A Lorenz/Boer energy budget for the atmosphere of Mars from a “reanalysis” of spacecraft observations

Tabataba-Vakili, Fachreddin; Read, Anna; Lewis, Stephen; Montabone, Luca; Ruan, Tao; Wang, Bo; Valeanu, Alexandru and Young, Roland M. B. (2015). A Lorenz/Boer energy budget for the atmosphere of Mars from a “reanalysis” of spacecraft observations. Geophysical Research Letters, 42(20) pp. 8320–8327.
Supporting Information for “A Lorenz/Boer energy budget for the atmosphere of Mars from a reanalysis of spacecraft observations”

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**Introduction**
The Lorenz budget equations presented in the paper (equations 1 - 4) are solved according to Boer [1989] by:

\[ K_Z = \int_M \frac{1}{2} [\Theta][V]_r \cdot [V]_r dm \] (1)

\[ K_E = \int_M \frac{1}{2} [\Theta V^* \cdot V^*] dm \] (2)

\[ A_Z = A_{Z1} + A_{Z2} \]

\[ = \int_M C_p \Theta N_Z[T]_r dm + \int_S (p_s - \pi_s Z) \Phi_s d\sigma / g \] (3)

\[ A_E = A_{E1} + A_{E2} \]

\[ = \int_M C_p \Theta (N - N_Z)Td\mu + \int_S (\pi_s Z - \pi_s) \Phi_s d\sigma / g \] (4)

\[ C_K = C_{K1} + C_{K2} \]

\[ = -\int_M a \cos \phi \left\{ \left( [\Theta u^* V^*] \cdot \nabla + [\Theta u^* \omega^*] \frac{\partial}{\partial p} \right) \right( \frac{[u]_r}{a \cos \phi} \right) \cdot \left( \left( [\Theta v^* V^*] \cdot \nabla + [\Theta v^* \omega^*] \frac{\partial}{\partial p} \right) \right) \right\} dm \] (5)

\[ + \int_M \left\{ \left[ \Theta \frac{\partial \Phi^*}{\partial t} \right] + [V]_r \cdot [\Theta \nabla \Phi^*] \right\} d\mu \]
\[ C_Z = C_{Z1} + C_{Z2} \]
\[ = - \int_M [\Theta] [\omega]_r [\alpha]_r dm - \int_S \left[ \frac{\partial p_s}{\partial t} \Phi_s \right] d\sigma/g \]  
(6)

\[ C_E = - \int_M [\Theta \omega^* \alpha^*] dm \]  
(7)

\[ C_A = - \int_M C_p \left( \frac{\theta}{T} \right) \left( [\Theta T^* V^*] \cdot \nabla + [\Theta T^* \omega^*] \frac{\partial}{\partial p} \right) \]
\[ \left( \frac{T}{\theta} N_Z \right) dm \]  
(8)

\[ G_Z = \int_M \Theta N_Z [Q]_r dm \]  
(9)

\[ G_E = \int_M \Theta (N - N_Z) Q dm \]  
(10)

\[ F_Z = \int_M [\Theta] [V]_r \cdot [F] dm \]  
(11)

\[ F_E = - \int_M [\Theta] [V^* \cdot F^*] dm \]  
(12)

with

\[ \Theta(p - p_s) = \Theta(\lambda, \phi, p, t) = \begin{cases} 
1, & p < p_s \\
0, & p > p_s 
\end{cases} \]  
(13)

\[ [X]_r = \begin{cases} 
[\Theta X]/[\Theta], & [\Theta] \neq 0 \\
[X], & [\Theta] = 0 
\end{cases} \]  
(14)

\[ X^* = X - [X]_r \]  
(15)

\[ N(\pi) = 1 - (\pi/p)\kappa \]  
(16)

\[ N_Z = N(\pi_Z) \]  
(17)

\[ \pi_Z = \pi([\Theta]_r, t) \]  
(18)

\[ [\theta]_r = [T]_r \left( \frac{p_{oo}}{p} \right)^\kappa \]  
(19)

\[ \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right) = \left( \frac{1}{a \cos \phi} \frac{\partial}{\partial \lambda} \right) \]  
(20)
and

\[ A_E: \text{eddy available potential energy} \]
\[ A_Z: \text{zonal available potential energy} \]
\[ C_A: \text{conversion term between } A_Z \text{ and } A_E \]
\[ C_E: \text{conversion term between } A_E \text{ and } K_E \]
\[ C_K: \text{conversion term between } K_Z \text{ and } K_E \]
\[ C_Z: \text{conversion term between } A_Z \text{ and } K_Z \]
\[ F_E: \text{dissipation of } K_E \text{ by friction} \]
\[ F_Z: \text{dissipation of } K_Z \text{ by friction} \]
\[ G_E: \text{generation of } A_E \text{ by diabatic heating} \]
\[ G_Z: \text{generation of } A_Z \text{ by diabatic heating} \]
\[ K_E: \text{eddy kinetic energy} \]
\[ K_Z: \text{zonal kinetic energy} \]
\[ [X]: \text{zonal mean of } X \]
\[ N: \text{efficiency factor} \]
\[ N_Z: \text{zonal efficiency factor} \]
\[ p_s: \text{surface pressure} \]
\[ \mathbf{V}: \text{horizontal velocity vector field} \]
\[ u: \text{zonal component of } \mathbf{V} \]
\[ v: \text{meridional component of } \mathbf{V} \]
\[ \alpha: \text{specific volume } (1/\rho) \]
\( \theta_s \): potential at \( k \)-th isentropic level of the reference state

