Estimating penetrometer resistance and matric potential from the velocities of shear and compression waves

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Estimating Penetrometer Resistance and Matric Potential from the Velocities of Shear and Compression Waves

It would be useful if the velocity of elastic waves could be used to deduce soil physical properties. We wanted to validate the suggestion that the small strain shear modulus has a relatively simple linear relationship with penetrometer resistance. We were also interested in testing published equations for predicting shear wave velocity with an independent data set. Three soils were investigated in this study: a loamy sand soil and two silty clay loam soils. The soils were packed into cores with vertical axial stresses of 30, 200, or 1000 kPa. Following saturation, they were drained to a range of matric potentials between −10 and −500 kPa. After equilibration, we measured the velocities of shear (S wave) and compression (P wave) waves as well as the penetrometer resistances. Our data confirmed a previous proposal that the penetrometer resistance was an approximately linear function of the small strain shear modulus but tested the relationship by direct measurement. The relationships were found to have some sensitivity to soil type. Nevertheless, we show for the first time that there is considerable potential for using S wave velocity to deduce penetrometer resistance with a calibration that is relatively insensitive to soil type. Although estimation of the matric potential with either shear or compression wave velocity was found not to be very accurate, the possibility for estimating the matric potential from an elastic wave velocity given a priori knowledge of the void ratio is an interesting opportunity.
Earlier work (Whalley et al., 2012) has shown that the velocity of an S wave in soil, \( V_s \), is given by the following empirical relationship:

\[
V_s = A \left( \frac{F - \varepsilon}{1 + \varepsilon} \right) \left( \frac{\psi}{\psi_{ae}} \right)^{0.55}\]

where \( \varepsilon \) is the void ratio, \( \psi \) is the matric potential (kPa), \( \psi_{ae} \) is the air-entry potential (kPa), and \( \sigma_s \) is the net stress applied to the soil (kPa); \( A, F, r, \) and \( \gamma \) are adjustable empirical parameters but for a relatively wide range of soils their values do not depend on soil type. Thus, provided \( \varepsilon, \sigma_s, \) and \( \psi_{ae} \) are known, in principle, it should be possible to estimate \( \psi \) from measurements of \( V_s \).

Previously, Gao et al. (2012a) proposed that the penetrometer resistance, \( Q \), is a linear function of the small strain shear modulus \( G \) (from Eq. [1]), \( G = \rho V_s^2 \) such that

\[
Q = \kappa G
\]

Gao et al. (2012b) estimated shear wave velocity with Eq. [3]; however, the validity of Eq.[4] has not been tested with measurements of both \( V_s \) and \( Q \) made on the same soil sample.

In this study, our objectives were (i) to provide an independent assessment of the ability of Eq. [3] to predict shear wave velocity in soils, (ii) for the first time to compare \( Q \) and \( G \) determined from \( V_s \) with measurements on the same soil cores, and (iii) to evaluate the scope for using shear and compression wave velocity data to predict matric potential.

**MATERIALS AND METHODS**

**Soils and Sample Preparation**

The three soil samples used in this study included one loamy sand soil and two silt clay loam soils (Table 1). The loamy sand soil was collected from an arable field located at Butt Close, Woburn, UK, as described by Whalley et al. (2008). The silt clay loam soils were collected from the laboratory in predetermined random order.

Following equilibration, the samples were taken off for measurements of both \( V_s \) and \( Q \) at five matric potentials: −10 and −30 kPa on a tension table and then drained to five matric potentials: −100, −300, −500 kPa using a pneumatic press. Nine samples (3 axial stresses × 3 treatments) of each soil where placed randomly on one of nine predetermined locations within the pressure vessel or tension table. Following equilibration, the samples were taken off for measurement in a predetermined random order.

The penetrometer resistance and the velocity of S and P waves (see below) were measured in the equilibrated soil samples. The final water content and void ratio were also determined. The experiment had 3 replications × 3 compression levels × 5 matric potentials = 45 treatment combinations for each of the three soils.

