even if Balmer decrement reddening corrections are performed (e.g. Serjeant, Gruppioni & Oliver 2001); secondly, the Tresse & Maddox (1998) survey area is sufficiently small to be affected by large-scale structure (e.g. Oliver et al. 2001). ELAIS is immune to these problems, as the cosmic variance over the PHOT survey area is ~<20 per cent at z < 0.3 (Paper I). These star formation rates are, however, consistent with extrapolations from those derived from mid-infrared data at higher redshift by Flores et al. (1999) and Rowan-Robinson et al. (1997).

The prospects for improving on the results presented here are excellent. A large-scale spectroscopic follow-up of the ELAIS northern areas is currently underway, which will have a major impact on the science analysis of ELAIS. In particular, the ELAIS 15-μm luminosity function will probe higher redshifts than accessible to PHOT at 90 μm, from which the 15-μm luminosity density can constrain the cosmic star formation history (e.g. Serjeant, in preparation).

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REFERENCES


Hughes D. et al., 1998, Nat, 394, 421


Telesco C. M., 1988, ARA&A, 26, 343


APPENDIX A: 1/V MAX WITH MULTIVARIATE FLUX LIMITS

In a volume-limited non-evolving sample, the comoving number density of objects can be calculated trivially:

\[ \phi = \frac{N}{V} = \frac{1}{V} \sum_{i=1}^{N} 1, \]  \hspace{1cm} (A1)

where \( V \) is the volume and \( N \) the number of objects. Suppose that the sample is incomplete in whatever way, and that the probability of the \( i \)th object being contained in the sample is \( p_i \). Provided that none of the \( p_i \)s in the underlying galaxy population is zero, the comoving number density in an observed sample of \( N_{\text{obs}} \) objects can be expressed as

\[ \phi = \frac{1}{V} \sum_{i=1}^{N_{\text{obs}}} p_i^{-1}. \] \hspace{1cm} (A2)

This will be an unbiased estimator of the underlying value.

Applying this formalism to our current data set, the incompleteness is due to the multivariate flux limits. We treat the (non-evolving) galaxies as sampling random redshifts within the volume, so that each \( p_i \) is the probability that such a galaxy would
lie above the flux limits. The statement that the underlying \( p_i \)s are all non-zero is equivalent to assuming that there are no galaxies (in the 90-\( \mu \)m luminosity range being considered) that would lie outside the selection criteria at all redshifts. We can be confident that this is the case, because any sufficiently local galaxy will have an optical identification passing criterion (i) of our selection function. A hypothetical population of far-infrared-luminous, optically faint galaxies can already be excluded from IRAS.

There are two equivalent ways of calculating the \( p_i \)s. If we embed our flux-limited sample in a larger volume of size \( V_0 \), we can use the assumption that the population is non-evolving and the fact that the flux limits are monotonic in \( z \) to express the \( p_i \)s as

\[
p_i = \frac{V_{\text{max},i}}{V_0},
\]

where \( V_{\text{max},i} \) is the volume enclosed by the maximum redshift \( z_{\text{max},i} \) at which the \( i \)th object is visible. Note that this is the smallest redshift at which the object fails any of the selection criteria. The number density in a sample of \( N_{\text{obs}} \) galaxies is then

\[
\phi = \frac{1}{V_0} \sum_{i=1}^{N_{\text{obs}}} \frac{V_0}{V_{\text{max},i}} = \sum_{i=1}^{N_{\text{obs}}} \frac{1}{V_{\text{max},i}},
\]

(A4)

The rms error is simply

\[
\Delta \phi = \sqrt{\sum_{i=1}^{N_{\text{obs}}} \frac{1}{V_{\text{max},i}^2}}.
\]

(A5)

An alternative but equivalent method of calculating the \( p_i \)s is to model the incompleteness arising from some or all of the multivariate flux limits by introducing a weighting factor to the differential volume elements:

\[
dV' = \gamma(z, \ldots) dV.
\]

(A6)

The \( z_{\text{max},i} \) values would then be calculated using the remaining unmodelled flux limits only. If all the flux limits have been modelled, then \( \forall i \), \( z_{\text{max},i} = z_0 \) where \( V(z_0) = V_0 \). The \( \gamma \) factor would drop to zero at some redshift if (under the previous method for calculating the \( z_{\text{max},i} \)) \( z_0 > \max(z_{\text{max},i}) \).

Here, our approach is to use the first method (equation A4) to treat the multivariate flux limits, as it is less model-dependent. However, we use a weighting function on the volume elements to correct for the ELAIS 90-\( \mu \)m completeness function (fig. 6 of Paper III):

\[
V'_{\text{max},i} = \int_0^{z_{\text{max},i}} \gamma(z, S_i) \frac{dV}{dz} dz,
\]

(A7)

where \( S_i \) is the 90-\( \mu \)m flux of the \( i \)th object.

To calculate the luminosity function in a redshift bin, an additional top-hat selection function is applied in redshift space and incorporated into the calculation of the \( z_{\text{max},i} \). The volume enclosed by the minimum redshift of the bin must also be subtracted from the \( V_{\text{max},i} \) in equation (A3). We are currently confident that none of the underlying \( p_i \)s is zero, but if we calculated the luminosity function in a series of redshift bins this would not necessarily be the case for all bins. For example, there are galaxies in Table 1 that only pass criterion (i) at the lowest redshifts, and these may be unobservable in principle in higher redshift bins. We would then need to make some model of the multivariate correlations to correct for the missing \( p_i = 0 \) galaxies. We therefore restrict ourselves to a single redshift range (de-evolving the galaxies if necessary), partly to avoid the model dependence of correcting for missing populations, but partly also because of our small sample size.

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