Measuring mathematical resilience: an application of the construct of resilience to the study of mathematics.

Mathematical Resilience: An application and exploration of motivational constructs related to resilience in the study of mathematics

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Abstract

Motivation, attitudes towards mathematics, and resilience theories are synthesized to develop a theory and instrument useful in studying malleable attributes predictive of persistence in mathematics in spite of adversity. The Mathematical Resilience Scale (MRS) is an affective instrument designed to measure student's attitudes in studying mathematics, summarized by four correlated factors: value, struggle, growth and resilience. The MRS was developed and validated using three samples utilizing the techniques of exploratory and confirmatory factor analysis. Evidence presented supports three factors: value, struggle, and growth, and provides a new approach for encouraging greater student participation and persistence in mathematics.

Keywords

Mathematical resilience, mathematical attitudes, mathematical motivation, STEM, self-determination theory, growth theory of learning.

Mathematical Resilience:

An application and exploration of the construct of resilience to the study of mathematics

A solid foundation in the study of mathematics is important for student success in many careers, and in particular those in Science, Technology, Engineering and Mathematics (STEM). Favorable current job opportunities, higher relative wages, and stronger projected demand are fueling the sharp increase in interest in the topic of education for STEM careers (Chairman's Staff of the Joint Economic Committee, 2012; Council of Economic Advisers, 2009; Rask, 2010). We are no longer preparing a sufficient number of math and science related majors to meet the needs of an increasingly technical workforce. (National Mathematics Advisory Panel, 2008). Persistence in mathematics coursework is a key factor in improving student access and success in STEM careers.

In order to encourage more students of all ability levels to persist in mathematics, we need to find new approaches to mathematics education. Mathematics is a key part of the STEM curriculum, arguably the most important discipline of the four since mathematics is foundational to the development of the majority of scientific theory and properties (Shaughnessy, 2012). New theories and interventions need to be identified and tested to encourage higher rates of student persistence in mathematics, enrollment in advanced mathematics, engagement in the content of STEM fields, and choice of STEM careers. Research has shown that affective traits such as motivation, beliefs, and attitude are linked to increased likelihood of taking advanced mathematics courses (Ma, 2006) and are significant predictors of improved cognitive activity and achievement (Buff, Reusser, Rakoczy & Pauli, 2011; Ethington & Wolfe, 1986).

Many influences in the life of a student converge in the mathematics classroom, resulting in some students continuing their pursuit of the mathematics despite experiencing setbacks while others do not. University students enroll in mathematics classes either because of interest or because the course is required for their major. Students in STEM fields typically are required to take more courses in mathematics than students in fields outside of STEM, and college level mathematics can present students with significant challenges. Students need to be able to overcome these challenges to remain engaged in STEM. The construct of psychological resilience provides a theoretical parallel, a language, and a research base for studying the variables that may be predictive of persistence in mathematics in spite of adversity. Although psychological resilience has been researched extensively (Luthar, Cicchetti, & Becker, 2000; Luthar, 2007) the study of resilience in an academic setting and specifically resilience in the study of mathematics represent a relatively new approach (Johnston-Wilder & Lee, 2010; Rivera & Waxman, 2011, Yeager & Dweck, 2012).

The purpose of this study is to develop and validate the Mathematical Resilience Scale (MRS), an affective instrument that measures student attitudes toward mathematics that are hypothesized as predictive of a positive response to challenges in the study of mathematics. This paper provides a discussion of theory supporting this conjecture as well as the results of the factor analysis of the MRS using three samples which included community college students, university undergraduates from a large state university, and professional actuaries. The resultant instrument is designed to be appropriate for college level students of any ability level. The instrument will be useful to support research on the effect of these attributes on persistence in mathematics; on their interaction with other attitudes, values and beliefs related to learning mathematics; and on the development of interventions to improve mathematical resilience. The ultimate goal of this research is to improve student prospects for persistence and success in mathematics at all levels.

Background

The review of the literature on motivation, psychological resilience, and attitudes towards mathematics informed this research leading to the identification of malleable attributes which are hypothesized as predictive of mathematical resilience. This study conceptualizes mathematical resilience as a positive adaptive stance to mathematics which allows students to continue to pursue learning mathematics despite adversity. The factors which represent the attitudes towards mathematics are value, struggle, growth and resilience. High levels of these factors would contribute to greater student persistence in mathematics in spite of difficulty, while lower levels would diminish student persistence in mathematics when faced with adversity.

Methods of Studying Resilience

The extant literature on psychological and academic resilience contains a strong theoretical basis for studying mathematical resilience. Research on mathematical resilience borrows from psychological resilience which represents a positive response to a significant threat or adversity (Luthar, Cicchetti, and Becker, 2000). Resilience is often studied in people who have been exposed to dangerous, potentially life threatening, or stressful environments such as areas of poverty and violence in some inner city neighborhoods. All individuals are considered at risk when exposed to these threats; some respond with positive adaptations such as academic achievement, high school graduation and mental health while others suffer maladaptive symptoms such as involvement in drugs, crime, and academic failure. "Resilience is a multidimensional construct regulating optimal human functioning and locates itself in a positive psychology which addresses mental wellness rather than mental illness," (Karairmak, 2010, p.