**Penetrometer Resistance Measurement**

The penetrometer resistance was measured with the method used by Gao et al. (2012a, 2012b). A 2-mm-diameter and 60° cone angle penetrometer was inserted into the soil core at a constant speed of 20 mm min\(^{-1}\) from the surface to the 30-mm depth using a universal test frame (Davenport-Nene Test Frame DN10). An electronic balance was used to record the force that was experienced by the penetrometer in each soil. Three vertical penetrometer measurements were made on each soil core.

**Measurement of Compression and Shear Wave Speeds**

Shear wave (S wave) velocity, \( V_s \), and compression wave (P wave) velocity, \( V_p \), were measured using piezo-ceramic devices at the top and bottom of the cylindrical samples as shown in Fig. 1. The electrical input signal was a single cycle of a sine wave with a 0.2-ms period. This signal was converted into an elastic wave by one
of the devices, which was then detected by the second device. The time taken for the P or S wave to travel the length of the core was determined by comparing the signal input to the soil core with the detected wave (Whalley et al., 2011). The sensors, related electronics, and software are available commercially (GDS Instruments).

RESULTS AND DISCUSSION

Soil Water Content and Density

The water release characteristics of the soil samples were consistent with those in the earlier data of Gao et al. (2012b) and are not shown here. The soil bulk densities resulting from the initial compression pressures (30, 200, and 1000 kPa) for three soil samples were: 1.53, 1.64, and 1.71 g cm\(^{-3}\) for the ALS soil; 1.16, 1.50, and 1.77 g cm\(^{-3}\) for the FZCL soil; and 0.94, 1.16, and 1.34 g cm\(^{-3}\) for the GZCL soil. In soils consolidated to a given pressure, the soil water content for ALS was lower than that of FZCL and GZCL. The GZCL soil retained more water than FZCL. Soil compaction decreased the water content at higher matric potentials, which was consistent with the data of Gao et al. (2012b) for these soils.

Soil Penetrometer Resistance

An ANOVA indicated that, for each soil type, the soil penetrometer resistance varied with matric potential and initial compression stress (\(P < 0.001\)). Generally, for each soil type, the penetrometer resistance increased with increasing initial compression stress and decreasing matric potential (Fig. 2). An exception to this was in the grassland soil (GZCL), where penetrometer resistance was not very sensitive to matric potential. No simple relationship was found between the initial soil compression, matric potential, and soil type and penetrometer resistance (Fig. 2). This observation is supported by previous studies (e.g., To and Kay, 2005; Vaz et al., 2011).

Compression and Shear Wave Speeds

The travel time of P and S waves was determined from the time delay between the first peak of the input signal and the first peak of the signal detected after it had traveled through the soil sample, as described by Whalley et al. (2011). Shear wave velocity, \(V_s\), and compression wave velocity, \(V_p\), are plotted against matric potential and void ratio for the three soils in Fig. 3 and 4, respectively.

The shear and compression wave velocities were influenced by the matric potential and void ratio (achieved by the initial soil compression). The velocities of the shear and
Compression and Shear Wave Speeds and Effective Stress

Assuming that in the packed soil cores $\sigma$, in Eq. (3) was zero, and with the values of parameters reported by Whalley et al. (2012), we were able to predict the shear wave velocity of the soils in this study (Fig. 6). In general, the degree of saturation is more easily estimated than the air-entry potential, thus the suction stress was calculated by replacing $(\psi/\psi_a)^{0.55}$ with the degree of saturation $S$. If $S < 0.5$, then it was fixed at 0.5 (Whalley et al., 2012). The use of $S$ to scale the matric potential to give a suction stress is likely to make this work easier to apply. With the exception of the GZCL soil, the measured shear wave velocities agreed well with the predicted values (Fig. 6; Table 4), although there was a tendency for Eq. (3) to overestimate $V_s$. The greater degree of aggregation in the GZCL soil may have resulted in $V_s$ and $V_p$ be-

\begin{align}
V_p &= mV_s + c \quad [5]
\end{align}

where $m$ is the slope and $c$ is the intercept, but different values of $m$ and $c$ were needed for the three different soils (Fig. 5; Table 3). Strictly, the relationship between the velocities of P and S waves is determined by Poisson’s ratio, $\nu$, where

\begin{align}
\frac{V_p}{V_s} &= \sqrt{\frac{2(1-\nu)}{1-2\nu}} \quad [6]
\end{align}

and if $V_p$ and $V_s$ are both known, then Poisson’s ratio can be calculated as

\begin{align}
\nu &= \frac{1}{2}\left(\frac{V_p}{V_s}\right)^2 - 1 \quad [7]
\end{align}

