An active mute for the trombone

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An active mute for the trombone

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Abstract

A mute is a device that is placed in the bell of a brass instrument to alter its sound. However, when a straight mute is used with a brass instrument, the frequencies of its first impedance peaks are slightly modified, and a mistuned, extra impedance peak appears. This peak affects the instrument’s playability, making some lower notes difficult or impossible to produce when playing at low dynamic levels. To understand and suppress this effect, an active mute with embedded microphone and speaker has been developed. A control loop with gain and phase shifting is used to control the damping and frequency of the extra impedance peak. The stability of the controlled system is studied and then the effect of the control on the input impedance and radiated sound of the trombone is investigated. It is shown that the playability problem results from a decrease in the input impedance magnitude at the playing frequency, caused by a trough located on the low frequency side of the extra impedance peak. When the extra impedance peak is suppressed, the playability of the note is restored. Meanwhile, when the extra impedance peak is moved in frequency, the playability problem position is shifted as well.
I. Introduction

Over the last 50 years, the acoustical properties of brass musical instruments\textsuperscript{1,2,3} and their mutes\textsuperscript{4,5} have been widely studied. When a brass instrument is played, a mute can be inserted into the instrument bell to reduce the level and to alter the timbre of the sound. An efficient mute should modify the instrument’s sound with no impact on the playability or the intonation. However, some mutes have a perturbing effect on the instrument’s input impedance. For example, introducing a straight mute into a trombone bell will lead to the appearance of a mistuned, extra impedance peak, usually between the two first peaks of the input impedance\textsuperscript{5}. This extra impedance peak has an effect on the playability of certain low notes on the instrument, specifically those that have a fundamental that is close in frequency to that of the extra impedance peak. In the case of the mute studied in this paper, it is the playability of some pedal notes\textsuperscript{6} that is affected, with the effect particularly noticeable at low dynamic levels.

In order to understand and also to suppress this particular effect of the mute on the trombone, an active mute with embedded sensor (a microphone) and actuator (a loudspeaker) linked by a controller composed of a phase shifter and a gain amplifier has been developed, inspired by Chen et al.\textsuperscript{8}. Over the past few decades, active control techniques have been applied to various musical instruments, starting with percussion and string instruments\textsuperscript{9,10,11} but progressing to wind instruments. In particular, active control
has been used to play a complete octave on a flute with no holes. To achieve this, the incident wave was absorbed by a loudspeaker at the end of the tube, and replaced by a chosen reflected wave. Active control using gain and phase shifting has also been used to globally modify the resonances of a simplified clarinet (with all resonances affected by the control). Meanwhile, modal active control has enabled individual resonances of a clarinet to be controlled separately in simulations and experimentally.

In Section II of this paper, the active mute, its effect on the trombone input impedance and playability, and the principles underpinning the control are described. Section III then presents simulations of the control of the mute. Finally, in Section IV, experimental results are presented which demonstrate the effectiveness of the control when the active mute is inserted in a real trombone. In particular, the effects of the active mute on the input impedance, the playability at low dynamic levels, and the radiated sound of the instrument are shown.

Frequency modifications are described in cents because of their musical meaning: 100 cents equals a semitone. The sounds referred to in Sections II.A and IV may be found at http://instrum.ircam.fr/?p=806. (It should be noted that the clicks which can be heard in the third, fourth and fifth sound clips are caused by the switch of the control system.)

II. The active mute
A. Effect of the mute on the trombone’s acoustics

A straight trombone mute may be considered as a 1-degree-of-freedom resonator\textsuperscript{4}. A Denis Wick straight trombone mute has been modified in order to apply active control to it (see Figure 1). A Tymphany Peerless PLS-P830983 loudspeaker, enclosed at the rear, and an electret microphone located at the end of a 65 mm long capillary tube, have been added to the mute (the capillary tube is used to decrease the acoustic pressure level between the mute and the microphone).

Figure 2 shows the transfer functions measured between loudspeaker and microphone when the mute was (i) positioned in free air, and (ii) positioned inside the bell of a trombone. These measurements were carried out by sending an electrical swept sine signal to the loudspeaker and recording the resultant signal measured by the microphone embedded in the mute. The transfer functions (for the mute in free air and for the mute positioned in the trombone bell) were then calculated by dividing the microphone signal by the swept sine excitation signal; essentially giving the ratio between the pressure measured by the microphone ($P_2$ on Figure 1) and the flow produced by the speaker ($U_3$ on Figure 1). The main peak in each transfer function is related to the Helmholtz resonance of the mute. As can be seen from Figure 2, this depends on the mounting; when in free air, the peak has a frequency $f_H = 114$ Hz. The phase exhibits the expected rapid change of angle close to the resonance frequency $f_H$, but this rapid change is part of a more extensive slope.
at low frequencies (below 200 Hz). This slope may be due to the distance between the speaker and microphone, to the capillary tube, or to the response of the loudspeaker (resonance frequency at 147.5 Hz).