350). Protective factors help to reduce the effects of negative stimuli by helping an individual to persist through the challenges they face. Protective factors include adaptive attitudes, supportive families, schools, communities, and peer groups that shield those at risk from negative outcomes. Luthar (2007) argued that since resilience is complex, it needs to be examined in the context of a particular domain, encouraging researchers to study many types of resilience including resilience in educational settings.

Although the psychological construct of resilience is considered a dynamic process rather than a personal trait, instruments have been developed to measure attributes predictive of resilience. Personal characteristics have been identified that are consistently present in adaptive populations and absent in maladaptive populations. The Connor-Davidson Resilience Scale (CD-RISC) identified three factors in the construct of psychological resilience which are attributes that allow the individual to be stress resistant: tenacity and personal competence, tolerance of negative affect, and tendency toward spirituality/strength (Karairmak, 2010). The factors of CD-RISC provided a comparative reference when considering potential factors associated with the construct of mathematical resilience.

Methods of Studying Mathematics Resilience

Studying the response of students to difficult or challenging experiences while in the mathematics classroom or while engaged in studying mathematics represents an area that has only begun to be researched. Yeager & Dweck (2012) defined resilience as any "behavioral, attributional, or emotional response to an academic or social challenge that is positive or beneficial for development." For resilience to exist there must be some type of adversity followed by a positive outcome. In the context of mathematics, the adversity can take many forms including a failing grade, struggle beyond the student's tolerance, boredom, embarrassment due to poor performance, poor quality curriculum or instruction, and non-supportive teacher-student or student-student interaction. The student may have the perception that they are unable to learn the mathematics beyond some degree of rote memorization; they may have performance anxiety during class or on a test, or math anxiety. To study resilience, we need to recognize that it is not the teacher's or researcher's perception of the experience that matters; it is the student perception of the adversity that is paramount (Yeager & Dweck, 2012). The positive outcome we are seeking can be proximal as in reengagement in mathematics evidenced by improved participation in the classroom, greater understanding of the material, enhanced interest and enjoyment, and improved performance on assessments, or distal as in persistence in taking higher levels of mathematics coursework and majoring in STEM careers.

Several studies identified mathematical resilience as important for success, and although there appears to be consensus regarding its definition, no measures of resilience in the context of learning mathematics have been rigorously developed or validated. Rivera and Waxman (2011) conducted semi-structured interviews of 118 English Language Learners (ELL) to describe student perceptions and attitudes towards mathematics. Students in the study were considered at risk of academic failure by virtue of their status as ELL. From this pool of students, teachers were asked to nominate students as resilient based upon high achievement test scores, strong attendance, high quality daily school work, and high level of motivation. Similarly, teachers were also asked to nominate students as non-resilient based upon low test scores, attendance, school work and motivation. The authors identified the use of teacher nomination of resilient students as a limitation of their study, further motivating development of an instrument.

Johnston-Wilder and Lee (2010) suggested that mathematical resilience include perseverance despite setbacks, working collaboratively, ability to articulate mathematical understanding and a growth theory of learning. Their study consisted of action research in an urban school in England in which the researchers were invited into the school to administer interventions for the purpose of improving student motivation and achievement levels in mathematics. Although Johnston-Wilder and Lee (2010) advocated the importance of mathematical resilience and provided introductory theory, they did not report quantitative results from their study. In addition, they utilized an instrument which had not been evaluated for its psychometric properties. This limitation further defined a need for an instrument that measures mathematical resilience.

Hypothesized Factor Structure and Theory

The key question to be addressed in this study is what attitudes, beliefs, and values separate the student who persists in mathematics from one that does not when all students will experience some type of difficulty? The review of the literature provided the theoretical foundation for identifying malleable attributes, similar to protective factors, which are hypothesized as predictive of positive outcomes in studying mathematics.

Value

The mathematical resilience factor labeled *v*alue refers to the extent to which students find studying mathematics valuable in attaining their current or future goals, how this relates to intrinsic and extrinsic motivation, and the role of self-regulation. For students to succeed in mathematics, they need to perceive math as valuable (Deci, Vallerand, Pelletere, & Ryan, 1991). The higher the student perceives the value of math in the world, the greater the motivation to study it (Eccles, 1983). Since this study is designed for college age students, the value component selected focused on utility since these students are making choices having proximal impact on coursework choices as well as distal impact on major and future career. These choices are motivated by their sense of value to perform tasks based upon intrinsic or extrinsic motivation. Intrinsic motivation refers to the desire to do something because it provides satisfaction and enjoyment to the individual. Extrinsic motivation refers to the desire to do something because it is a means to an end. Although it is well accepted that intrinsic motivation facilitates learning, extrinsic motivation has been overlooked in its strength in promoting achievement. As reported in Ryan and Deci (2000), extrinsic motivation does not necessarily lead to disengagement and disinterest; rather when the learner highly values the end, the extrinsic goal may drive the individual's behavior as much as an intrinsic goal. Whether, students are seeking outcomes related to intrinsic or extrinsic motivation, the amount of value the student ascribes the class and grade will fuel their level of persistence.