(Santamarina et al., 2001). The ratio $V_p/V_s$ is constant only for the grassland soil (GZCL) because the intercept, $c$, in Eq. [5] was not significantly different from zero ($P = 0.194$). Thus for this soil, the applied treatments did not have any effect on Poisson’s ratio. For the other two soils (ALS and FZCL), Poisson’s ratio depended on the applied compaction and matric potential treatments, but no pattern could be identified. The range of values of Poisson’s ratio (0.15–0.44 with a mean of 0.34) is similar to values reported in the literature (e.g., Nakagawa et al., 1997).
ing less sensitive to matric potential, especially in the low compaction treatments (30 kPa) (Fig. 3 and 4). In aggregated soil, there is a dual pore structure, and simple models for elastic wave velocity such as Eq. [3] do not take this into account (Bagherieh et al., 2009), which may be responsible for the overestimation of $V_s$.

Both P and S wave velocities ($V$) could be described by a modified form of Eq. [3]:

$$V = \frac{A_1 (F - \epsilon)^2}{1 + \epsilon} \left( s, S^* \right)^{\gamma} \quad [8]$$

where $S^* = S$ for $S > 0.5$; otherwise $S^* = 0.5$ (Fig. 7; Table 5).

**Estimation of Matric Potential**

There is no doubt that $V_s$ and $V_p$ correlate well with the matric potential (Lu and Sabatier, 2009; Whalley et al., 2012; Yang et al., 2008) and can account for a high percentage of the variance (Table 2); however, these relationships are calibrations that apply to a given soil type packed to a given condition. It would be useful if the void ratio data could be used to develop calibrations that needed only different parameters to distinguish between soil types. Provided that the void ratio, $\epsilon$, the degree of saturation, $S$, and the elastic wave velocity, $V$, are known, then in principle, Eq. [8] can be solved for the matric potential (Table 5; Fig. 8) for a given soil. The use of $V_s$ appeared to give a better estimation of soil moisture than $V_p$. This may, in part, be because compression waves are less sensitive than shear waves to changes in matric potential when the soil is tension saturated. In saturated soils, the P wave velocity is determined by the bulk modulus of water and only increases when air enters the soil (Mitchell and Soga, 2005). In contrast, S waves are very sensitive to changes in the matric potential of the saturated soil (Whalley et al., 2012). The relationship between fitted and experimental log$_{10}(S\psi)$ values for the grassland soil were poor in comparison with the other two soils (Fig. 8), explaining only about 50% of the variance compared with >80% for ALS and FZCL. As already noted, the grassland soil has a strong dual-porous structure, which may require a more complex model than Eq. [8] for shear or compression wave velocity (Bagherieh et al., 2009).

Compression and shear wave velocity data could be combined to eliminate the need for a priori void ratio data when the soil moisture status is predicted. Assuming a common value of $F$ in Eq. [8] allows the ratio of shear and compression wave speeds to be written as

$$\frac{V_s}{V_p} = A_{ls} (S\psi)^{\gamma_p}$$

and, following inversion, $S\psi$ is given as

Fig. 4. Compression (P) wave velocity plotted as a function of matric potential and void ratio for arable loamy sand (ALS), fallow silty clay loam (FZCL), and grassland silty clay loam (GZCL) soils in various consolidation states.
The subscripts s and p refer to parameters for S and P waves, respectively, in Eq. [8]. To test the feasibility of this approach, we plotted log10($V_s/V_p$) against log10($S_y$). We found a significant relationship only for the A LS soil (Fig. 9), and log10($S_y$) accounted for 66% of the variance in log10($V_s/V_p$). Again, the difficulty is that Eq. [10] is highly nonlinear (Fig. 9), and the variance increases with the mean value of $S_y$. The most appropriate approach for developing a calibration function that can be applied generally remains to be determined.