When the mute is placed in the bell of the trombone with the slide in the Bb\textsubscript{1} position (closed position), the peak in the transfer function moves to $f_{pp} = 66$ Hz (see Figure 2). This modification results from the addition of an inductance parallel to the mute as a result of the opening at the input of the bell\textsuperscript{4}. The slope in phase at low frequency observed when the mute is in free air is also visible when the mute is put inside the bell of the trombone. The remainder of the paper focuses on the situation where the mute is positioned inside the bell of the trombone.

The input impedance of a trombone has been measured with the impedance sensor of the CTTM (Centre de Transfert de Technologie du Mans)\textsuperscript{16}. Figure 3 shows the input impedance of the trombone with slide in the Bb\textsubscript{1} position, measured (i) without mute, and (ii) with the active mute placed in the bell of the instrument but without control applied. With the mute present, an extra impedance peak appears at a frequency of $f_{pp}$ (the same as the peak of the transfer function when the mute is positioned inside the bell of the trombone), the frequencies of the next three peaks are slightly increased (by less than 2 Hz or 20 cents) and the frequency of the first peak is slightly decreased (by 0.7 Hz). The phase of the input impedance is also modified by the mute, with a local phase shifting between
45 Hz and 90 Hz and a maximum shift of 1.4 rad at 63 Hz.

Figure 4 shows a spectrogram of a descending musical sequence played on a trombone at low dynamic level by a professional musician with a normal straight mute. The note $Bb_1$ (58 Hz) is unstable, and the note $A_1$ (55 Hz) cannot be played. Note that $A_1$ can sometimes be played but is always perturbed by the presence of the mute. These two notes are the pedal notes when the slide is in the $Bb_1$ and $A_1$ positions respectively. They do not present any problem when played without a mute. The difficulties in playing these notes are due to the presence of the mute.

A control system using a phase shifter $\phi$ and a gain $G$ is added to the mute in order to modify the extra impedance peak (see Figure 1). The phase shifter uses a phase inverter (which applies a $\pi$ phase shift at all frequencies) coupled to an operational amplifier phase shifter. Figure 5 shows the electrical circuit of the coupled phase shifter and phase inverter, and Table 1 gives the values used for its different components. The operational amplifier phase shifter does not have constant phase shifting at all frequencies. The phase shift is defined by:

$$\phi(j\omega) = -2\arctan(RC_1\omega)$$

with $\omega = 2\pi f$ the angular frequency, $R$ a variable resistance, and $C_1 = 10 \mu F$ a capacitance. The phase shift $\phi = \pi/2$ used in Section IV.B is then the phase shift at the resonance frequency of the transfer function ($\phi(\omega_0) = \pi/2$).
For the sake of simplicity, the phase shift will be considered constant in Sections II.B and III.

Table 1: Values of the components used in Figure 5.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>1 kΩ</td>
</tr>
<tr>
<td>$R_2$</td>
<td>15 kΩ</td>
</tr>
<tr>
<td>$R_3$</td>
<td>3.9 Ω</td>
</tr>
<tr>
<td>$C_1$</td>
<td>10 µF</td>
</tr>
<tr>
<td>$C_2$</td>
<td>1 µF</td>
</tr>
</tbody>
</table>

B. Principle of the active control

An analogue proportional feedback active control employing gain and phase shifting is applied to the mute.

Let $H$ be the transfer function of a resonator described by a second order band-pass filter:

$$H(s) = \frac{H_0 s}{s^2 + 2\xi \omega_0 s + \omega_0^2} \quad (2)$$

where $H_0$ is the gain of the resonator, $s = j\omega$ is the Laplace variable, $\xi = 1/2Q$ the damping of the resonator with $Q$ its quality factor and $\omega_0 = 2\pi f_0$ with $f_0$ the resonance frequency of the resonator. Table 2 shows the values for these parameters, adjusted so that the resulting curves match the measured transfer functions of the mute outside and inside the bell of the trombone (see Figure 2). (Note that eq.(2) is used to represent the coupled system [mute + loudspeaker], assuming the loudspeaker dynamics to be of little influence.)
Figure 2 shows that such a simplification is acceptable, although a more complete model is presented in Appendix A.)

Table 2: Mute parameters obtained through fitting using eq.(2) and the measured transfer functions of Figure 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mute outside the trombone bell</th>
<th>Mute inside the trombone bell</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$</td>
<td>120</td>
<td>90</td>
</tr>
<tr>
<td>$f_0$</td>
<td>114 Hz</td>
<td>66</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.035</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Figure 6 shows the control loop applied to $H$. The closed-loop (i.e. controlled) transfer function $H_{CL}$ can be written as:

$$H_{CL} = \frac{y}{w}$$  \hspace{1cm} (3)

with $w$ the excitation (often referred to as a disturbance in the context of active control) applied to the resonator and $y$ the output of the resonator, so that

$$y = H(u + w) = Hu + Hw$$  \hspace{1cm} (4)

where $u$ is the command produced by the controller and applied to the resonator, with

$$u = y Ge^{j\phi}.$$  \hspace{1cm} (5)
where $G$ is the control gain and $\phi$ the phase shifting.