As a final component of the theory supporting the factor of value, it is important to consider the role of self-regulated learning which reflects the degree that students can direct their efforts, skills, and abilities to achieve their desired academic goals (Zimmerman, 2008). Self-determination theory suggests that people will take actions that are self-initiating and self-regulating, contributing to their ability to meet their needs (Ryan & Deci, 2000). Students at the university level have freedom to choose their major, and for students considering STEM careers, this choice may lead to requirements to take extensive mathematics coursework. If a student greatly values either studying mathematics or values the career choice requiring mathematics, they will be more persistent even in the face of challenges.

Struggle

The factor labeled struggle refers to the student belief that because mathematics can be a difficult subject to learn, there is an expectation that the process of learning it will require overcoming challenges. The experience of struggling with mathematics is not unusual; even exceptional mathematicians make errors and struggle with learning and solving mathematics problems. Struggle refers to student perception and tolerance of the level of difficulty in studying mathematics, especially relative to others. Struggle is related to the theory of human agency, which purports that individuals assess and control their own thoughts, motivations and actions (Bandura,1989). Human agency is often exercised through the collective experiences and culture of a group. Bandura (2000) found that when the motivational investment of the group is high, the staying power in the face of setbacks is strengthened, hence enhancing performance. Students need to be challenged at the appropriate degree in order to continue to engage in the study of mathematics. Yet when they are challenged, it is important that they not interpret the challenge as an indication that something is wrong with them. When a student believes that struggle is inherent to the study of mathematics, she attributes the reason for the challenge to the mathematical content rather than on limitations in her ability. Students who understand that struggle in math is common to their peer group, to all mathematics students, and even to experts in mathematics will have more tolerance and stronger staying power in the face of setbacks.

Growth

A growth theory of learning mathematics refers to the belief that level of knowledge of mathematics is a malleable attribute that can grow. Implicit theories of intelligence as reported in Dweck (1986) and Yeager and Dweck (2012) provide a model of either an incremental (growth) theory of intelligence or an entity (fixed) theory of intelligence. Students with a growth theory of intelligence believe that if they work at it, they can learn more. Students with a fixed theory believe that their level of intellect is static, and they are limited in developing this fixed ability level. In recent research, there has been more emphasis on the incremental theory of intelligence, the need for a certain “mindset” which helps overcome the challenges that are common in the life of a student (Dweck, (2006); Yeager & Dweck , 2012). In studies of resilience in academic and social settings, an incremental theory of intelligence paired with the mastery goal orientation made significant differences in student’s ability to respond favorably to social and academic challenges (Yeager & Dweck, 2012). Research has also shown that a student’s theory of intelligence is malleable, it predicts academic performance over time, and that a growth theory of learning improves achievement (Blackwell, Trzesniewski & Dweck, 2007; Yeager & Dweck, 2012).

Resilience

The foundation for the fourth hypothesized factor, resilience, is based on the literature on psychological resilience defined as positive response to a significant adversity, where the positive adaptation and adversity are related to the process of learning mathematics. Resilience reflects an adaptive response to challenges which are common to existence. Students in mathematics can experience a great deal of stress. Contrasting with math anxiety, which has been studied extensively (Hembree, 1990; Richardson & Suinn, 1972; Tobias, 1978), mathematical resilience considers the qualities inherent to those who experience the negative stimuli but rather than develop anxiety, respond by functioning optimally. Resilience was included as a separate factor in the first portion of this research, referred to as EFA1, and was found to be multidimensional.

In summary, mathematical resilience has been defined as a set of attitudes which theory suggests are predictive of a positive response to studying mathematics despite negative experiences. The extant literature provided support of the view of mathematical resilience as a construct with four dimensions: value, struggle, growth and resilience.

Method

After completion of the literature review and development of the theoretical model for mathematical resilience, items were developed using a seven point Likert scale (1 = *completely disagree* and 7 = *completely agree*). Items then underwent content validation to determine their degree of fit to the constructs of interest, using guidelines from McKenzie, Wood, Kotecki, Clark, and Brey (1999). A jury of eleven subject matter experts met the criteria of either knowledge of mathematics, education, or the actuarial profession, of which two did not reply. The remaining nine subject matter experts were asked to function as content validation jurors. These nine jurors were comprised of five professors, of which two were professors of mathematics education, one of mathematics, one of gifted education, and one of assessment; one Ph.D in gifted education; two graduate students and one actuary.

The instrument was pilot tested in three stages. First, the instrument was administered to a convenience sample of 262 including the 34 items along with 5 demographic questions. This sample was used to complete an exploratory factor analysis (EFA1). The results of EFA1 served to identify the factor structure, reduce the number of items on each scale, and identify other items that needed fine tuning. A revised instrument was administered to 603 participants, and the sample was randomly divided into two approximately equal sets. One sample was used to perform an external replication of the exploratory factor analysis, EFA2, as recommended by Osborne & Fitzpatrick (2012) to determine if the different sample and item revision affected the factor structure. The other sample was used to complete a confirmatory factor analysis (CFA) to test the fit of the data to the theoretical structure using Amos 18.0.0 (Arbuckle, 2009; Thompson, 2010).