### Relationship between Penetrometer Resistance and the Small Strain Shear Modulus

Penetrometer resistance was closely correlated to the small strain shear modulus (Fig. 10). This is consistent with the findings of Gao et al. (2012b). The implicit assumption is that the penetrometer resistance in these soils is dominated by elastic failure because the small strain shear modulus is a measure of the elastic properties of the soil.

### Table 2. Relationships between shear wave velocity, $V_s$, or compression wave velocity, $V_p$, and soil matric potential for the arable loamy sand (A LS), fallow silty clay loam (FZCL), and grassland silty clay loam (GZCL) soils presented in Fig. 3 and 4.

<table>
<thead>
<tr>
<th>Soil</th>
<th>Initial compression stress</th>
<th>Relationship between $V_s$ or $V_p$ and matric potential, $\psi$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A LS</td>
<td>30</td>
<td>$V_s = 55.476\psi^{0.2461}$</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>$V_p = 175.69\psi^{0.1376}$</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>$V_s = 64.627\psi^{0.2468}$</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_p = 183.88\psi^{0.1759}$</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_s = 92.37\psi^{0.1913}$</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_p = 230.44\psi^{0.1423}$</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_s = 40.848\psi^{0.2464}$</td>
<td>0.97</td>
</tr>
<tr>
<td>FZCL</td>
<td>30</td>
<td>$V_p = 87.471\psi^{0.256}$</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>$V_s = 66.715\psi^{0.2728}$</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>$V_p = 179.41\psi^{0.208}$</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_s = 150.44\psi^{0.1742}$</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_p = 513.85\psi^{0.101}$</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_s = 33.363\psi^{0.1119}$</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_p = 80.264\psi^{0.1014}$</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_s = 73.528\psi^{0.062}$</td>
<td>0.85</td>
</tr>
<tr>
<td>GZCL</td>
<td>200</td>
<td>$V_p = 125.71\psi^{0.0874}$</td>
<td>0.86</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>$V_s = 113.53\psi^{0.1402}$</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$V_p = 198.57\psi^{0.1495}$</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Fig. 5. A comparison of compression (P) and shear (S) wave velocities ($V$) for the arable loamy sand (A LS), fallow silty clay loam (FZCL), and grassland silty clay loam (GZCL) soils subject to a range of initial stresses and matric potentials. The curves are fitted to Eq. [5] and the fitted parameters are listed in Table 3.

Table 3. Parameter values of Eq. [5] relating compression (P) and shear (S) wave velocities to each other with a linear function, where $m$ is the slope and $c$ is the intercept. The fitted curves are shown in Fig. 5.

<table>
<thead>
<tr>
<th>Soil†</th>
<th>Parameter</th>
<th>$m$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A LS</td>
<td></td>
<td>1.394 ± 0.126</td>
<td>126.6 ± 4.71</td>
</tr>
<tr>
<td>FZCL</td>
<td></td>
<td>2.019 ± 0.075</td>
<td>48.8 ± 19.7</td>
</tr>
<tr>
<td>GZCL</td>
<td></td>
<td>1.744 ± 0.123</td>
<td>23.0 ± 17.6 NS‡</td>
</tr>
</tbody>
</table>

† A LS, arable loamy sand; FZCL, fallow silty clay loam; GZCL, grassland silty clay loam.
‡ NS, not significant ($P > 0.05$).

### Table 4. Predicted and measured shear wave velocities ($V_s$) compared as $V_s$ predicted = $kV_s$ measured in Fig. 6.

<table>
<thead>
<tr>
<th>Soil†</th>
<th>$k$</th>
<th>Variance accounted for</th>
</tr>
</thead>
<tbody>
<tr>
<td>A LS</td>
<td>1.3007 ± 0.0361</td>
<td>81.6%</td>
</tr>
<tr>
<td>FZCL</td>
<td>1.1383 ± 0.0282</td>
<td>89.8%</td>
</tr>
<tr>
<td>GZCL</td>
<td>1.3395 ± 0.0725</td>
<td>49.7%</td>
</tr>
</tbody>
</table>

† A LS, arable loamy sand; FZCL, fallow silty clay loam; GZCL, grassland silty clay loam.