Substituting eq.(4) and eq.(5) into eq.(3) leads to

$$H_{CL} = \frac{y}{w} = \frac{H}{1 - HGe^{j\phi}}.$$  \hspace{1cm} (6)

Figure 6 also shows the resonator $H$ in open-loop. Let $H_{OL}$ be the open-loop transfer function of the resonator, so that

$$H_{OL} = \frac{u}{w}$$  \hspace{1cm} (7)

with

$$u = wHGe^{j\phi}$$  \hspace{1cm} (8)

then

$$H_{OL} = \frac{u}{w} = HG e^{j\phi}.$$  \hspace{1cm} (9)

The phase at the resonance of the open-loop transfer function of the resonator gives information about the effect of the control on the closed-loop transfer function. Modifying the phase value at resonance leads to modifications of the frequency and damping of the resonance. The role of the gain is then to accentuate these effects; the higher the gain, the bigger the effect. Four phase zones may be seen:\n
\begin{itemize}
  \item $-\pi < \phi < 0$, the resonance frequency is decreased,
  \item $0 < \phi < \pi$, the resonance frequency is increased,
\end{itemize}
Meurisse et al., JASA, p. 11

- $-\pi/2 < \phi < \pi/2$, the damping of the resonance is decreased (its amplitude is increased),

- $-\pi < \phi < -\pi/2$ and $\pi/2 < \phi < \pi$, the damping of the resonance is increased (its amplitude is decreased).

For most phase values, both the frequency and the damping of the resonance are affected. However, for four phase values there is just a single effect:

- $\phi = 0$, only the damping of the resonance is decreased (its amplitude is increased),

- $\phi = \pi$, only the damping of the resonance is increased (its amplitude is decreased),

- $\phi = \pi/2$, only the resonance frequency is increased,

- $\phi = -\pi/2$, only the resonance frequency is decreased.

As an example, the parameters of $H(s)$ in eq.(2) are chosen to be those obtained through fitting of the transfer function of the mute when put inside the bell of the trombone (see Table 2).

Substituting $H(s)$ from eq.(2) into eq.(6), the closed-loop transfer function of this resonator may be calculated. The effects of the control on $H(s)$ for different $G$ and $\phi$ values ($\phi = [-\pi/2; 0; \pi/2; \pi]$) are shown in Figure 7. The observed effects are those which were predicted previously: only the amplitude of the resonance increases when $\phi = 0$ and
decreases when $\phi = \pi$, and only the frequency of the resonance increases when $\phi = \pi/2$ and decreases when $\phi = -\pi/2$. These effects are reported in Table 3, where the case without control ($\phi = 0$ and $G = 0$) is also included.

Table 3: Control effects on the resonator and differences between controlled cases and uncontrolled (ie. $G = 0$ and $\phi = 0$) case.

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0</th>
<th>$\pi$</th>
<th>$\pi/2$</th>
<th>$-\pi/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>0</td>
<td>0.45</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Amplitude (dB)</td>
<td>6.7</td>
<td>37</td>
<td>-7.8</td>
<td>6.7</td>
</tr>
<tr>
<td>Difference (dB)</td>
<td>NA</td>
<td>+30.3</td>
<td>-14.5</td>
<td>0</td>
</tr>
<tr>
<td>Frequency (Hz)</td>
<td>66</td>
<td>66</td>
<td>66</td>
<td>82</td>
</tr>
<tr>
<td>Difference (Hz, cents)</td>
<td>NA</td>
<td>0, 0</td>
<td>0, 0</td>
<td>+16, +113</td>
</tr>
</tbody>
</table>

III. Simulations of the control

Simulations can be used to predict the stability of the controlled system as well as to explore the possibilities of the control. Using the transfer function measured when the mute is inserted in the bell of the trombone (Figure 2), and using the root locus method\textsuperscript{17;18;19}, the effects of the control are studied. This involves using eq.(6), together with the measured transfer function $H_m(s)$, and varying the phase shift $\phi$ from $-\pi$ to $\pi$ and the gain $G$ from 0 to 5, in order to map the resultant changes to the amplitude and
frequency of the peak observed in the measured transfer function.

Figures 8 and 9 show maps of the transfer function modifications in amplitude and frequency when control in gain and phase shifting is applied. On these maps, each point represents the amplitude or the frequency of the closed loop transfer function.

The white zone on Figure 8 and the black zone on Figure 9 represent controls for which the system is unstable. These zones have been determined using the root locus method. As a comparison, the theoretical limit of the unstable zone for a resonator described by eq.(2), determined using the root locus method, is added to Figure 8. The differences between the theoretical limit and the shape of the white zone may be due to the slope in the phase of the measured transfer function.

To make Figure 8 more readable, the amplitudes have been limited to 30 dB. It can be seen that the amplitude of the resonance varies from -11 dB, at \(G = 5; \phi = 2.5 \text{ rad}\), to infinity (here, 30 dB) between \(\phi = -2.35 \text{ rad}\) and \(\phi = 1.7 \text{ rad}\). The minimum gain value for instability is \(G = 0.36; \phi = -0.1 \text{ rad}\).

