A New Polarimetric Change Detector in Radar Imagery

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Abstract

In modern society, the anthropogenic influences on ecosystems are central points to understand the evolution of our planet. A polarimetric SAR (synthetic aperture radar) may have a significant contribution in tackling problems concerning land use change, since such data are available with any-weather conditions. Additionally, the discrimination capability can be enhanced by the polarimetric analysis.

Recently, an algorithm able to identify targets scattering an electromagnetic wave with any degree of polarization has been developed, which makes use of a vector rearrangement of the elements of the coherency matrix. In the present work, this target detector is modified in order to perform change detection between two polarimetric acquisitions, for land use monitoring purposes. Regarding the selection of the detector parameters, a physical rationale is followed, developing a new parameterization of the algebraic space where the detector is defined. As it will be illustrated in the following, this space is 6 dimensional complex with restrictions due to the physical feasibility of the vectors. Specifically, a link between the detector parameters and the angle differences of the eigenvector model is obtained. Moreover, a dual polarimetric version of the change detector is developed, in case that quad-polarimetric data are not available.

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With the purpose of testing the methodology, a variety of datasets were exploited: quad-polarimetric airborne data at L-band (E-SAR), quad-polarimetric satellite data at C-band (Radarsat-2), and dual-polarimetric satellite data at X-band (TerraSAR-X). The algorithm results show agreement with the available information about land changes. Moreover, a comparison with a known change detector based on the maximum likelihood ratio is presented, providing improvements in some conditions. The two methodologies differ in the analysis of the total amplitude of the backscattering, where the proposed algorithm does not take this into consideration.

**Keywords**

Synthetic Aperture Radar, Polarimetry, Change Detection.

I. INTRODUCTION

In order to understand comprehensively the evolution of our planet, accurate land monitoring is indispensable. Satellite systems seem to suit better these requirements due to their extensive and constant coverage. Additionally, radar images can be acquired with reasonable independence on the weather conditions. The latter property is revealed as a winning advantage to achieve time series of data. Moreover, the analysis of the polarization of the electromagnetic waves scattered by targets in the scene [1], [2], [3] enhances significantly the discrimination capabilities with respect to single polarization acquisitions. The work presented in this paper is aimed at developing a polarimetric change detector (PCD). The PCD would be able to detect areas (or generally targets) in the scene for which the observed polarimetric characteristics suffered a change between the two acquisitions. The more direct application of a PCD is the identification of areas that suffered changes due to human or natural interventions. Classical examples could be forest logging, woodland fires or flooding. Operatively, a PCD may be exploited in conjunction with a GIS (Geographical Information System) database to perform disaster or illegal activities monitoring. On the other hand, a different application of a PCD (as it will be better explained in the following) is as a pre-
processing step for methodologies exploiting time series of polarimetric SAR acquisitions that require a stability of the target characteristics in the scene. In the case of Polarimetric and Interferometric SAR (POLInSAR) this requirement is often referred as Equi Scattering Mechanism (ESM).

In the following, a very brief introduction to target polarimetry is presented, focusing mainly on the mathematical tools exploited in the development of the proposed change detector.

A single target is any target scattering an electromagnetic (EM) wave having a fixed polarization in time/space. It can be characterized using the scattering (Sinclair) matrix \[1, 2\]:

\[
[S] = \begin{bmatrix}
HH & HV \\
VH & VV
\end{bmatrix},
\]

(1)

where \(H\) stands for linear horizontal and \(V\) for linear vertical (therefore the HV image is obtained transmitting a linear vertical polarization and receiving the linear horizontal one). Equivalently, a formalism based on the scattering vector can be exploited:

\[
k = \frac{1}{2} \text{Trace} ([S] \Psi_2) = [k_1, k_2, k_3, k_4]^T.
\]

(2)

where \(\text{Trace}(.)\) is the sum of the diagonal elements of the matrix inside, and \(\Psi_2\) is a complete set of 2x2 basis matrices under a Hermitian inner product [1][2]. In the case that the transmitter and receiver antennas are the same (i.e. monostatic) and the observed target is reciprocal the scattering matrix is symmetric (i.e. \(HV = VH\)) and only 3 complex numbers are needed to characterize a single target. Therefore, the four dimensional scattering vector can be substituted with a three dimensional one. For the sake of brevity, in this paper, the hypothesis of a symmetric scattering matrix will be assumed. However, the latter hypothe-
sis is not necessary for the derivation of the detector and an extended version, covering the nonreciprocal and bistatic case, can be easily obtained. Finally, it is possible to define a scattering mechanism as a normalized vector \(\omega = k/||k||\). The latter unitary vector is defined in a special unitary 3 dimensional complex space (SU(3)) and represents an ideal polarimetric target where the total amplitude backscattered (the span of the scattering matrix) is unitary.

Generally, the targets observed by a SAR system are not ideal scattering mechanisms, but a combination of different objects which we refer to as partial targets [4]. In order to characterize a partial target, a single scattering matrix \([S]\) is not sufficient, since this is a stochastic process and second order statistics are required [1], [2], [3]. In this context the target covariance matrix can be estimated:

\[
[C] = \langle k k^* T \rangle,
\]

where \(\langle \rangle\) is the finite averaging operator. In general, if the scattering vector in a generic basis is \(k = [k_1, k_2, k_3]^T\), where \(k_1, k_2, k_3 \in \mathbb{C}\), the covariance matrix is:

\[
[C_3] = \begin{bmatrix}
\langle |k_1|^2 \rangle & \langle k_1 k_2^* \rangle & \langle k_1 k_3^* \rangle \\
\langle k_1^* k_2 \rangle & \langle |k_2|^2 \rangle & \langle k_2 k_3^* \rangle \\
\langle k_1^* k_3 \rangle & \langle k_2^* k_3 \rangle & \langle |k_3|^2 \rangle
\end{bmatrix},
\]

In the literature, when \(k\) is expressed in the Pauli basis (i.e. \(k = \frac{1}{\sqrt{2}}[HH + VV, HH - VV, 2*HV]^T\)), the covariance matrix takes the name of coherency matrix \([T]\) [1], [2], [3]. However, the matrix \([T]\) and \([C]\) are unitarily similar, therefore they contain the same information and they can be both exploited in the following formulation.

The methodology proposed in this paper takes advantage of the polarimetric coherence. If two different scattering mechanisms, \(\omega_1\) and \(\omega_2\), are considered, the polarimetric coherence
is:
\[
\gamma_p = \frac{\omega_1^T[C]\omega_2}{\sqrt{(\omega_1^T[C]\omega_1)(\omega_2^T[C]\omega_2)}}.
\]

(5)

In the case of two polarimetric acquisitions (i.e. two polarimetric images acquired from different positions and/or different times), two scattering vectors can be retrieved: \(k_1\) and \(k_2\). The polarimetric coherence in (5) can then be substituted by a mixed polarimetric and interferometric coherence as shown in equation 6 [5], [6]:
\[
\gamma_{12} = \frac{\omega_1^T[\Omega_{12}]\omega_2}{\sqrt{(\omega_1^T[T_{11}]\omega_1)(\omega_2^T[T_{22}]\omega_2)}},
\]

where \([T_{11}] = \langle k_1 k_1^T \rangle, [T_{22}] = \langle k_2 k_2^T \rangle\) and \([\Omega_{12}] = \langle k_1 k_2^T \rangle\).

In the following, the symbol \([T]\) will be used exclusively for the coherency matrix (i.e. Pauli basis), while \([C]\) denotes a generic covariance matrix in any basis. In this work, the coherence in (6) will be estimated exploiting coherency matrices due to their wide utilization in the literature (but again other bases could be exploited).