Fig. 6. Prediction of shear (S) wave velocity with the model described by Eq. [3] for the arable loamy sand (A LS), fallow silty clay loam (FZCL), and grassland silty clay loam (GZCL) soils. A net stress ($s_s$) of zero was used and the suction stress ($S\psi$) was computed using the degree of saturation. The values of the parameters were those suggested as a general solution for a wide range of soils by Whalley et al. (2012). A statistical comparison of the predicted and measured values is given in Table 4.

$$S\psi = \left( \frac{A_s}{A_p} \right)^{\left( \frac{V_s}{V_p} \right)}$$

[10]

where the subscripts s and p refer to parameters for S and P waves, respectively, in Eq. [8]. To test the feasibility of this approach, we plotted log10($V_s/V_p$) against log10($S\psi$). We found a significant relationship only for the A LS soil (Fig. 9), and log10($S\psi$) accounted for 66% of the variance in log10($V_s/V_p$). Again, the difficulty is that Eq. [10] is highly nonlinear (Fig. 9), and the variance increases with the mean value of $S\psi$. The most appropriate approach for developing a calibration function that can be applied generally remains to be determined.
the soil elastic properties. The empirical relationships in Fig. 10 are linear (Table 6) but differ between different soil types; however, they do provide a promising basis for inferring penetrometer resistance from shear wave velocity. We found that an improved fit between the penetrometer resistance ($Q$, kPa) and the small strain shear modulus ($G$, kPa) for soils could be obtained for two of the soils (FZCL and GZCL) by including a constant term such that

$$Q = mG + c$$

Regressions against penetrometer resistance based on the P wave velocity were less successful than those that used the shear wave velocity. Table 6. The parameters of a linear fit of penetrometer resistance (kPa) against the small strain shear modulus (kPa) Eq. [11] , where $m$ is the slope and $c$ is the intercept.

<table>
<thead>
<tr>
<th>Soil†</th>
<th>Parameter</th>
<th>$m$ (kPa)</th>
<th>$c$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALS</td>
<td>$A$</td>
<td>0.0286 ± 0.0013</td>
<td>−53.00 ± 12.9 NS‡</td>
</tr>
<tr>
<td></td>
<td>$F$</td>
<td>0.0159 ± 0.0006</td>
<td>694.32 ± 10.8</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>0.03377 ± 0.0013</td>
<td>439.57 ± 54.6</td>
</tr>
<tr>
<td>FZCL</td>
<td>$A$</td>
<td>0.0286 ± 0.0013</td>
<td>−53.00 ± 12.9 NS‡</td>
</tr>
<tr>
<td></td>
<td>$F$</td>
<td>0.0159 ± 0.0006</td>
<td>694.32 ± 10.8</td>
</tr>
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<td></td>
<td>$\gamma$</td>
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<td>439.57 ± 54.6</td>
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<tr>
<td>GZCL</td>
<td>$A$</td>
<td>0.0286 ± 0.0013</td>
<td>−53.00 ± 12.9 NS‡</td>
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<td></td>
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</tr>
</tbody>
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† ALS, arable loamy sand; FZCL, fallow silty clay loam; GZCL, grassland silty clay loam.

‡ NS, not significant ($P > 0.05$).
Fig. 9. The log10($V_s/V_p$), where $V_s$ is the shear wave velocity and $V_p$ is the compression wave velocity, plotted against log10(−$S$), where $S$ is the suction stress. Only data for the arable loamy sand (ALS) is shown; for the other soils, the fits were not significant.

CONCLUSIONS

There is potential for using the S wave velocity to deduce penetrometer resistance with a linear calibration that has some sensitivity to soil type. Prediction of the matric potential from P or S wave speeds with a general calibration function was found to be not very accurate because the variance was a function of the mean value and a logarithmic transformation was required. It may be possible to estimate the matric potential in soil from elastic wave speeds with more accuracy, provided a calibration for each soil at a given density is determined.

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