It can be seen from Figure 9 that the frequency of the resonance varies from 32 Hz, at \(G = 5; \phi = -2.6 \text{ rad}\), to 99 Hz at \(G = 5; \phi = 2 \text{ rad}\). This is a difference of 1955 cents (more than 19 semitones), which corresponds to a perfect twelfth. At low gains, the modifications appear first around \(\phi = \pm \pi/2\). When the gain is higher, shifts appear both
when the frequency decreases and increases. These shifts may be due to the slope in the phase in Figure 2.

IV. Experimental study

A control system using a phase shifter $\phi$ and a gain $G$ is added to the mute in order to modify the extra impedance peak (see Figure 1). The effects of the control on the Helmholtz resonance of the mute when it is put inside the bell of the trombone, on the input impedance, and on the sound produced by the instrument are studied using two phase shifts: $\phi(\omega_0) = \pi$ (see Section IV.A), causing a decrease in the amplitude of the resonance, and $\phi(\omega_0) = \pi/2$ (see Section IV.B), causing an increase in the frequency of the resonance. The aim of the first control is to suppress the extra impedance peak, in order to restore the playability of the notes. The second control is designed to study the effect obtained when the frequency of the extra impedance peak is changed. Finally, Section IV.C uses two criteria - the sound level and the spectral centroid - to characterize the differences that occur in the radiated sounds.

A. Suppressing the extra impedance peak

Figure 10 shows the control of the resonance measured between the speaker and microphone of the mute when $\phi(\omega_0)$ is set equal to $\pi$ and $G$ is varied. The damping increases with increasing gain, leading to a decrease in the amplitude (by as much as 15 dB
Figures 11 and 12 show the input impedances of the trombone with the slide in $Bb_1$ and $A_1$ positions respectively, without mute, with a normal straight mute inserted, and with the active mute with control set at $\phi = \pi$ and $G = 2$. Meanwhile, Figure 13 zooms in on the extra impedance peak of Figures 11 and 12. On all the figures, the extra impedance peak appears at a frequency close to that of the resonance in the transfer function of Figure 2 (i.e. $f_{pp}$). The small variations in frequency are due to environmental changes (temperature, humidity) between measurement sessions. On each side of the extra impedance peak, there are small troughs. Due to these troughs, the magnitude of the impedance at 58 Hz (playing frequency of $Bb_1$) in Figures 11 and 13 (top) is 4.1 dB lower when there is a mute present, compared with the case when there is no mute. Similarly, the magnitude of the impedance at 55 Hz (playing frequency of $A_1$) in Figures 12 and 13 (bottom) is 3.7 dB lower when there is a mute present. With the control applied, the peak disappears in both figures, and the impedance magnitudes at the playing frequencies are closer to those measured with no mute present; 1 dB higher at 58 Hz in Figures 11 and 13 (top) and 0.9 dB lower at 55 Hz in Figures 12 and 13 (bottom). The control also has an effect on the other peaks of the impedance magnitude curve of Figure 11, compared with the normal mute (-2 dB on the second peak, -2 dB and +4 cents on the third peak and +2.5 dB and +5 cents on the fourth peak, no frequency modification and an amplitude
modification of less than 1 dB for the next peaks).

These figures also show modifications in the phase of the input impedance. A local shift in the phase appears when the mute is inserted into the bell of the instrument, with a maximum shift at 63 Hz. At 58 Hz in Figures 11 and 13 (top), the phase angle is 0.32 rad greater when the mute is present, and at 55 Hz in Figures 12 and 13 (bottom), it is 0.25 rad greater. With control applied, the phase value is closer to that of the impedance with no mute present at 58 Hz in Figure 11 (the phase angle is only 0.15 rad greater when control is applied). However, at 55 Hz in Figure 12, with control applied, the phase value is further from that of the impedance with no mute present (the phase angle is 0.4 rad greater when control is applied).

For both notes, the behavior with respect to the magnitude of the input impedance is the same; when the mute is present with no control applied, the amplitude at the playing frequency is decreased and there are difficulties in playability, and when the control is applied, the amplitude at playing frequency is increased and the playability is restored. Conversely, the behavior with respect to the phase of the input impedance is different for each note when the control is applied; in one case it becomes closer to the phase value without mute ($Bb_1$) and in the other case it becomes further away ($A_1$). This suggests that the primary reason for the difficulties in the playability of the notes when the mute is inserted is the decrease in impedance magnitude at the playing frequency rather than the
phase modification.

Figure 14 shows a spectrogram of the same musical sequence as depicted in Figure 4, but with control configuration $\phi = \pi$ and $G = 2$ applied. This time, all the notes, including $Bb_1$ and $A_1$, can be played without difficulty. This is explained by the suppression of the extra impedance peak in the input impedance (see Figure 13), and the increase of the impedance magnitude at the playing frequency when the control is applied (+4.7 dB at 58 Hz for the note $Bb_1$ and +2.8 dB at 55 Hz for the note $A_1$, compared to when no control is applied when the mute is inside the bell). With this control gain, the effective power sent by the amplifier to the speaker varies from 0.05 Watts (low level) to 2 Watts (high level). As the trombone is played at a low dynamic level here, the effective power is close to 0.05 Watts.