An exploratory factor analysis was completed for both the EFA1 and EFA2 samples to identify the unobserved latent factor structure. Both EFA studies used oblique rotation to identify the structure of the instrument and to allow for correlated factors. Two different extraction methods were utilized: principal axis factoring (PAF) and principal components analysis (PCA). Eight different estimates for the number of factors were identified: the Kaiser-Guttman rule criteria of eigenvalues greater than 1 (Pett, Lackey and Sullivan, 2003, p. 115), scree plot, minimum average partial (MAP) squared and fourth power, parallel analysis principal components analysis (PCA) means and percentiles, and principal axis factoring (PAF) means and percentiles. Preference was given to the estimates from the PCA and MAP procedures based upon recommendations from Pett et al. (2003). A principal axis factoring extraction, again using oblique rotation (direct oblimin with $\delta = 0$) provided pattern and structure coefficients along with factor correlations. Following recommendations from the literature (McCoach, Gable, & Madura, 2013; Pett et al. 2003) items were retained with pattern coefficients of .40 or greater on one factor and less than .30 on a second factor. The results were then analyzed to determine what actions needed to be taken if any on the instrument.

Using the factor structure identified by the EFA2, a CFA was completed on the second half of the sample, with factor loadings estimated using maximum likelihood estimation. Fit statistics were selected and evaluated following recommendations by Netemeyer et al. (2003). These were the Chi Square, the comparative fit index (CFI >.90), the Tucker-Lewis Index (TLI>.90), and the root mean square error of approximation (RMSEA<.08).

Item Generation and Content Validation

The literature review guided the development of forty-six items using a seven point Likert scale (1 = *completely disagree* and 7 = *completely agree*). These items were then submitted to jurors for content validation. Completion of the content validation of the instrument involved identifying subject matter experts to serve as jurors in validation of the instrument, requesting the jurors complete a questionnaire on the items, analyzing the results and applying the analysis to decision making and final item selection. Jurors were asked to judge whether the items represent the complete range of content regarding mathematical resilience as outlined in McKenzie, et al. (1999).

The content validity of the instrument was evaluated using a questionnaire which provided content validators with information on mathematical resilience to assist them in assessing item quality. The questionnaire contained a description of mathematical resilience and definitions of the four hypothesized factors of value, struggle, growth, and resilience. The forty-six items from the four hypothesized factors were masked in terms of hypothesized factor structure and then scrambled in order of presentation. Jurors were asked to indicate the best category fit for each item, including the option of no category, and to indicate the certainty of the placement (1=*not very sure*, 2=*pretty sure*, or 3=*very sure*. Jurors were also asked to indicate how relevant they felt each item was for the category (L=*low/no relevance*, M=*mostly relevant*, to H=*highly relevant*). Finally, jurors were provided with five questions which encouraged them to provide qualitative feedback on such things as the definition of the construct or wording of items. The data from the questionnaire were compiled and studied for the purpose of identifying which items should be excluded, included, and possibly reworded for the final survey instrument.

The initial criteria was to eliminate items that did not receive at least 80% consensus from experts on the placement in the proper factor, and to obtain the goal of a minimum of 8 items, maximum of 10 items for each factor. For factors resilience, struggle, and growth, this process eliminated all but eight items, but for value, no items were eliminated. Additional criteria were applied in a stepwise fashion, using juror quantitative and qualitative feedback, which supported the identification of items that were confusing, multidimensional, or unclear. The final set of items included eight items in struggle, growth, and resilience and nine items for value bringing the total to 33 items. For a list of the initial 33 pilot items, refer to Table 2 which provides the items as well as the results from EFA1.

Study 1: EFA1

A convenience sample of 262 was collected comprised mostly of insurance actuaries (63.2%) from across the United States and community college math students (23.3%) living in the northeast. After listwise deleting subjects for missing values, the total came to 253. The demographic breakdown of the EFA1 sample as well as the samples used for EFA2 and the CFA are shown in Table 1. In researching the line of distinction between those who continue to pursue the study of mathematics despite experiencing difficulties, the profession of the insurance actuary presented itself as of interest. An insurance actuary has to pass a series of as many as 10 exams, on topics including complex mathematical and statistical content with the pass rate typically less than 50% (Casualty Actuarial Society, 2011). Although many professionals have rigorous challenges to overcome to achieve set goals, actuaries provided a unique population since most of the exams cover rigorous statistics and mathematical content. To provide variation, we also sampled community college mathematics students. We hypothesized that on average community college mathematics students are more likely to have struggled with mathematics and perhaps have not developed the qualities needed and related to mathematical resilience. The remainder of the sample was comprised of undergraduate students and adults in a variety of professions. The original sampling plan provided for more community college students, but due to a low response rate, the sample was not well balanced. Although the sample of actuaries provided a convenient and interesting group to study, full validation of the scale would require a sample that was heterogeneous with respect to the construct in question, homogeneous with respect to other characteristics, and demographically similar to the target population (Lackey & Wingate, 1998). For this reason, additional samples were gathered at later steps in the study.

Descriptive statistics were run to study the mean and standard deviation to determine if sufficient variation exists to run a factor analysis model. Items G3 to G8 were reverse scored to align with an interpretation of higher scores as more favorable towards mathematical resilience. The mean scores ranged from 4.38 to 6.49, and standard deviations ranged from .908 to 1.708, indicating a reasonable range and good variation. The sample size was "fair" according to the guidelines in Comrey and Lee (as cited in Pett et al., 2003, p. 48), resulting in a sample size to item ratio of 8 respondents per item.