\( \kappa \): \( R/c_p \)

\( \pi \): pressure of the reference state

\( \pi_{sZ} \): zonal-mean of surface pressure of the reference state

\( \rho \): density

\( \Phi \): geopotential

\( \Phi_s \): surface geopotential

\( \omega \): vertical velocity vector in pressure coordinates

\( Q \): heating rate

\( F \): dissipation of kinetic energy

\( \Theta \): Heaviside step function

**Text S1.**

Figure S4 shows the integrands of the altitude-dependent energy and conversion terms in longitudinal and temporal mean, plotted over latitude and pressure. \( K_Z \) shows strong contributions in mid-latitudes, coinciding with jet activities. \( K_E \) values seem to be correlated with \( K_Z \), except for a region in the upper atmosphere of the south-polar region. Further analysis shows that this maximum receives most of its contribution during the GDSE of MY 25. The integrands of \( A_{Z1} \) mirror the behaviour (and sign) of the zonal-mean efficiency factor \( N_Z \) by showing negative regions in the in the pole regions, this pattern is also observed in the Earth atmosphere [see e.g. *Siegmund*, 1994, , their Figure 3]. Please note that the effect of using the efficiency factor to determine APE integrands is signifi-
cant [see Boer, 1975]. $A_{E1}$ shows non-zero values at the surface, extending latitudinally towards the upper mid-latitudes. A temporal analysis (not shown) reveals that large $A_{E1}$ integrands are found to coincide with strongly negative $A_{Z1}$ integrands, revealing maxima in the difference between $N$ and $N_Z$.

The bottom half of Figure S5 shows the integrands of the conversion terms. Please note that most of the visible patterns displayed have only small contributions to the integrated values presented above because the mass element $dm$ strongly favours lower altitudes. $C_A$, just like $K_Z$ and $A_{Z1}$, is stronger in the northern hemisphere. The maximum of $C_A$ in the north polar region coincides with NH winter, and thereby with the extrema of $K_Z$ and $A_{Z1}$. $C_E$ has positive ($A_E$ to $K_E$) integrands in the equatorial region and negative ($K_E$ to $A_E$) values polar regions and high altitudes. The negative values seem slightly correlated to regions with high $K_E$. The visible pattern of $C_{K1}$ is strongly correlated with the south polar maximum in $K_E$ that occurs during the GDSE. The global-mean values of $C_{K1}$ originate from lower altitudes because these are more strongly weighted. We find that maxima of $C_{K2}$ coincide with high values of $A_Z$ in the southern hemisphere. $C_{Z1}$ shows thermally direct circulation in the summer hemisphere and thermally indirect circulation in the winter hemisphere. The annual mean shows that the southern summer circulation dominates.

An evaluation of the diurnal components (Fig. S5) of these integrands shows correlating patterns for those terms that are strongly affected by diurnal timescales (i.e. $K_E$, $A_{E1}$, $C_A$, $C_E$, $C_{K1}$) and generally negligible diurnal integrands for the rest.
References


Table S1. Lorenz energy budget of Mars in seasonal and hemispheric decomposition. Seasons are given in solar longitudes, where $L_s = 0^\circ$ is northern hemisphere spring equinox. Hence e.g. $L_s = 0^\circ - 90^\circ$ is spring on the northern hemisphere, followed by summer, autumn and winter. Annual values were averaged over two full years (MY 25 and MY 26). Seasonal values are the mean of either two ($L_s = 0^\circ - 180^\circ$) or three ($L_s = 180^\circ - 360^\circ$) full seasons.

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Figure S1. Mean values of energy and conversion terms per unit area of Earth (top, Boer and Lambert [2008]) from NCEP (white/blue) and ERA (black) reanalysis data over 17 years (1979-1995); of Mars over almost 3 martian years (bottom, current work). All Energies ($A_Z, A_E, K_Z, K_E$) are given in $10^5 J m^{-2}$ and all conversion terms in W m$^{-2}$. 
Figure S2. Mean values of energy and conversion terms per unit mass of Earth (top, Boer and Lambert [2008]) from NCEP (white/blue) and ERA(black) reanalysis data over 17 years (1979-1995); of Mars over almost 3 martian years (bottom, current work). All Energies ($A_Z, A_E, K_Z, K_E$) are given in J/kg and all conversion terms in $10^{-4}$ W/kg.
Figure S3. Total (a), daily averaged (b) and tidal (c) component of the conversion terms of the Lorenz energy budget of the Mars atmosphere given in 30-sol mean values from $L_s = 141^\circ$ MY 24 to $L_s = 82^\circ$ MY 27. The tidal component (c) shows the contribution ($X_{\text{tidal}}/X \cdot 100\%$) of the tidal component to the total energies in percent.
Figure S4. Integrands of energy and conversion terms resolved over latitude and pressure coordinates. Displayed data is a mean over two full martian years.
**Figure S5.** Diurnal components of the integrands of energy and conversion terms resolved over latitude and pressure coordinates. Displayed data is a mean over two full martian years.
Figure S6. Total (a), daily averaged (b) and tidal (c) component of the conversion terms of the Lorenz energy budget of the northern hemisphere of the Mars atmosphere given in 30-sol mean values from $L_s = 141^\circ$ MY 24 to $L_s = 82^\circ$ MY 27.
Figure S7. Total (a), daily averaged (b) and tidal (c) component of the conversion terms of the Lorenz energy budget of the southern hemisphere of the Mars atmosphere given in 30-sol mean values from $L_s = 141^\circ$ MY 24 to $L_s = 82^\circ$ MY 27.