To demonstrate the influence of the control on the playability of the instrument, a small musical sequence consisting of two repetitions of the note $A_1$ is played (see Figure 15). For each repetition, the control is successively switched ON, OFF and finally ON.

The first note is from 0 to 3 seconds, with the control applied from 0 to 1.5 seconds, then switched OFF and then applied again from 2.8 seconds. At first, with the control applied, the pitch $A_1$ is played without problem. When the control is switched OFF, the self-sustained oscillations stop, and there is no more sound produced by the instrument. When the control is applied again, the self-sustained oscillations start again, sounding at
pitch $A_2$ (110 Hz) for 25 ms until the musician stops blowing.

The second note is from 4.5 to 8 seconds, with the control stopped between 5.3 and 6.4 seconds. When the control is applied, the note is played without any problem, but when it is stopped, the note is unstable, with frequency variations. But this time, as the self-sustained oscillation is retained, the stable note still sounds at pitch $A_1$ when the control is switched back ON. This control case is referred to as case 1 in Table 4.

These two examples show that using active control to suppress the extra impedance peak restores the playability of the notes which were influenced by the mute. Moreover, it seems that the difficulties in producing the notes when a straight mute is inserted are due to a decrease in the magnitude of the input impedance at the playing frequency.

**B. Moving the extra impedance peak**

Figure 16 shows the control of the resonance measured between the speaker and microphone of the mute when $\phi = \pi/2$ and $G = 0.7$, and when $\phi = 3\pi/4$ and $G = 3$. The second control case was determined according to Figure 9; when increasing the gain $G$, greater changes in frequency can be achieved when a phase shift of $3\pi/4$ is used (compared with a phase shift of $\pi/2$), although the amplitude of the peak is then also affected. The control causes the frequency of the resonance to increase, from 66 Hz to 73 Hz (increase of 175 cents) in the first case, and from 66 Hz to 115 Hz (increase of 961 cents) in the second
case. Only the first control case, with $\phi = \pi/2$ and $G = 0.7$, is considered in the remainder of this section.

Figure 17 shows the input impedance of the trombone with the slide in the $C_2$ position without mute, with a normal straight mute inserted and with the active mute with control set at $\phi = \pi/2$ and $G = 0.7$. To play a $C_2$, a tenor trombone with a F valve has been used. Note that the $C_2$ is not a pedal note, as it has a frequency corresponding to the second impedance peak when there is no mute. For the normal mute without control, the extra impedance peak appears at the same frequency as the resonance of the transfer function of Figure 2 (i.e. $f_{pp}$). As in Section IV.A, on both sides of this peak, there are little troughs. These troughs do not modify the impedance magnitude at 65 Hz, the playing frequency of the note $C_2$, when there is no control. With the control applied, the peak is moved in frequency, from 66 Hz to 71 Hz. The troughs on both sides of the peak move with it, so that the magnitude at 65 Hz is decreased by 4.6 dB when the control is applied.

The local shift in the phase also moves, along with the peak, when the control is applied, with a maximum shift of 0.95 rad at 68.5 Hz. At 65 Hz, the shift is of 1.28 rad when no control is applied, which doesn’t lead to difficulties in playability. When the control is applied, the phase shift is reduced to 0.65 rad.

Figure 18 shows a spectrogram of two repetitions of the note $C_2$ played separately on a trombone, first with the control applied (from 0 to 2.5 seconds), and second without
control (from 3 to 5 seconds). Remember that without control, there is no difficulty in playing $C_2$. When the control is applied, the note exhibits instabilities, particularly around 1 second where the harmonic richness of the emitted sound is reduced for about 200 ms, with no harmonics higher than 500 Hz. According to the musician, the note $C_2$ is difficult to play when the control is applied, but not as difficult as $Bb_1$ and $A_1$ when there is no control. The second $C_2$ note, without control applied, does not show any problem in terms of its playability. To check that the dynamic level was the same for these two notes, the spectral centroid (see Section IV.C) has been calculated for the two notes when they are stable, as the value of the spectral centroid depends on the harmonic richness of the sound, which for a brass instrument depends on the playing level due to nonlinear effects. For the first note, between 1.5 and 2 seconds, the spectral centroid is at 731 Hz, while for the second note, between 4 and 4.5 seconds, it is at 730 Hz. Therefore, it can be concluded that both notes were played at the same dynamic level.

Moving the extra impedance peak in frequency results in the playability issues affecting different notes (for instance, $C_2$). Once again, this is explained completely by the decrease in the magnitude of the input impedance at the playing frequency (see Figure 17 and Section IV.A). This control case is referred to as case 2 in Table 4.