An exploratory factor analysis using an oblique rotation was completed to identify the structure of the instrument. The results of the first step of running the MAP, PCA and PAF provided evidence of a 4 factor structure, with PCA means and percentiles and MAP squared and fourth power extracting four factors. Based upon these results, a four factor model was selected. A SPSS Inc. (2009) dimension reduction procedure was completed using all items and a principal axis factoring with 4 factors. Based upon the results of measures of sampling adequacy, Bartlett's test of sphericity, $\chi^2(528,253) = 3948.41$, $p < .001$, Kaiser-Mayer-Okin (KMO) statistic (.883), the diagonals of the anti-image correlation matrix (all greater than .70), and a non-zero determinant of the correlation matrix, an exploratory factor analysis could be completed (Pett et al., 2003).

The resultant pattern matrix of the PAF solution with oblimin rotation is presented in Table 2 along with the structure matrix and the extraction communalities. The results were encouraging in that the patterns generally followed theory with a four factor model explaining 44.5% of the variance. In order to distinguish which items loaded on which factor, we only selected items with pattern coefficients of .40 or greater on one factor and less than .30 on other factors (McCoach, Gable, & Madura, 2013; Pett et al. 2003). Any item with pattern coefficients greater than .30 on more than one factor was considered multidimensional. Based upon these criteria, three items appeared to be multidimensional from the resilience factor as follows: R4 (“When I fail or do poorly on a math test, I know I have to work harder”), R5 (“When I struggle with math, I return to it until I get it”) and R8 (“When I don’t do as well as I hoped on a math task or test, I keep trying until I can do it”). In addition four items that were hypothesized as associated with resilience loaded on other factors. These were R1 (“When I have done poorly on something related to math, I know how to adapt”) and R3 (“I have strategies to use when I get stuck trying to solve math problems”) which loaded on Value; R2 (“I sometimes get discouraged by difficulties in mathematics but I bounce back”) and R7 (“I sometimes find math confusing, but I stick with it”) which loaded on Struggle.

Internal Consistency Reliability Analyses for Initial Study and Resulting Decisions

Value scale. Nine items were hypothesized on the value factor, which refers to the extent to which students find studying mathematics valuable for current and future goals. An example item is V1,(“Math is essential for my future”). The factor analysis extracted 12 items on a single factor including all nine value items, two resilience items, and one growth item. The item with the highest pattern coefficient was V4,(“Knowing math contributes greatly to achieving my goals”), with a pattern coefficient of .905. Item V6, (“People who are good at math have more opportunities than those who aren’t good at math”) loaded weakly on two factors, indicating that it may reflect some other construct beyond just valuing math, so it was deleted. One of the growth factor items, G9 (“I believe I can grow in my knowledge of math”) loaded on the value factor. Two items from the hypothesized resilience factor also loaded on value as follows: R1

("When I have done poorly on something related to math, I know how to adapt") and R3 ("I have strategies to use when I get stuck trying to solve math problems"). Although similar to some of the value items which mention goals, skills, and knowledge, this further indicated that the factor labeled resilience was multidimensional.

The final value scale from EFA1 contained 11 items including G9, the two resilience items R1 and R3, all the hypothesized value items except V6. The empirical evidence indicates that the value scale is internally consistent with coefficient alpha at .909, 95% C.I. [.891, .925].

Struggle scale. Eight items were hypothesized on the struggle factor, which refers to student perception and tolerance of difficulty in studying mathematics. An example item is S1, ("Everyone struggles with math at some point"). The factor analysis for struggle extracted all eight original struggle items and two resilience items bringing the total number of items to ten. The two resilience items are R2 ("I sometimes get discouraged by difficulties in mathematics, but I bounce back") and R7 ("I sometimes find math confusing, but I stick with it"). It could be argued that R2 and R7 align with the struggle factor since discouragement and confusion are related to struggle.

The final struggle scale contained 10 items, including all the original struggle items and the R2 and R7. Coefficient alpha for this ten item scale was .818, 95% C.I. [.783, .849] providing evidence for the scale's internal consistency reliability.

Growth scale. Eight items were hypothesized on the growth factor, G1-G7 and G9, which refers to the belief that level of knowledge of mathematics, is a malleable attribute that can grow. An example item is G2, ("Math can be learned by anyone"). Six items from the original growth factor loaded on one factor. G1 ("Everyone can get better at math if they try") and G9 ("I believe I can grow in my knowledge of math") also had a pattern coefficient of greater than .290 on value. In addition, G6 ("I believe a person's math ability is determined at birth") was of some concern since it loaded at -.252 on value. Since G1, G6 and G9 had relatively strong loadings on value, we revised the wording of each of them for the next sample. The final scale contained 7 growth items with coefficient alpha of .829, 95% C.I [.795, .859].