Many algorithms have been developed to tackle the problem of change detection [7], [8]. Among the procedures exploiting radar polarimetry, one of the most utilized is the Maximum Likelihood ratio (LR) of the two acquisitions [9]. If \([T_{11}]\) and \([T_{22}]\) are the two coherency matrices, the test can be easily built defining \([T] = \frac{|T_{11}| + |T_{22}|}{2}\) and calculating the ratio:
\[
R_\Delta = \sqrt{\frac{\text{det}([T_{11}])\text{det}([T_{22}])}{\text{det}([T])}},
\]

(7)

where \(\text{det}([T])\) is the matrix determinant, and \(R_\Delta\) is defined between 0 and 1. If the ratio is close to unity then no change has occurred.

The latter methodology was selected since it is fast and beneficial and therefore, provides a simple way to compare the results of the proposed approach with detection masks obtained with a methodology fundamentally different from the one exploited in this paper. The differences between the LR methodology and the proposed change detector are mainly two. First
of all, the LR approach is based on the statistical distribution of the polarimetric data and is focused on differences on the distributions, whereas the proposed one exploits the polarimetric information of the data. As a consequence, differences are expected between the two detections results. Secondly, in the calculation of the matrix determinants the overall amplitude of the backscattering (i.e. $\text{Trace}(T)$) has a significant contribution. This last aspect is helpful when differences in amplitude correspond to target changes. In such a case, the proposed detector will not be able to pick the amplitude differences and can be complemented with the LR methodology (or a threshold on the Span of the scattering matrix). However, in general the amplitude is very sensitive to other factors as changes in the dielectric constant (e.g. in vegetation due to wetness of the weather conditions) and inaccurate radiometric calibration. Therefore, a change detector relying excessively on amplitude may present some undesired false alarms. In conclusion, with the proposed detector there is an opportunity to exclude these changes and benefit from a separated treatment between polarimetry and amplitude. The change detector proposed in this paper is based on a partial target detector, previously developed by the authors [10], [11], [12], and depends exclusively on the polarimetric information, excluding the overall amplitude. For this reason, the proposed algorithm is independent of the overall amplitude as well. In this sense, it is complementary to the LR ratio mentioned above. As a final remark, it has to be said that the LR methodology exploited here is originally derived from early works in [13] (which takes also into account the issue related with the number of looks employed). Clearly, the full maximum likelihood approach as presented in [13] can be used for quantitative decision boundaries, however it will not change the main ideas and conclusions derived from the comparison.

The organization of the text is as follows. Firstly, the partial target detector based on a perturbation analysis will be presented. Having obtained a general formulation of the detector,
the algorithm is modified in order to accomplish change detection. Specifically, two versions are proposed to take into account quad and dual polarization polarimetry. Subsequently, the issue of setting the selectivity of the detector is tackled in order to detect changes of different magnitude. A mathematical formulation is pursued that is able to link the differences of the scattering mechanism with the detector parameters. In this work, a physical rationale was followed and a parameterization of the change detector is devised. Specifically, the test is set on the base of changes on the parameters of the eigenvector model [14]. Once the parameters can be set, the algorithm is applied to real data in order to test its performance. In particular, a relatively large variety of data, ranging from airborne to satellite, and with different frequencies, is employed for this purpose. A comparison with the LR ratio is also discussed.

II. PARTIAL TARGET DETECTOR

The aim of this section is to outline the development of a general detector for partial targets, exploiting the same methodology (i.e. perturbation analysis) of the single target detector presented in [15]. More details on the partial target detector can be found in [10], [11]. The first step in order to obtain an extension of the single target detector is to introduce a vector formalism where each partial target can be uniquely defined with one vector. Starting from (4), a feature partial scattering vector is introduced [11], [10]:

\[
\mathbf{t} = \text{Trace}([C]\Psi_3) = [t_1, t_2, t_3, t_4, t_5, t_6]^T = \\
= [\langle |k_1|^2 \rangle, \langle |k_2|^2 \rangle, \langle |k_3|^2 \rangle, \langle k_1^* k_2 \rangle, \langle k_1^* k_3 \rangle, \langle k_2^* k_3 \rangle]^T,
\]

where \( \Psi_3 \) is a complete set of 3x3 basis matrices under a Hermitian inner product. \( \mathbf{t} \) represents all the information of a coherence matrix \([C]\) (6 independent complex numbers) in a vector formalism. Therefore, \( \mathbf{t} \) is defined in a subspace of \( \mathbb{C}^6 \) and has the first three elements...
real positive and the second three complex, since the \([C] \) matrix is Hermitian semi positive definite. In the following, the norm of the vector \( t (\sigma = |t|) \) will be referred to as the \textit{overall amplitude}. Any partial target can be characterized by a unique partial vector \( t \), therefore the partial target that we are interested in the detection can be characterized with this formalism with the symbol \( t_T \). The latter can be any vector of \( \mathbb{C}^6 \) in the subspace of the physically feasible ones (again its first three elements real positive and the second three complex). More formally, \( t_T \) could be expressed as

\[
\mathcal{L}_T = \text{Trace}([C_T] \Psi_3) = [t_{T1}, t_{T2}, t_{T3}, t_{T4}, t_{T5}, t_{T6}]^T
\]

where \([C_T]\) represents the covariance matrix of the target of interest. In order to apply the same processing introduced in [15] for single targets, a perturbed version of \( t_T \) can be obtained \( \hat{t}_T \). This operation can be accomplished by slightly rotating the \( t_T \) in the space of physically feasible targets. The subsequent step is to perform a change of basis which makes the target of interest to lie only on 1 component: \( \mathcal{L}_T = \sigma_T [1, 0, 0, 0, 0, 0]^T \), where \( \sigma_T \) represents the norm of \( t_T \) \( (\sigma_T = ||t_T||) \). The latter operation can again be accomplished by multiplications with rotation matrices in the 6 dimensional space. Please note, care must be taken selecting the rotation matrix, since there are rotation matrices that can transform the vector outside the space of physical feasibility. However, it can be demonstrated that any vector in the subspace of physical feasibility can be transformed by a rotation into the first axis of the basis. In the following, we will make use of a normalized version of the vector \( \mathcal{L}_T \), defined as \( \hat{\mathcal{L}}_T = \frac{\mathcal{L}_T}{\sigma_T} \). In such basis the unitary perturbed target is expressed as \( \hat{\mathcal{L}}_P = [a, b, c, d, e, f]^T \), where \( a, b, c \in \mathbb{R}^+ \), \( |d| \leq \sqrt{ab} \), \( |e| \leq \sqrt{ac} \), \( |f| \leq \sqrt{bc} \) and \( a \gg b, c, |d|, |e|, |f| \). In [15] the detector is built as a weighted inner product between \( \varphi_T \) and \( \varphi_P \) (the two scattering mechanisms). In order to obtain this in the 6 dimensional space.
of the partial targets, we can build a matrix $[A]$ exploiting a basis of $C^6$ built employing a Gram-Schmidt ortho-normalization where the first axis is represented by the vector $\hat{t}_T$ [16]. If $u_1 = \hat{t}_T$, $u_2$, $u_3$, $u_4$, $u_5$, and $u_6$ represent the ortho-normal basis, we can build a diagonal matrix $[A]$ with the observed partial target $t$ expressed in the new Gram-Schmidt basis:

$$[A] = \text{diag}(\hat{t}_T^T t, \hat{u}_2^T t, \hat{u}_3^T t, \hat{u}_4^T t, \hat{u}_5^T t, \hat{u}_6^T t)$$