To verify that the playability problem has been moved and not enhanced, Figure 19 shows a musical sequence in which the note $A_1$ is played on a trombone, with the same
control alternately switched ON and OFF (it is ON from 0 to 1.2 seconds and from 2.3 to 3.5 seconds, and OFF from 1.2 to 2.3 seconds and from 3.5 to 4 seconds). As discussed previously, without control, the $A_1$ note is difficult to produce. When the control is not applied, the sound produced by the instrument endures a loss of richness between 1.2 and 2.3 seconds (no harmonic content beyond 1500 Hz), and the instrument fails to sound between 3.5 and 4 seconds. When the control is applied, the sound is richer harmonically and more stable, despite a few instabilities at the beginning of the sequence. This control case is referred to as case 3 in Table 4.

Increasing the frequency of the extra impedance peak results in a modification of the playability problem; the issue is encountered on higher notes. This confirms that the playability problem results from a decrease in the magnitude of the input impedance at playing frequency.

C. Radiated Sound characterization

In this section, a comparison is made between the sound radiated by the instrument when the mute is uncontrolled and the sound radiated when the mute is controlled. The comparison is made for three different control cases:

- the control with $\phi = \pi$ and $G = 2$ of Section IV.A, focusing on the note $A_1$ (Figure 15), referred to as case 1,
the control with $\phi = \pi/2$ and $G = 0.7$ of Section IV.B, focusing on the note $C_2$
(Figure 18), referred to as case 2,

- the control with $\phi = \pi/2$ and $G = 0.7$ of Section IV.B, focusing on the note $A_1$
(Figure 19), referred to as case 3.

In case 1, the comparison is made between the $U$ (uncontrolled) part and the second $C$ (controlled) part of the second note of Figure 15 (the note played from 4.5 to 8 seconds). In case 3, the comparison is made between the first $U$ (uncontrolled) part and the second $C$ (controlled) part of Figure 19. For all three cases, the comparison is made between notes that could actually be played (for example, in the first $U$ part of Figure 15, the note cannot be played, so no comparison may be made).

The comparisons between sounds, measured one metre away from the output of the instrument, are made using two criteria:

- the sound level,
- the spectral centroid, which is related to the “brightness” of the sound$^{20}$.

The sound level $L$ is calculated with

$$L_{dB_{SPL}} = 10\log_{10} \left( \frac{p^2}{p_0^2} \right),$$

(10)
where \( p \) is the measured pressure signal, and \( p_0 \) the reference value. As the comparison is between uncontrolled and controlled instruments, \( p_0 \) is the mean value of the RMS pressure amplitude during the period with no control applied, and \( p \) is the mean value of the RMS pressure amplitude during the period when the instrument is being controlled.

The frequency of the spectral centroid \( f_{sc} \) is calculated using

\[
\begin{align*}
    f_{sc} &= \frac{\sum_{f=1}^{N} f x(f)}{\sum_{f=1}^{N} x(f)},
\end{align*}
\]

where \( f \) is the frequency, \( N \) is the Nyqvist frequency for the measured signal (here, \( N = 22050 \) Hz) and \( x(f) \) is the amplitude of the spectrum of the sound at frequency \( f \).

Table 4: Differences in sound level and spectral centroid for the sound radiated by the instrument when control is applied to the mute (for three cases of control).

<table>
<thead>
<tr>
<th>Control cases</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound level (dB_{SPL})</td>
<td>+2.77</td>
<td>-1.48</td>
<td>+8.22</td>
</tr>
<tr>
<td>Spectral Centroid (Cents)</td>
<td>+160</td>
<td>-72</td>
<td>+185</td>
</tr>
</tbody>
</table>

Table 4 presents these differences. The sound level is increased for cases 1 (+2.77 dB) and 3 (+8.22 dB). In both cases, the intention of the control is to restore the playability of the notes, therefore as the notes are more stable and easier to play, they sound louder. In case 2, the sound level is decreased because the control has a negative impact on the playability of the note \( C_2 \) for 200 ms (see Figure 18).
The differences in spectral centroid for the three control cases are shown in cents. The spectral centroid changes in value depending on how the player blows the instrument. If the musician blows harder, the high frequency harmonics in the sound are enhanced, and the spectral centroid becomes higher in frequency. For these three cases, the spectral centroid varies between 700 Hz (case 2) and 900 Hz (case 3), which correlates with Sandell\textsuperscript{21}. For control cases 1 and 3, the spectral centroid increases in frequency when the control is applied, which indicates a “brighter” sound, while in control case 2, it decreases in frequency when the control is applied, which indicates a “darker” sound.

When the control is applied so that the playability of the note is restored, the note is played more easily, and therefore the sound level is higher and there is more harmonic content in the produced sound. Conversely, when the playability of a note is reduced, both its sound level and its harmonic content are reduced.

V. Conclusion and perspectives

When a mute is used on a brass instrument, an extra impedance peak is introduced that affects the playability of some of the low notes (the pedal notes of the instrument in the case studied in this paper). Using active control to suppress the extra impedance peak enables the playability of these notes to be restored. Meanwhile, moving the peak in frequency shifts the playability problem to other notes.
To further investigate the effect that the mute has on the magnitude of the input impedance (and hence the playability of certain notes on the instrument), future work could include carrying out a full simulation of the behavior of the self-sustained dynamics of a trombone with mute inserted. With this fuller understanding, it may become possible in the future to create a single universal mute, employing active control to mimic the variety of effects provided by the different types of mute in current usage.