Resilience scale. Eight items were hypothesized on the resilience factor, which refers to the attitude that one will react with a positive response to a significant adversity when learning mathematics. An example item is R1, ("When I have done poorly on something related to math, I know how to adapt"). Items from the factor of resilience were problematic in that two loaded on the struggle factor, two loaded on the value factor, two were multidimensional, and two loaded weakly on a fourth factor. These problems may have been due the theoretical broadness of the items defining the construct of resilience as an attitude. In retrospect, the factor of resilience was almost self-referential, with items representing a positive outcome following the occurrence of something negative. The resilience items referred to outcomes, and the implication was that positive outcomes included outcomes such as improvements in mathematics achievement or enrollment in more advanced mathematics coursework. Since measurement of outcomes was not a part of this instrument and the EFA indicated that most of the resilience items were multidimensional, the factor called resilience was dropped along with all items associated with it.

Summary: EFA1

The results of the content validation and the exploratory factor analysis of the EFA1 sample provided initial evidence of the validity of the value, struggle and growth as three factors reflected in the items. Based upon the results, we decided to remove the resilience factor and all items associated with it due to the self-referential nature of the definition. The resultant model from EFA1 contained three factors with 27 items, eleven on value, ten on growth and six on struggle.

Study2: EFA2

Early on in this study, we recognized that full validation of the scale would require a replication of the analysis with a sample that was heterogeneous with respect to the construct in question, homogeneous with respect to other characteristics, and demographically similar to the target population (Lackey & Wingate, 1998; Osborne & Fitzpatrick, 2012). We took this opportunity to revise the instrument to improve wording and content coverage, using the results of EFA1. The wording on three items, G1, G6, and S8 was revised in an attempt to more concisely reflect the full breadth of the content of the construct, and item G9 was dropped. Three additional items were written to ensure full coverage of content, and these were G8 (“Only smart people can do math”), S10 (“When someone struggles in math, it doesn’t mean they have done something wrong”), and S11 (“Making mistakes is necessary to get good at math”). A second convenience sample of 603 was collected from university undergraduate students attending a research-intensive state university located in the northeast. These students were enrolled in large lecture hall courses, including Philosophy, Art History, and Introductory Statistics. The sample was divided using the SPSS random split into two random sample of approximately 50%, (N=293 and N=310). The first sample was drawn for EFA2 and the second for the CFA, with 280 and 290 remaining after listwise deletion respectively. A demographic breakdown of all three samples is shown in Table 1.

Descriptive statistics were run to study the means and standard deviations. Items G3 to G8 were reverse scored to align with an interpretation of higher scores as more favorable towards mathematical resilience. The mean scores ranged from 3.83 to 6.09, and standard deviations ranged from 1.10 to 1.75, indicating a reasonable range and good variation. The sample size was "fair" according to the guidelines in Comrey and Lee (as cited in Pett et al., 2003, p. 48), resulting in a sample size to item ratio of 10 respondents per item.

Following the same procedure as the EFA1, an exploratory factor analysis using an oblique rotation was completed to identify the structure of the instrument. The MAP, PCA and PAF were examined with preference given to the results of the PCA means and percentiles and MAP squared fourth power techniques. The PCA and MAP techniques all provided evidence of a 3 factor structure. Three factors were extracted using a dimension reduction procedure was completed using all items and a principal axis factoring technique. Similar to the procedure followed in EFA1, Bartlett's test of sphericity ($\chi 2($351,280) =360.94, p < .001), KMO statistic (.883), the diagonals of the anti-image correlation matrix (all greater than .70) and a non-zero determinant of the correlation matrix, indicated that the sample was suitable for an exploratory factor analysis (Pett et al., 2003).

The resultant pattern matrix of the PAF solution with direct oblimin rotation is shown in Table 3 along with the structure matrix and the extraction communalities. The criteria used for identifying items on each factor was once again to retain items with pattern coefficients of .40 or greater on one factor and no more than .29 on other factors (McCoach, Gable, & Madura, 2013; Pett et al. 2003). Based upon these criteria, all items loaded on factors as hypothesized, an encouraging result indicating that the factor structure replicated successfully in a second sample (Osborne & Fitzpatrick, 2012). The communalities which represent the proportion of variance in the item explained by the factors once again indicated that the items with the strongest communalities loaded on Value. The total amount of variance explained by this factor structure was 42%.

Internal Consistency Reliability Analyses for EFA2 and Resulting Decisions

Although all items loaded on a single factor as hypothesized, some factor loadings were lower than our desired threshold of .40. Pattern and structure coefficients along with communalities are provided in Table 3.

Value scale. The hypothesized value factor contained 9 items, and the factor analysis extracted all 9 items on one factor with the coefficient alpha equal to .901, 95% CI=[883, .917]. Primary factor pattern coefficients ranged from .421 to .867, and no secondary factor coefficient was greater than .29.

Struggle scale. In order to strengthen the content coverage for the struggle scale, two items were written for inclusion in the EFA2 factor analysis. Item S9 was rewritten from ("Math teachers have difficulties sometimes when answering a math question") to ("Math teachers are sometimes stumped by a hard math problem") in an attempt to reflect having difficulties to the point of not being able to make progress to the solution. The pattern coefficient for this factor in EFA1 was .449, but it deteriorated in EFA2 to .395. There is no way to determine if this is due to the item rewording or the differences in the samples. Items S10 ("When someone struggles in math, it doesn't meant they have done something wrong") and S11 ("Making mistakes is necessary to get good at math") were new items added to EFA2. The pattern coefficient for S11 was at a satisfactory level of .533, but S10 had an unsatisfactory pattern coefficient of .309. In retrospect, we recognized that S10 contained multiple negative references which may have caused confusion among the respondents.