The linear product between the target and its perturbed version can be simply defined as $\hat{t}_T \hat{t}_P^T$. In order to weight the inner product for the target observed in the data we can multiply each vector for the matrix $[A]$. To conclude, the weighted inner product can be defined as $\langle [A] \hat{t}_T \rangle \langle [A] \hat{t}_P \rangle$:

$$\gamma_q = \frac{\hat{t}_T^T [P] \hat{t}_P}{\sqrt{\langle [A] \hat{t}_T \rangle^T [P] \langle [A] \hat{t}_T \rangle \langle [A] \hat{t}_P \rangle}} = \frac{1}{\sqrt{1 + \frac{b^2 P_2}{a^2 P_1} + \frac{c^2 P_3}{a^2 P_1} + \frac{|d|^2 P_4}{a^2 P_1} + \frac{|e|^2 P_5}{a^2 P_1} + \frac{|f|^2 P_6}{a^2 P_1}}}$$

where $[P] = \langle [A] \rangle^T \langle [A] \rangle = \text{diag} \left( |\hat{t}_T^T t|^2, |\hat{u}_2^T t|^2, |\hat{u}_3^T t|^2, |\hat{u}_4^T t|^2, |\hat{u}_5^T t|^2, |\hat{u}_6^T t|^2 \right)$. In order to make this more readable the following formal substitution is performed:

$$[P] = \text{diag}(P_1, P_2, P_3, P_4, P_5, P_6)$$. $\gamma_q$ is a real number defined between 0 and 1. In order to have unbiased results the elements of $\hat{t}_P$ have to be chosen as $b = c = |d| = |e| = |f|$. With bias it is meant an imbalance in the weight assigned to each of the covariance matrix elements. In such case, some elements of $[C]$ will have more importance in the evaluation of changing than others. This is clear from (10), where $b = c = |d| = |e| = |f|$ appear in the detector as factors multiplied to the elements of the covariance matrix. We leave as a future work an analysis of whether different weighting of the elements may improve the detection capabilities.
If the clutter components are renamed as $P_c = P_2 + P_3 + P_4 + P_5 + P_6$, the target as $P_T = P_1$, and the ratio $b^2/a^2 = RedR$, the detector can be simplified:

$$
\gamma_q = \frac{1}{\sqrt{1 + RedR \frac{P_c}{P_T}}} = \frac{1}{\sqrt{1 + RedR \left( \frac{t^T \hat{t}}{|t^T \hat{t}|^2 - 1} \right)}}.
$$

The Signal to Clutter Ratio (SCR) can be defined as:

$$
SCR = \frac{P_T}{P_c} = \left( \frac{t^T \hat{t}}{|t^T \hat{t}|^2 - 1} \right)^{-1}.
$$

The detector is finalized by setting a threshold on $\gamma_q$ as:

$$
H_0 : |\gamma_q(P_T, P_c)| \geq T \text{ and } H_1 : |\gamma_q(P_T, P_c)| < T.
$$

Details regarding the selection of the parameters $RedR$ and $T$ for a single-acquisition partial target detector can be found in [10].

### III. CHANGE DETECTOR

The aim of this section is to modify the previous algorithm in order to perform change detection between two different acquisitions. The algorithm presented in the previous section can detect a specific polarimetric target (i.e. a reference target), selected on the base of a theoretical model or trained on real data. The main idea of the change detector is to employ the partial vector of the first acquisition as a reference target for a detection executed on the second acquisition. Therefore, if $[T_{11}]$ and $[T_{22}]$ correspond to the coherency matrices of first and second acquisitions, the target to detect is $t_1 = \text{Trace}([T_{11}] \Psi_3)$ (with $\hat{t}_1$ its normalized version), and the target under test is $t_2 = \text{Trace}([T_{22}] \Psi_3)$. 
The polarimetric change detector (PCD) expression can be easily obtained starting from 11 and formally submitting $\hat{\boldsymbol{t}}_T = \hat{\boldsymbol{t}}_1$ and $\hat{\boldsymbol{t}} = \hat{\boldsymbol{t}}_2$:

$$\Gamma_q = \frac{1}{1 + RedR \left( \left| \frac{\hat{\boldsymbol{t}}_2^T \hat{\boldsymbol{t}}_1}{|\hat{\boldsymbol{t}}_2|^2} \right|^2 - 1 \right)} \geq T. \tag{14}$$

The detector parameters $RedR$ and $T$ still have to be set on the base of some rationale. The subsequent sections deal with this issue.

A. Dual polarimetric change detector

Before proceeding with the setting of the detector parameters, in this subsection a particularization of the test is introduced. The algorithm proposed in the previous section exploits quad polarimetric SAR data. In actual fact, quad polarimetric data are needed in order to characterize uniquely a partial target. However, in some practical instances the acquisition of quad polarimetric data can be excessively expensive or impracticable. Often in such cases a single transmit polarization is used with dual coherent receivers measuring orthogonal components of the scattered wave. Such cases we will refer to as dual polarimetric [2], [1].

The methodology exploited to derive the proposed change detector is based on a generic algebraic manipulation (a weighted and normalized inner product), therefore it can be easily adapted to different typologies of data, as long as the observables can be represented with vectors in an Euclidean space [16]. Moreover, the following mathematical development is presented as an example of the general methodology exploited to derive a detector working on a different algebraic linear space. For instance, a different formalism for the quad polarimetric detector could obtained by rearranging the elements of the Muller matrix in a $9$ dimensional real vector (in case of symmetric scattering matrix).
When only two polarimetric channels are available, a two-dimensional scattering vector can be observed: $k_d = [k_1, k_2]^T$, with $k_1, k_2 \in \mathbb{C}$ [1], [2]. Subsequently, $k_d$ can be exploited to build a 2x2 covariance matrix:

$$
[C_d] = \begin{bmatrix}
\langle |k_1|^2 \rangle & \langle k_1^* T k_2 \rangle \\
\langle k_2^* T k_1 \rangle & \langle |k_2|^2 \rangle
\end{bmatrix}.
$$

(15)

A 3-dimensional partial feature vector can then be built:

$$
t_d = \text{Trace}([C_d] \Psi_2) = \begin{bmatrix}
\langle |k_1|^2 \rangle, \\
\langle |k_2|^2 \rangle, \\
\langle k_1^* T k_2 \rangle
\end{bmatrix}^T.
$$

(16)

Again, the partial vectors must be measured in the two acquisitions: $l_{d1}$ and $l_{d2}$, and the first partial vector must be normalized as $\hat{t}_{d1} = \frac{l_{d1}}{|l_{d1}|}$.

The dual polarimetric change detector is then expressed as

$$
\Gamma_d = \frac{1}{1 + RedR_d \left( \frac{t_{d2}^* T t_{d2}}{|l_{d2}|^2} - 1 \right)} \geq T_d.
$$

(17)

The problem of setting the parameters $RedR_d$ and $T_d$ is treated in the following sections.

IV. PARAMETERIZATION AND PARAMETERS SETTING

A. Parameterization

The change detector developed in the previous sections has two free parameters, $RedR$ and $T$, which allow the tuning of the detector selectivity. The aim of this section is to find a criterion able to set them starting from some requirement on the minimum change to detect.

One of the advantages of utilizing partial vectors in $\mathbb{C}^6$ is that any partial target can be uniquely identified with a vector rather than a 3x3 matrix. However, due to the novelty of the detector, a parameterization of such space has not been devised yet, and we cannot
link vectors in $\mathbb{C}^6$ with scattering mechanisms in SU(3). With such a link, a change in
SU(3) would be easily linked with a change of the partial vector, allowing the derivation of
a rationale for setting the detector parameters.

Considering that there are two free detector parameters (i.e. $\text{RedR}$ and $T$), one of them, the
threshold $T$, can be theoretically set arbitrarily in the interval $]0, 1[$ (and without any particular rationale). The value chosen in this paper is $T = T_d = 0.9$. Therefore, the following part of
the section will be focused on the setting of $\text{RedR}$ (after the value $T = T_d = 0.9$ is selected).