I. Appendix A: Model with mute and loudspeaker

In Section II.B, the coupled system [mute + loudspeaker] is described by a second order band-pass filter (eq.(2)), which is a simplification neglecting some of the loudspeaker dynamics. In this section, a model combining a second order band-pass filter, representing the mute, and a model of the speaker is developed. The loudspeaker model is based on Lissek et al.\textsuperscript{22} (case 0).

Let \( H \) be as defined by eq.(2), using the parameters obtained through fitting of the transfer function for the mute positioned inside the bell of the instrument (see Table 2), and \( H_L \) be the acoustic impedance of the loudspeaker, defined as:

\[
H_L(s) = \frac{1}{Z_{mc}} \left( R_{ms} + sM_{ms} + \frac{1}{sC_{mc}} \right), \tag{12}
\]

where \( Z_{mc} = \rho c S_d \) is the mechanical equivalent to characteristic medium impedance, with \( \rho \) the density of the air, \( c \) the velocity of sound in the air and \( S_d \) the surface of the membrane.
of the loudspeaker, $R_{ms}$ the mechanical resistance of the membrane, $M_{ms}$ its mass, $s = j \omega$ the Laplace variable and

$$C_{mc} = \frac{C_{ms} V_b}{V_b + C_{ms} \rho c^2 S_d^2},$$

(13)

with $C_{ms}$ the suspension compliance and $V_b$ the closed volume behind the loudspeaker. The values used for these parameters are reported in Table 5.

$H$ and $H_L$ are in parallel, so that

$$H_{tot} = \frac{H(s)H_L(s)}{H(s) + H_L(s)},$$

(14)

Figure 20 shows the transfer functions $H$ and $H_{tot}$. Both are very similar, with differences of only 0.4 Hz (3 cents) in frequency and of less than 0.1 dB in amplitude. This confirms that it is not necessary, in this particular case, to model the loudspeaker separately from the mute.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$340 , m.s^{-1}$</td>
<td>$\rho$</td>
<td>$1.2 , kg.m^{-3}$</td>
</tr>
<tr>
<td>$S_d$</td>
<td>$13.10^{-4} , m^2$</td>
<td>$R_{ms}$</td>
<td>$0.47 , kg.s^{-1}$</td>
</tr>
<tr>
<td>$M_{ms}$</td>
<td>$1.5.10^{-3} , kg$</td>
<td>$C_{ms}$</td>
<td>$0.76.10^{-3} , kg.m^2.s^{-2}$</td>
</tr>
<tr>
<td>$V_b$</td>
<td>$1.10^{-4} , m^3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Acknowledgements**
This work was carried out during the PhD of Thibaut Meurisse, funded by Agence Nationale de la Recherche (ANR IMAREV project) and Université Pierre et Marie Curie (UPMC, Paris 6). We thank gratefully Alain Terrier and Gérard Bertrand for their precious help in building the system. We thank the Newton International Fellowship scheme for funding the collaboration between France and UK.

REFERENCES


6. For the trombone without mute, in any given slide position, the first resonance is not part of the harmonic series formed by the other resonances. As a result, the harmonic series formed by the higher resonances has a missing fundamental. The pitch that corresponds to this missing fundamental lies an octave below the second resonance and is known as a pedal note\(^7\).


Figure Captions

Figure 1. Top left: Schematic diagram of a straight mute with embedded microphone and speaker, and control system (phase shifter and gain), inserted into the bell of a trombone. Right: Photograph of the active straight mute (phase shifter and gain are not shown here). Bottom left: Equivalent electric circuit of the trombone coupled to the mute with control system, with $U_1$ and $P_1$ respectively the flow and the pressure at the input of the trombone, $Z_1$ the impedance of the trombone, $U_2$ the flow at the input of the mute, $P_2$ the pressure measured inside the mute by the microphone (Mic), $Z_2$ the impedance of the mute and $U_3$ the flow induced by the loudspeaker (LS) inside the mute, $\phi$ the phase shifting and $G$ the control gain.

Figure 2. Top: Transfer functions measured between the speaker and the microphone for the mute outside (solid black line) and inside (solid gray line) the bell of the trombone, and fitted transfer functions for the mute outside (dash black line) and inside (dash gray line) the bell of the trombone (see Table 2 and eq.(2)). Bottom: Phase of the measured transfer functions for the mute outside (solid black line) and inside (solid gray line) the bell of the trombone.

Figure 3. Top: Input impedance magnitude of a trombone in $Bb_1$ position without mute (dash grey line) and with the active mute with no control applied (solid black line). Bottom: Phase of the input impedance.
Figure 4. Spectrogram of a musical sequence played on a trombone with an ordinary straight mute. This spectrogram corresponds to the first of the online sound clips.