The hypothesized struggle factor for EFA2 contained 10 items, and the factor analysis extracted all 10 items on a single factor with the coefficient alpha equal to .788, 95% CI= [749,

.823]. Primary factor pattern coefficients for all items ranged from .309 to .612, and no secondary factor coefficient was greater than .30.

Growth scale. The hypothesized growth factor contained 8 items, and the factor analysis extracted all 8 items on one factor with the coefficient alpha equal to .819, 95% CI= [786, .849]. Primary pattern coefficients ranged from .469 to .681, and no secondary factor coefficient was greater than .30.

The replication of the factor structure from EFA1 in the EFA2 factor analysis provides us with evidence that the items tested represent three factors, which we have labeled value, struggle and growth. For all three scales, coefficient alpha ranged from .749 to .917. Alpha levels are considered large enough to use the instrument in research according to Clark and Watson's (1995) guidelines after accounting for sampling error.

Summary: EFA2

We decided to retain all the items tested in EFA2 for the confirmatory factor analysis; however items with low coefficients in EFA1 and EFA2 received closer scrutiny in the CFA step to determine whether they should be retained or removed to improve the theoretical meaning and fit of the final model. The resultant model from EFA2 contained three factors with 27 items, nine on value, ten on growth and eight on struggle.

Study 3: CFA

A CFA was employed using Amos 18.0.0 (Arbuckle, 2009; Thompson, 2010) on the third sample using the hypothesized three correlated factor structure with the same items and factor structure extracted from the EFA2 study, consisting of three factors with 27 items, eleven on value, ten on growth and six on struggle.

The CFA technique utilizes maximum likelihood estimation to confirm the factor structure identified in an EFA study, with the assumption that each item loads on a single factor and error variances are uncorrelated (Kline, 2011). When the model fits well, relatively high standardized factor loadings provide evidence of convergent validity, and relatively low correlations between the factors provide evidence of discriminant validity (Kline, 2011). The chi-square is not an optimal measure of model fit for a CFA since it is commonly significant when testing large samples, so alternate fit indices were used to measure degree of fit. The Tucker-Lewis Index (TLI) values above .90, comparative fit index (CFI) values above .90, and root mean-square-error-of-approximation (RMSEA) values below.08 were used as indicators of acceptable model fit (Netemeyer et al., 2003). When multiple nested models were compared, the chi-square difference test was used to identify the preferred model (Kline, 2011).

The CFA was completed to test the fit of the initial measurement model with items V1 through V9 on value, items S1, S3 through S11 on struggle, and items G1 through G8 on growth. Path coefficients were all statistically significant ($p<.05$), but the model fit statistics indicated room for improvement: $\chi 2$ (322, 290)=737.6, $p<.001$, RMSEA = .067, CFI=.88, and TLI=.87. Steps to improve model fit started with an examination of the regression weights and modification indices. For the growth factor, items G1 through G8 had satisfactory standardized regression weights of .54 to .73, but the error variance for G1 had a large modification index with both the value factor and the struggle factor. Combining this result with earlier evidence of a large secondary factor loading on value for G1 in EFA1 caused us to be concerned that G1 may be multidimensional. Therefore, we decided to remove G1 from the instrument. This iteration improved model fit, as can be seen by the significant Chi Square difference test ($\chi 2$ (49, 290)=148.6, $p<.001$) . For the value factor, items V1 through V9 all had standardized regression weights of .65 or higher except V6 ("People who are good at math have more opportunities than those who aren't good at math") which had a standardized regression coefficient of .28. Since V6 also had low pattern coefficients in EFA1 and EFA2, it was removed resulting in improved model fit ($\chi2$ (25, 290)=57.1, p<.001). For the struggle factor, items S8 and S11 had low standardized regression coefficients from .34 for item S8 ("People who are good at math may fail a hard math test") to .36 for item S10 ("When someone struggles in math, it doesn't mean they have done something wrong"). When S8 was removed, the model significantly improved ($\chi2$ (22, 290)=47.0, p<.001) , but when S10 was also removed it did not improve the model ($\chi2$ (23,290)=20.3, p=.061). In addition, when S8 was retained and S10 removed, this also did not improve model fit over the model with S8 and S10 ($\chi2$ (22, 290) =19.0, p=.064). The selected best fitting model contained all items except G1, V6, and S8.

The final measurement model contained 23 items: It met the target criteria for model fit, $\chi2$ (273, 290)=628.9, p<.001, RMSEA = .062, and CFI=.92, and TLI=.91. The covariances between the factors were all statistically significant (p<.05). The correlation between value and growth, growth and struggle, and value and struggle are .432, .244, and .403. These small to moderate and statistically significant correlations between the factors provided evidence of discriminant validity, that the three factors measure unique dimensions of the construct under investigation. Internal consistency reliability was sufficiently high for affective survey research based upon coefficient alpha, .942, 95% CI [.931, .952] for value, .726, 95% C.I.[.676, .771] for struggle, and .829, 95% C.I. [.80, .86] for growth. Standardized pattern coefficients, which were all statistically significant (p<.05) are presented in Table 4.