First, a parameterization for quad polarimetric data is devised, leaving the treatment of
dual polarimetric data for a later section. The modification of the scattering mechanism can
be represented with a rotation of the single target in SU(3) (since the scattering mechanism
are unitary) [17]. Given $\omega_1$, a generic single target in the first acquisition, the basis of the
space can be chosen in order to express $\omega_1 = [1, 0, 0]^T$. The scattering mechanism observed
in the second acquisition is defined as $\omega_2$. Considering $\omega_2$ is unitary, an easy way to represent
it in the new basis is by polar coordinates:

$$
\omega_2 = [\cos \theta, \sin \theta \cos \varphi e^{j\nu}, \sin \theta \sin \varphi e^{j\tau}]^T.
$$

(18)

It is important to note that this formalism matches with the Cloude-Pottier eigenvector
model in case that $\omega_1$ is a surface or odd bounce scattering mechanism [1]. The first step in
developing an algorithm able to detect changes of scattering mechanism in two acquisitions
is to select an algebraic operator able to evaluate differences between two scattering mechan-
isms, $\omega_1$ and $\omega_2$, in SU(3). The normalized inner product is selected for its generality
[16]. Additionally, it is invariant with respect to the vector norm, keeping the advantages of
a treatment based exclusively on polarimetry.

$$
\Delta = |\omega_1^T \omega_2| = \cos \theta \in \mathbb{R}.
$$

(19)
A parameterization able to express $\Delta$ in the $C^6$ partial target space (where the detector works) is required. The expressions of $\hat{t}_1$ and $t_2$ can be derived from the coherency matrices of the two scattering mechanisms. These are calculated exploiting the outer product of $\omega_1$ and $\omega_2$:

$$\omega_1 \rightarrow \omega_1 \omega_1^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \hat{t}_1 = \hat{t}_1 = [1, 0, 0, 0, 0, 0]^T,$$  \hspace{1cm} (20)

$$\omega_2 \rightarrow \omega_2 \omega_2^T \rightarrow t_2 = \begin{bmatrix} \cos^2 \theta \\ \sin^2 \theta \cos^2 \varphi \\ \sin^2 \theta \sin^2 \varphi \\ \cos \theta \sin \theta \cos \varphi e^{-j\nu} \\ \cos \theta \sin \theta \sin \varphi e^{-j\tau} \\ \sin^2 \theta \cos \varphi \sin \varphi e^{j(\nu-\tau)} \end{bmatrix}. \hspace{4cm} (21)$$

Clearly, the coherency matrices derived from $\omega_1$ and $\omega_2$ have unitary rank, since they represent coherent (or deterministic) targets, therefore they do not need averaging.

Substituting $\hat{t}_1$ and $t_2$ in the expression of the $SCR$ we obtain

$$SCR = \frac{\cos^4 \theta}{det};$$  \hspace{1cm} (22)

$$det = \sin^4 \theta \cos^4 \varphi + \sin^4 \theta \sin^4 \varphi + \cos^2 \theta \sin^2 \theta \cos^2 \varphi$$

$$+ \cos^2 \theta \sin^2 \theta \sin^2 \varphi + \cos^4 \theta \cos^2 \varphi \sin^2 \varphi.$$  \hspace{1cm} (23)

After some algebraic manipulations, the $SCR$ can be simplified to:

$$SCR = \frac{\cos^4 \theta}{\sin^4 \theta (\cos^4 \varphi + \sin^4 \varphi) + \cos^2 \theta \sin^2 \theta + \cos^4 \theta \cos^2 \varphi \sin^2 \varphi}.$$  \hspace{1cm} (24)

The $SCR$ depends on $\theta$ and $\varphi$ (two rotation angles in polarization space). $\theta$ is selected by the requirements on $|\omega_1^T \omega_2|$, whereas there are no requirements concerning $\varphi$ (rotation
around \( \omega_1 \) that does not change the distance of the two scattering mechanisms. Our approach consists in adopting the worst scenario, in this way any random choice of \( \varphi \) will return an equal or smaller detector \( \Gamma_q \) (therefore these points will be automatically discarded). The maximization of \( \Gamma_q \) can be accomplished maximizing the \( SCR \) over \( \varphi \) [18]:

\[
\varphi_M = \max_{\varphi \in [0, 2\pi]} \Gamma_d = \max_{\varphi \in [0, 2\pi]} SCR, \tag{25}
\]

\[
\forall \varphi \neq \varphi_M : SCR(\varphi) < SCR(\varphi_M). \tag{26}
\]

The minimization of the reciprocal of \( SCR \), defined as \( CSR \), can be preferred since it is more straightforward:

\[
\varphi_M = \max_{\varphi \in [0, 2\pi]} SCR = \min_{\varphi \in [0, 2\pi]} \frac{1}{CSR} = \min_{\varphi \in [0, 2\pi]} CSR. \tag{27}
\]

The search for the minimum is accomplished with the rule of the derivatives:

\[
\frac{\partial CSR}{\partial \varphi} = 0 \Rightarrow \sin(4\varphi_M) = 0 \Rightarrow \varphi_M = n\frac{\pi}{2}. \tag{28}
\]

The second derivative is used to assess when a critical point is a maximum or a minimum. In particular, for \( \tan^4 \theta < 0.5 \) (i.e. \( \theta \) smaller than about 40 degrees) the maxima of \( SCR \) are \( \varphi_M = 0, \varphi_M = \frac{1}{2} \pi, \varphi_M = \pi \) and \( \varphi_M = \frac{3}{2} \pi \). On the other hand, for \( \tan^4 \theta > 0.5 \) the maxima of \( SCR \) are \( \varphi_M = \frac{1}{4} \pi, \frac{3}{4} \pi, \frac{7}{4} \pi \) and \( \frac{9}{4} \pi \). For \( \theta \sim 40 \) degree the \( SCR \) is constant with respect to \( \varphi \).

Figure 1.a shows the plot of the \( SCR \), where \( \varphi \) is varied for a fixed \( \theta \). The plots confirm that the maxima are located in \( \varphi_M \) and the values of \( SCR \) is constant for \( \theta \sim 40 \) degree.

Once fixed \( \varphi_M = 0 \), the expression of the \( SCR \) can be simplified to:

\[
SCR = \frac{\cos^4 \theta}{\sin^4 \theta + \cos^2 \theta \sin^2 \theta} = \frac{\cos^4 \theta}{\sin^2 \theta}. \tag{29}
\]
which is an expression depending only on $\theta$. Subsequently, starting from (14) the expression of the $RedR$ can be set utilizing the values of $SCR$ and $T$:

$$RedR = SCR \left(\frac{1}{T^2} - 1\right).$$  \hspace{1cm} (30)

In order to illustrate common parameter values, Table I shows the $RedR$ and $SCR$ for four choices of angle $\theta$ (degrees). Again the threshold is set as $T = 0.9$.

Finally, Figure 1.b depicts the trend of $RedR$ varying angle $\theta$. Therefore, if particularly small changes are of interest (small $\theta$ angle), the value of RedR to be chosen becomes very high. In the detector, this will increase the components perpendicular to $\hat{t}_1$, therefore a detection will be accomplished only if these components are very small (i.e. $t_2$ and $\hat{t}_1$ very similar, for exception of the overall amplitude). The plot also shows a symmetric behavior with a reflection around angle $\pi/2$. This is expected, since the direction of the scattering
mechanism can be included in its absolute phase and hence neglected in the characterization of the target. For instance, two vectors with an angle difference of $\pi$ (i.e. a minus sign) represent the same scattering mechanism.