Figure 5. Electrical circuit of the coupled phase shifter and phase inverter. The values of the different components are given in Table 1.

Figure 6. Schematic diagram of the active control applied to a system $H$ in closed-loop (dotted $u$ arrow) and in open-loop (solid $u$ arrow), with a gain $G$ and a phase shifting $\phi$. $y$ is the output of the system, $u$ the command and $w$ the disturbance applied to the system.

To obtain an open loop, $u$ is not added to $w$ and there is no actuator.

Figure 7. Top : Calculated transfer functions of a resonator without control (solid black line), and with control so that $\phi = 0$ and $G = 0.45$ (dashed black line), $\phi = \pi$ and $G = 2$ (solid light grey line), $\phi = \pi/2$ and $G = 2$ (dashed dark grey line) and $\phi = -\pi/2$ and $G = 2$ (solid dark grey line). Bottom : Phase of the transfer functions.

Figure 8. Amplitude map for different controls of gain (ordinates) and phase (abscissa), when the mute is in the bell of the trombone. The lighter the colour, the higher the amplitude. The white zone is unstable. The -o- curve shows the theoretical limit between stable and unstable zone for a resonator described by eq.(2).

Figure 9. Frequency map for different controls of gain (ordinates) and phase (abscissa), when the mute is in the bell of the trombone. The lighter the colour, the higher the frequency. The black zone is unstable.
Figure 10. Top: Transfer functions of the mute inside the bell of the trombone without control (solid black line) and controlled with $\phi = \pi$ and different $G$ values: $G = 0.1$ (solid grey line), $G = 0.4$ (dash black line), $G = 0.8$ (dash grey line), $G = 1$ (-x-) and $G = 2$ (-o-). Bottom: Phase of the transfer functions.

Figure 11. Top: Input impedance magnitude of a trombone in $Bb_1$ position without mute (solid grey line), with a normal mute (solid black line) and with an active mute with $\phi = \pi$ and $G = 2$ (dash black line). Bottom: Phases of the input impedance.

Figure 12. Top: Input impedance magnitude of a trombone in $A_1$ position without mute (solid grey line), with a normal mute (solid black line) and with an active mute with $\phi = \pi$ and $G = 2$ (dash black line). Bottom: Phases of the input impedance.

Figure 13. Top: Zoom on the extra impedance peak of the input impedance magnitude of a trombone in $Bb_1$ position without mute (solid grey line), with a normal mute (solid black line) and with an active mute with $\phi = \pi$ and $G = 2$ (dash black line). The vertical dotted arrow is at 58 Hz, the playing frequency of a $Bb_1$. Bottom: Zoom on the extra impedance peak of the input impedance magnitude of a trombone in $A_1$ position without mute (solid grey line), with a normal mute (solid black line) and with an active mute with $\phi = \pi$ and $G = 2$ (dash black line). The vertical dotted arrow is at 55 Hz, the playing frequency of a $A_1$.

Figure 14. Spectrogram of a musical sequence played on a trombone with an active mute,
\( \phi = \pi, G = 2 \). This spectrogram corresponds to the second of the online sound clips.

Figure 15. Spectrogram of the note \( A_1 \) played two times, from 0 to 3 seconds and from 4.5 to 8 seconds, with \((C)\) and without \((U)\) control applied, \( \phi = \pi, G = 2 \). Between the two notes, the musician does not blow \((NB)\). This spectrogram corresponds to the third of the online sound clips.

Figure 16. Top : Transfer functions of the mute inside the bell of the trombone without control (solid black line), controlled with \( \phi = \pi/2 \) and \( G = 0.7 \) (dash black line) and with \( \phi = 3\pi/4 \) and \( G = 3 \) (dash grey line). Bottom : Phase of the transfer functions.

Figure 17. Top : Input impedance magnitude of a trombone in \( C_2 \) position without mute (solid grey line), with a normal mute (solid black line) and with an active mute with \( \phi = \pi/2 \) and \( G = 0.7 \) (dash black line). Bottom : Phases of the input impedance.

Figure 18. Spectrogram of the note \( C_2 \) played two times, first with control applied from 0 to 2.5 seconds and second without control applied from 3 to 5 seconds, with \( \phi = \pi/2, G = 0.7 \). This spectrogram corresponds to the fourth of the online sound clips.

Figure 19. Spectrogram of the note \( A_1 \) played with \((C)\) and without \((U)\) control applied, \( \phi = \pi/2, G = 0.7 \). This spectrogram corresponds to the fifth of the online sound clips.

Figure 20. Top : Transfer function of the mute inside the bell of the trombone, calculated with eq.(2) (black line) and calculated taking the loudspeaker in account with eq.(14) (gray line). Bottom : Phases of the transfer functions.
Impedance (dB)

Phase (rad)

Frequency (Hz)

--- No mute
--- No control
--- $\phi = \pi$, $G = 2$
Impedance (dB)

Phase (rad)

Frequency (Hz)

No mute
No control
\( \phi = \pi/2, G = 0.7 \)