**B. Interpretation of the distance between two $\omega$.**

In the previous section, the detector parameters were linked to the scattering mechanisms differences through a parameterization. However, the angular distance between two scattering mechanisms does not yet have a straightforward physical interpretation. The aim of this section is to provide a physical/algebraic interpretation of the $\theta$ angle. The model selected to represent the scattering mechanism is the Cloude Pottier eigenvector model, due to its straightforward geometrical significance [1], [2], [19]. However, other parameterizations could be exploited [19].

The two scattering mechanisms are:

\[
\omega_1 = [\cos \alpha_1, \sin \alpha_1 \cos \beta_1 e^{j\mu_1}, \sin \alpha_1 \sin \beta_1 e^{j\epsilon_1}]^T,
\]

\[
\omega_2 = [\cos \alpha_2, \sin \alpha_2 \cos \beta_2 e^{j\mu_2}, \sin \alpha_2 \sin \beta_2 e^{j\epsilon_2}]^T.
\]

(31)

$\theta$ corresponds to the modulus of the inner product $|\omega_1^T \omega_2|$. After the scalar product is
performed, the trigonometric product-to-sum formulas can be exploited to find the expression:

\[
\omega_1^T \omega_2^* = \frac{1}{2} \left[ \cos \Delta \alpha + \cos \Sigma \alpha + \frac{1}{2} (\cos \Delta \alpha - \cos \Sigma \alpha)(\cos \Delta \beta + \cos \Sigma \beta)e^{j\Delta \mu} + \frac{1}{2} (\cos \Delta \alpha - \cos \Sigma \alpha)(\cos \Delta \beta - \cos \Sigma \beta)e^{j\Delta \epsilon} \right] = \cos \theta e^{j\zeta}, \tag{32}
\]

where \( \Delta \alpha = \alpha_2 - \alpha_1 \), \( \Delta \beta = \beta_2 - \beta_1 \), \( \Sigma \alpha = \alpha_2 + \alpha_1 \) and \( \Sigma \beta = \beta_2 + \beta_1 \). After a few manipulations, we can obtain the expression:

\[
\cos \theta e^{j\zeta} = \frac{1}{2} \left[ \cos \Delta \alpha + \cos \Sigma \alpha + \frac{1}{2} (\cos \Delta \alpha - \cos \Sigma \alpha) \right.
\]
\[
\times \left[ \cos \Delta \beta (e^{j\Delta \mu} + e^{j\Delta \epsilon}) + \cos \Sigma \beta (e^{j\Delta \mu} - e^{j\Delta \epsilon}) \right] , \tag{33}
\]

In order to simplify the calculations, the hypothesis \( \Delta \mu = \Delta \epsilon = \Delta \phi \) is assumed. In fact, \( \mu \) and \( \epsilon \) do not have a straightforward physical interpretation, and it is reasonable to consider a change which does not prefer one phase difference with respect to the other. Please note, this position does not state that \( \mu \) and \( \epsilon \) have the same value, but merely that their variations must be in both cases higher than \( \Delta \phi \) in order to set a detection.

\[
\cos \theta e^{j\zeta} = \frac{1}{2} \left( \cos \Delta \alpha + \cos \Sigma \alpha + \cos \Delta \alpha \cos \Delta \beta e^{j\Delta \phi} - \cos \Sigma \alpha \cos \Delta \beta e^{j\Delta \phi} \right) = A. \tag{34}
\]

The final purpose of these calculations is to find an expression of \( \theta \) depending exclusively on the angle differences of the eigenvector model and not on their actual value. In this way, the test would be general and applicable to any \( \omega \). The expression in (34) depends on \( \alpha_1 \), since \( \Sigma \alpha \) is a function of \( \alpha_1 \), even when the difference \( \Delta \alpha \) is kept constant (i.e. \( \Sigma \alpha = 2\alpha_1 + \Delta \alpha \)). To remove this dependence, an averaged expression over all the possible \( \alpha_1 \) angles can be considered [18]. This is performed analytically exploiting the normalized
integral varying the angles but keeping their difference constant (i.e. $\Delta \alpha = \text{const}$):

$$\cos \theta e^{j\zeta} = \frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} A d\alpha_1,$$

(35)

since $\alpha_1 \in [0, \frac{\pi}{2}]$. In order to simplify the formalism, in the following the symbol $\theta$ will be employed to refer to $\tilde{\theta}$. Considering that

$$\frac{2}{\pi} \int_{0}^{\frac{\pi}{2}} \cos(2\alpha_1 - \Delta \alpha) \, d\alpha_1 = \frac{2}{\pi} \sin \Delta \alpha,$$

(36)

the result of the integral is:

$$\cos \theta e^{j\zeta} = \frac{1}{2} \left( \cos \Delta \alpha + \frac{2}{\pi} \sin \Delta \alpha + \cos \Delta \alpha \cos \Delta \beta e^{j\Delta \phi} - \frac{2}{\pi} \sin \Delta \alpha \cos \Delta \beta e^{j\Delta \phi} \right).$$

(37)

We are interested in the modulus of the inner product. To derive this expression the following equalities can we can consider that

$$| \cos \theta e^{j\zeta} | = \cos \theta = \sqrt{(\Re(\cos \theta e^{j\zeta}))^2 + (\Im(\cos \theta e^{j\zeta}))^2}$$

(38)

$$e^{j\Delta \phi} = \cos \Delta \phi + j \sin \Delta \phi$$

$$\cos \theta = \frac{1}{2} \left[ \left( \cos \Delta \alpha + \frac{2}{\pi} \sin \Delta \alpha \right)^2 + \cos^2 \Delta \phi \cos^2 \Delta \beta \left( \cos \Delta \alpha - \frac{2}{\pi} \sin \Delta \alpha \right)^2 \right. $$

$$+ \left. 2 \cos \Delta \beta \cos \Delta \phi \left( \cos \Delta \alpha - \frac{2}{\pi} \sin \Delta \alpha \right) \left( \cos \Delta \alpha + \frac{2}{\pi} \sin \Delta \alpha \right) \cos \Delta \alpha \right]^{\frac{1}{2}}.$$

(39)

Collecting terms and exploiting the relationship $(a - b)(a + b) = a^2 - b^2$, a final expression can be derived:

$$\cos \theta = \frac{1}{2} \left[ \left( \cos \Delta \alpha + \frac{2}{\pi} \sin \Delta \alpha \right)^2 + \cos^2 \Delta \beta \left( \cos \Delta \alpha - \frac{2}{\pi} \sin \Delta \alpha \right)^2 \right. $$

$$+ \left. 2 \cos \Delta \beta \cos \Delta \phi \left( \cos^2 \Delta \alpha - \frac{4}{\pi^2} \sin^2 \Delta \alpha \right) \right]^{\frac{1}{2}}.$$

(40)
TABLE II

VALUES OF $\theta$ VARYING THE ANGLES (DEGREE), FOR QUAD POLARIMETRIC DATA.

<table>
<thead>
<tr>
<th>$\Delta \alpha$</th>
<th>$\Delta \beta$</th>
<th>$\Delta \phi$</th>
<th>$\theta$</th>
<th>RedR</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>5.25</td>
<td>27.50</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>9</td>
<td>11.70</td>
<td>5.25</td>
</tr>
<tr>
<td>16</td>
<td>16</td>
<td>16</td>
<td>20.41</td>
<td>1.49</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>25</td>
<td>30.89</td>
<td>0.48</td>
</tr>
<tr>
<td>27</td>
<td>27</td>
<td>27</td>
<td>34.16</td>
<td>0.35</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
<td>30</td>
<td>36.26</td>
<td>0.28</td>
</tr>
</tbody>
</table>

With this final expression we have obtained a link between the model angle differences and the $\theta$ angle, which enables a physical rationale for setting $\theta$. The latter is strictly related with the detector through the $SCR$ (as shown in the previous section). Table II represents some values of $\theta$ obtained with the model angle differences.

C. Dual-pol parameterization

This section is concerned with the setting of the parameter $RedR_d$ for the dual polarimetric change detector. Again the threshold is arbitrarily set as $T_d = 0.9$.

In order to relate the values of $\Gamma_d$ to physical changes, a parameterization of the $C^3$ space of the partial targets $t_d$ is needed. A change of the dual polarimetric scattering mechanism $\omega_d$ can still be represented with a rotation. Given $\omega_{d1}$, a generic single target in the first acquisition, the basis can be chosen in order to express $\omega_{d1} = [1, 0]^T$. A generic scattering mechanism in the second acquisition is defined as $\omega_{d2}$. Again $\omega_{d2}$ is represented in the new
basis by polar coordinates: $\omega_{d2} = [\cos \theta, \sin \theta e^{j\zeta}]^T$. The normalized inner product is chosen to evaluate the differences between $\omega_{d1}$ and $\omega_{d2}$:

$$\Delta = |\omega_{d1}^T \omega_{d2}| = \cos \theta \in \mathbb{R}. \quad (41)$$

A parameterization able to link $\Delta$ to the change detector is required. The dual polarimetric coherency matrices can be defined with the outer product of $\omega_d$ as $[C_d] = \langle \omega_d \omega_d^* \rangle$. The expressions of $\hat{t}_{d1}$ and $t_{d2}$ is derived as $\hat{t}_{d1} = [1, 0, 0]^T$ and $t_{d2} = [\cos^2 \theta, \sin^2 \theta, \cos \theta \sin \theta e^{j\zeta}]^T$.

Substituting $\hat{t}_{d1}$ and $t_{d2}$ in the expression of $SCR$ we obtain

$$SCR = \frac{\cos^4 \theta}{\sin^4 \theta + \cos^2 \theta \sin^2 \theta} = \frac{\cos^4 \theta}{\sin^2 \theta}, \quad (42)$$

which is interestingly the same expression found in (29). The expression in (42) provides a link between $SCR$ and the angle differences between two generic dual polarimetric scattering mechanism. Subsequently, $RedR_d$ can be set utilizing (30).

**D. Interpretation of the dual polarimetric distance for HH/VV**

In this section, a physical interpretation of $\theta$ in the case of dual polarimetric $HH/VV$ data is provided. The model selected to represent the scattering mechanism is again the eigenvector model [1]. Please note, the model can only be applied if the two co-polarizations are acquired. In dual polarimetric data only a subspace of the entire SU(3) can be observed and part of the parameters of the quad polarimetric eigenvector model can not be inverted unambiguously. The solution exploited in this paper is to neglect these parameters. Specifically, neglecting $\beta$ a reflection symmetric space is assumed. This hypothesis does not constitute a loss of generality of the proposed algorithm, since it is intrinsic of dual polarimetric acquisitions (in our case $HH/VV$). If physical changes take place in the complex line orthogonal to the complex plane spanned by the two vectors (e.g. $HH$ and $VV$), they can not be observed
anyway (i.e. quad polarimetry is required to observe these changes). The two scattering mechanisms are:

\[
\begin{align*}
\omega_{d1} &= [\cos \alpha_1, \sin \alpha_1 e^{j\zeta_1}]^T, \\
\omega_{d2} &= [\cos \alpha_2, \sin \alpha_2 e^{j\zeta_2}]^T.
\end{align*}
\]

The derivation proceed as for the quad polarimetric case. After the scalar product, the trigonometric product-to-sum formulas and the integration can be utilized to find the expression:

\[
|\omega_{d1}^T \omega_{d2}^*| = \frac{1}{2} \left[ \left( \cos \Delta \alpha + \frac{2}{\pi} \sin \Delta \alpha \cos \Delta \zeta \right)^2 + \left( \cos \Delta \alpha - \frac{2}{\pi} \sin \Delta \alpha \right)^2 \sin^2 \Delta \zeta \right]^{\frac{1}{2}},
\]

where \(\Delta \alpha = \alpha_2 - \alpha_1\) and \(\Delta \zeta = \zeta_2 - \zeta_1\). After some more manipulations a simpler expression can be found:

\[
|\omega_{d1}^T \omega_{d2}^*| = \frac{1}{2} \left[ 4 \cos^2 \Delta \alpha + 2 \left( \cos^2 \Delta \alpha - \frac{4}{\pi^2} \sin^2 \Delta \alpha \right) \left( \cos \Delta \zeta - 1 \right) \right]^{\frac{1}{2}}.
\]

Finally, considering that \(|\omega_{d1}^T \omega_{d2}^*| = \cos \theta\) a link between \(RedR\) and the angle differences \(\Delta \alpha\) and \(\Delta \zeta\) is found. Table III presents \(RedR\) for different values of \(\Delta \alpha\) and \(\Delta \zeta\).

V. REAL DATA VALIDATION

In this section, the proposed algorithms are tested over real data. In order to provide a relatively large validation, different typologies of data were considered, including airborne and satellite systems, with different resolutions, frequencies, and polarimetric modes.
A. Polarimetric data description

Firstly, airborne data are exploited, since they provide higher resolution and Signal to Noise Ratio (SNR) compared to the other datasets. The data were acquired in 2006 by German Aerospace Center (DLR) with the E-SAR airborne system, during the SARTOM campaign [20]. The acquisitions employed in this validation are in L-band with a resolution of 1.5 m in slant range and 0.9 m in azimuth (the pixel sampling is 1.5 m and 0.44 m). The L-band dataset was chosen since it is quad polarimetric and presents a low frequency foliage penetration capability when compared to other sensors exploited in the following sections. One of the aims of the SARTOM project was target detection under forest cover. For this reason, several targets were deployed inside and outside the forest. Additionally, some of the targets were removed in later acquisitions. Therefore, the dataset represents an ideal scenario for testing the proposed algorithm.

The second dataset considered was acquired by RADARSAT-2 (MDA) in 2009. This is

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**TABLE III**

**DETECTOR PARAMETERS VARYING THE ANGLES (DEGREE) FOR DUAL POLARIMETRIC HH/VV DATA**

<table>
<thead>
<tr>
<th>$\Delta \alpha$</th>
<th>$\Delta \zeta$</th>
<th>$\theta$</th>
<th>$R_{ed} R_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5.59</td>
<td>24.28</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>11.16</td>
<td>5.81</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>16.68</td>
<td>2.39</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>22.18</td>
<td>1.21</td>
</tr>
<tr>
<td>25</td>
<td>25</td>
<td>27.53</td>
<td>0.68</td>
</tr>
</tbody>
</table>
in C-band with resolution of 5.4 m in slant range and 8.0 m in azimuth (the sampling is 4.7 m in range and 5.1 m in azimuth). The exploitation of Radarsat-2 data allows to test a different frequency with coarser resolution. The data were acquired in Canada (Indian Head, Saskatchewan) in the framework of the AgriSAR2009 campaign [21]. This was a ESA campaign to study polarimetric time-series on agricultural fields. Accurate ground measurements of the fields were carried out during the acquisitions. The images employed in this particular test were acquired on 11\textsuperscript{th} of August and 04\textsuperscript{th} of September 2009 (24 days of temporal baseline) with an incidence angle of 34 degrees.

Finally, TerraSAR-X (DLR) dual-pol $HH/VV$ X-band data are employed. The resolution here is 1.18 m in slant range and 6.6 m in azimuth, while the sampling is 0.91 m in range and 2.39 m in azimuth. As pointed out previously, the use of quad polarimetric data is generally preferred due to the higher discrimination capability. However, in some instances only dual polarimetric data are available. In particular, the combination of the co-polar channels $HH$ and $VV$ seems to be more advantageous than the use of the cross-polarized channel $HV$, at least with this sensor. The scene consists of a rural area (mainly rice fields) close to Seville (Spain). Two acquisitions are used here, gathered on the 17\textsuperscript{th} and 28\textsuperscript{th} of May 2009, hence at the beginning of the rice cultivation season. The incidence angle is 30 degrees.

B. E-SAR L-band quad polarimetric data

The first experiment considers two acquisitions where the slave was acquired 4 days after with a zero spatial baseline. This is an ideal test for the proposed PCD, since only temporal effects are visible. Figures 2.a and 2.b present the corresponding Pauli RGB color composite image (Red: $|HH - VV|$; Green: $2|HV|$; Blue: $|HH + VV|$). A ground truth campaign was carried out during the acquisitions. The scene is a mix of forest stands and open field areas.
Specifically, the bright points (and geometrical shapes) in open field areas are man-made targets (e.g. vehicles and metallic nets). The open field area was mainly covered by grass with sparse bushes. Moreover, weather conditions were slightly different at these two dates: a rainy day in the master acquisition, and just a wet day (without rain) for the slave. The targets that were not present in the second acquisition are indicated with circles on the RGB images.

The coherency matrices for the two acquisitions $T_{11}$ and $T_{22}$ were estimated via spatial averaging. In this example, 121 samples were averaged, which corresponds to an Equivalent Number of Looks (ENL) of 59. The result of the algorithm is a coherence mask ($\Gamma_q$), where the values are linearly scaled between the threshold ($T = 0.9$) and 1. The detection mask is presented in Figure 3.a, where the angle differences is $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 27^\circ$. Consequently, all the black areas experienced changes that modified the model angles more than 27 degrees. All the removed targets appear as dark spots in the mask. Clearly, several other points are below the threshold. This is because the purpose of the algorithm is to detect all the areas that suffered changes in the scattering mechanisms between the two acquisitions and not just man-made targets.

In order to have an idea about the performances of the proposed PCD, another widely utilized test, the LR ratio of coherency matrices determinants is illustrated in Figure 3.b. The threshold employed here is 0.8, which is a lower value compared to the commonly exploited in the literature (which is 0.9) [9]. This is because here the algorithm is focused exclusively on changes due to the removal of point targets, which are supposed to be relatively significant. Therefore, the use of a lower threshold would reduce the number of false alarms. However, in the following experiments the more common value of 0.9 will be exploited. Again, it is important to keep in mind that the two detectors are focused on different aspects
of the data, therefore we expect some differences. Now, the open field presents the largest changes, and most areas are below the threshold. In this area there were not changes in the scattering mechanisms during the 4 days, however the overall backscattered amplitude is different due to changes in wetness conditions of the ground (i.e. dielectric constant). As expected, the change in amplitude is detected by the LR algorithm. Instead, this effect is not present in the proposed PCD (recall Figures 2.a and 2.b). Regarding the detection of changes to point targets, the LR test does not seem to be suited as well, due to its sensitivity to the overall intensity.

C. Radarsat-2 Fine quad polarimetric data

Figure 4 presents the RGB Pauli composite images of the two acquisitions. In order to make the pixel of the RGB image almost square on the ground, a preliminary polarimetric multilook of 4x2 (azimuth x range) is performed. Before to run the algorithm a coregistration
Fig. 3. Polarimetric change detection on E-SAR L-band data. ENL : 59. (a) Proposed PCD with $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 16^\circ$. (b) LR ratio test with threshold 0.8

procedure for the images was performed. The scene is composed of agricultural parcels of different sizes where different crop types are cultivated, with a lake in the upper left corner. The small dark spots in the right hand side are small ponds.

Figures 5.a and 5.b show the detection masks for angle differences of $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 16^\circ$ and $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 27^\circ$, respectively. Coherency matrices were estimated by averaging 200 samples, resulting in an ENL of 110. Most of the crop fields are above the threshold, but there is also a significant number with a dark or black mask. This is in line with the preliminary results of the campaign discussed in [21], since at the date of these acquisitions part of the crops in the area were developing quickly, whereas other maintained similar phenological stages. The fields below the threshold are easily identified on the RGB images, since it is clear how their color (hue) has changed significantly. Please note, on the RGB image only changes in color are related to polarimetric changes (i.e. changes in the scattering mechanisms), whereas changes in brightness are directly related to amplitude
A comparison with the LR ratio of matrix determinants is also provided. The mask in Figure 6.a is linearly scaled between 0.9 and 1 (i.e. equivalent to a threshold equal to 0.9). The test shows that almost all the fields are under the 0.9 requirement. In order to have a better understanding of this result, the LR ratio is also presented without any threshold, simply scaled between 0 and 1, in Figure 6.b. Most of the scene is relatively dark and only few bright fields are visible.

In order to have a better look at the changes, a smaller area is analyzed next. The RGB Pauli images of this area are presented in Figure 7. From the images it is clear that some of the parcels exhibit a visible change whereas others seem to remain stable. To have an additional insight of the polarimetric properties, the entropy (scaled between 0 and 1) and $\alpha$ angle (scaled between 0 and $\pi/2$) are shown in Figures 9 and 10, respectively. The detection mask for 16 degree angle difference with the PCD and the LR ratio without threshold are also presented in Figure 8.

For illustration purposes, we focus our analysis in three rectangles with different behavior on this area (see location in Figure 7):

1) Green rectangle on the upper left part. Here, the PCD does not detect consistent changes whereas it is relatively low for the LR ratio (around 0.4). In order to check if polarimetric changes took place between the two acquisitions, the entropy and $\alpha$ images are exploited, estimating an average value within the considered parcel. Interestingly, entropy remains stable in both acquisitions, with values of 0.61 and 0.67, respectively, for the lower right corner (bluish parcel, which corresponds to a lentil field), and 0.80 to 0.81 for the rest (which correspond to parcels cultivated with flax and spring wheat). Regarding the $\alpha$ angle, this passes from 25 to 27 degrees in the bluish part and stay stable at 38 degrees in the rest.
These changes are very small and much below the limit of 16 degrees that we imposed. In order to understand the reason of the low LR ratio, a closer look at the RGB images reveals a quite significant change in the overall amplitude. The span (i.e. sum of the diagonal terms of the coherency matrix) varies from about 1 dB to -6 dB in the bluish parcel and from 1.6 dB to -5 dB in the rest.

2) Red rectangle in the lower left corner. As for the previous case the LR ratio is particularly low: 0.3 for the right parcel (flax) and 0.21 for left parcel (fallow), while the proposed algorithm does not detect consistent changes. A closer look at the entropy reveals that it stays stable to 0.82 in the right parcel and varies from 0.49 to 0.59 in the left one. Regarding the $\alpha$ parameter, this changes from 40 to 38 degrees for the right and from 22 to 18 degrees for the left parcel. As a conclusion, the polarimetric characteristic of the two fields are not changing significantly. Regarding the analysis of the span, the second acquisition presents a consistent lower scattering from 1.7 dB to -6 dB for the right and from 0.98 dB to -8 dB for the left parcel.

3) Gray rectangle in the left side (spring wheat). In this situation both the PCD and the LR ratio detect changes (the latter is approximate 0.16). The study of this parcel is interesting since the PCD detector seems to be proximal to the threshold, with several parcel areas above and other below. In order to understand this result, entropy and $\alpha$ angle can be analyzed. Its entropy changes from about 0.85 to 0.73, and the $\alpha$ angle passes from 45 to 31 degrees. These large changes in $\alpha$ (14 degrees) and entropy justify the partial detection of these spots within the parcel. In fact, the $\alpha$ difference is just around the threshold of 16 degrees and the statistical variation along the parcel returns the fluctuating behavior of the detector.
Fig. 4. RGB Pauli image of the Radarsat-2 scene in Indian Head: (a) 1\textsuperscript{st} acquisition; (b) 2\textsuperscript{nd} acquisition

Fig. 5. Change detection with Radarsat-2 data in Indian Head. $\text{ENL} = 110$ (a) $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 16^\circ$; (b) $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 27^\circ$
(a) $T = 0.9$  
(b) No threshold

Fig. 6. LR test for Radarsat-2 data in Indian Head. $ENL = 110$ (a) Threshold 0.9; (b) No threshold

(a) 1$^{st}$ acquisition  
(b) 2$^{nd}$ acquisition

Fig. 7. RGB Pauli image of the Radarsat-2 data in Indian Head (small area) (a) 1$^{st}$ acquisition; (b) 2$^{nd}$ acquisition
(a) $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 16^\circ$ (b) LR: no threshold

Fig. 8. Change detection with Radarsat-2 data in Indian Head (small area). $ENL = 110$ (a) $\Delta \alpha = \Delta \beta = \Delta \mu = \Delta \epsilon = 16^\circ$; (b) LR without threshold

D. TerraSAR-X dual polarimetric data HH/VV

In this section, the algorithm is tested with dual polarimetric $HH/VV$ TerraSAR-X data. The RGB images of the two acquisitions are presented in Figures 11.a and 11.b, respectively. The color coding of the images is Red: $C_{11} = \langle |HH|^2 \rangle$, Green: $T_{22} = \langle |HH - VV|^2 \rangle$ and Blue: $C_{22} = \langle |VV|^2 \rangle$. The scene presents mainly flooded rice fields (dark areas), with other crops in the lower part, and the Guadalquivir river crossing diagonally. Ground measurements are available over the rice fields. In both the acquisitions the fields are flooded and without plants emerging. However, in the first acquisition some of the fields are still not flooded (they appear as brighter parcels and are indicated with green rectangles in the second RGB). Interestingly, the wind condition in the first acquisition was much stronger than in the second one. Due to the larger roughness of the water surface, the flooded fields look
Fig. 9. Entropy images with Radarsat-2 data in Indian Head (small area), scaled between 0 and 1. $ENL = 110$

(a) 1st acquisition; (b) 2nd acquisition

much brighter, giving an overall intensity difference of about 10 dB between the two acquisitions. More importantly, the backscatter from large parts of the rice fields in the second image was around or below the noise level of the SAR system (with Noise Equivalent Sigma Zero, $NESZ \approx -19$ dB).

In this case, 28x14 pixels were averaged for a total of 392 samples and $ENL = 109$. Compared to the Radarsat-2 data, the ENL is about the same, but the number of samples is much higher due to the large oversampling of the data. Fortunately, the resolution provided by TerraSAR-X data is higher than Radarsat-2, therefore the final resolution is not excessively influenced by the use of many samples. Clearly, the number of samples necessary is dependent on the application. In our case, we are analyzing agricultural fields where the targets of interest are supposed to be relatively distributed on a large area. On the other hand,
in urban areas where the targets are supposed to be more coherent and smaller, less samples can be considered.

As in the previous case, the entropy and $\alpha$ angles images, obtained with the dual-pol version of the eigendecomposition, are presented in Figure 12 and 13 to provide an insight about the polarimetric information content of both acquisitions. The resulting change detection masks are presented in Figure 14.a and 14.b. The angle differences chosen here are (a) $\Delta \alpha = \Delta \zeta = 18^\circ$ and (b) $\Delta \alpha = \Delta \zeta = 28^\circ$. Although the physical interpretation of $\alpha$ angle is the same, variations of $\alpha$ produce different effects on the angle between two generic scattering mechanisms in the two spaces (i.e. $\theta$). The evaluation of the angular difference between the two scattering mechanisms is the focal point of the detector (especially if this result has to be used as a test and exported to other algorithms). In order to compare the
results obtained with dual and quad polarimetric data the test has to be performed utilizing
the same $\theta$ and therefore the same $SCR$ (as illustrated in (29) and (42)). This is the reason
why the angle differences were selected as 18 and 28 degree in this case.

Again, the masks are linearly scaled between the threshold $T = 0.9$ and 1, therefore
all the gray points passed the test. Several changes can be detected especially with the more
restrictive angle difference of 18 degrees. A comparison with the LR is provided in Figure 15.
The LR test is below the threshold almost everywhere in the scene (except for stable isolated
scatterers). As introduced before the first acquisition exhibits on average a backscatter level
around 10 dB over the second one on the rice areas, and the LR test is strongly affected by
this change.

The fields indicated by the red rectangle on the right side are analyzed first. The algo-
rithm detect these pixels as suffering polarimetric changes. An accurate observation of the
area reveals that part of the rice parcels start acquiring a different color. Specifically, en-
tropy increases strongly and average $\alpha$ angle goes from low values to 45 degrees. The same
comments hold for other similar fields in the scene (not indicated here to do not overfull the
image with markers). This change is most probably due to backscattering coefficients being
below the noise floor during the second acquisition. In such noisy conditions entropy ap-
proaches 1, and for that entropy the average $\alpha$ angle is constrained to be close to 45 degrees
by definition in the dual-pol case.

The yellow rectangles (i.e. right side and upper right corner) exhibit clear changes already
in the RGB images. In this case these parcels were not flooded during the first acquisition
yet, and they corresponded to bare surfaces (without vegetation). Entropy and $\alpha$ angle can be
very beneficial to interpret this effect. Both images present a low entropy (around 0.5) and a
similar $\alpha$ angle (although the second is closer to 0). Therefore, the two targets (bare ground
and rough water surface) appears quite similar from the polarimetric point of view, and hence the PCD is able to see this similarity (this area is well above the threshold). Clearly, this does not happen for the LR ratio test, since the difference in amplitude makes the ratio very close to 0.

VI. CONCLUSION

In this paper, a new polarimetric change detector (PCD) is proposed. The algorithm is based on a previous partial target detector which exploits perturbation filters. The partial targets observed in the first acquisition are employed as reference targets for the detection performed on the second acquisition. In order to be able to set the threshold, a parameterization is introduced linking the outputs of the detector with the angle differences from the eigenvector model for the coherency matrix. The proposed change detector was tested with
Fig. 12. Entropy images with TerraSAR-X. ENL = 109: (a) 17\textsuperscript{th} May 2009; (b) 28\textsuperscript{th} May 2009.

Fig. 13. $\alpha$ angle images with TerraSAR-X. ENL = 109: (a) 17\textsuperscript{th} May 2009; (b) 28\textsuperscript{th} May 2009.
Fig. 14. Change detection masks. Linear scaling from 0.9 to 1. \( ENL = 109 \): (a) \( \Delta \alpha = \Delta \zeta = 18^\circ \). (b) \( \Delta \alpha = \Delta \zeta = 28^\circ \).

Fig. 15. Change detection with LR ratio. \( ENL = 109 \): (a) \( T = 0.9 \). (b) absence of threshold.
a variety of datasets, including airborne and spaceborne sensors with different resolutions, frequencies and polarimetric modes. This detector has been compared with a widely used polarimetric detector, the Maximum Likelihood (LR) ratio of the two acquisitions.

The main advantage of the proposed algorithm seems to be its capability to neglect the overall amplitude of the backscattering, taking into account exclusively the polarimetric information. Mathematically, changes in the overall amplitude are not related to changes of the polarimetric target and generally they are not related to macroscopic physical changes of the target. However, the question regarding the importance of the overall amplitude is still under analysis. One remarkable advantage of the proposed PCD is the possibility to separate the polarimetric information from the overall amplitude. Clearly, if the amplitude is essential in the characterization of the temporal characteristics of a target, a simple threshold detector on the amplitude can be used jointly with the polarimetric one.

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