After analyzing response patterns from 3 samples using exploratory and confirmatory factor analysis, we wanted to compare scale scores across subpopulations to determine if there are differences. Utilizing the demographic questions used in the instrument and the combined sample from EFA2 and CFA, further validity evidence was sought by examining scale scores by demographic groups to see if there were differing levels of the traits (McCoach et al., 2013). We expected to see higher scores for students who have taken more mathematics, for students who are pursuing STEM careers, and for students who have high math self-efficacy. Student mean scale scores were compared on the demographic traits of gender, race (white vs. all other), major (STEM and medical related vs. all other), and highest level of mathematics coursework (Calculus or higher vs. lower than Calculus) and above or below average self-reported knowledge of math. Our expectation would be that students who favor mathematics as indicated by taking Calculus or higher, choosing a STEM related career or rating high in knowledge of mathematics would have higher scores on the three subscales of the MRS.

Descriptive statistics for the mean subscale scores for value, struggle and growth are shown in Table 5. The combined resilience mean scale score was also calculated for each group by averaging the value, struggle and growth mean scores. A series of ANOVA analyses were completed on the combined resilience scale mean to determine if there were statistically significant differences between groups. The scale means were significantly higher for STEM majors, $F(1, 571) = 20.187$, $p < .001$, and for students with Calculus or higher, $F(1,571)=6.597$, $p<.001$. There were no significant differences by gender, $F(1,571)=.196$, $p=.658$, or race, $F(1,571)=1.094$, $p=.296$. In addition, the mean scale scores were significantly higher for students whose self-reported knowledge of mathematics was above average than for those below average, $F(1,579)=71.995$, $p<.001$. These results provided further evidence of the validity of the MRS in measuring attitudes that are predictive of student persistence of mathematics.

Discussion

This research synthesized the literature on motivational constructs in academic environments, attitudes towards mathematics, and psychological resilience to develop a unified theory and an instrument which measures the constructs we hypothesize are predictive of mathematical resilience. Mathematical resilience has been defined as a positive response in the study of mathematics despite setbacks. Recently there has been a heightened interest in ways to improve participation and persistence of students in STEM majors in universities. The results of this study would be of interest to researchers in motivation and mathematics education, as well as to policy leaders seeking ways to improve representation in STEM careers for all students. The current research provides a first step towards improving our understanding of attitudes and beliefs which theory suggests are malleable and predictive of resilience in mathematics.

The evidence provided in this study supports the reliability and validity of the MRS as a measure of a construct defined by three correlated factors: value, struggle and growth. The factor labeled value consists of the belief that math is essential for future success. The items with the highest pattern coefficients all have some aspect of the belief that being good at math is important for future success. The factor labeled struggle consists of the belief that experiencing challenges and difficulties is a normal part of working on math. The items with the highest pattern coefficients reflect the belief that struggle is a normal part of working on math. Finally, growth refers to the belief in a growth theory of learning when it comes to math. The items with the highest coefficients reflect the belief that math can be learned by anyone and is not limited to those with a predisposition or “math gene”. We attempted to include a resilience factor, but the items in EFA1 were multidimensional, indicating to us that the attitude represented by responding with grit, stamina, or determination to stay with it when facing significant challenges was the overarching construct. Since our goal for this instrument was to measure attitudes and beliefs, it was eliminated as a separate factor. However, it may be the case that the remaining factors load on a second order factor, providing further construct validity evidence for a single overarching construct. Future research will focus on investigation of the higher order or bi-factor model of mathematical resilience. Finally, the struggle subscale had lower than targeted standardized regression coefficients and coefficient alpha. Nonetheless, the content on this subscale is an important component of mathematical resilience and should be retained. In future studies, the items in the struggle subscale may benefit from wording revisions.

This instrument will be useful in future research on whether the triad of motivational factors of value, struggle, and growth is a significant predictor of mathematical resilience and what interventions would increase these factors. In order to test if the attitudes measured are predictive of resilience in mathematics, further study is needed. First we would need to operationalize resilience. Measures of persistence would be useful, but resilience is something more than just persistence. Students who successfully pursue rigorous mathematics coursework in college know how to respond and cope during periods of struggle in a math course. They are not just persisting; they are persisting while experiencing substantial challenges. In addition, the relationship between the MRS and other affective variables such as mathematical anxiety, mathematical self-efficacy, and other motivation variables such as those reported in Eccles (1983) would provide an opportunity to study the convergent and discriminant validity of the MRS and expand upon theory of the determinants of and relationships between different attitudes towards mathematics.

Limitations of These Studies

These results had a number of limitations in the realms of generalizability and the strength of factor loadings. The samples used in this study were convenience samples and thus limit the generalizability of the results. Although the members of the sample used for EFA1 came from multiple institutions of higher education, it contained a large proportion of respondents who were professional insurance actuaries. The samples used in EFA2 and the CFA were taken from a single university and included only students in one of eight large lecture halls. Furthermore, the MRS has only been validated on college and university students and adults, so we would suggest limiting its applicability to students entering college or older. Nonetheless, the theory developed herein may, with small adjustments, be just as relevant for younger students to persist in the study of mathematics in spite of difficulties. An area of future research would include conducting a validity study on students of other age groups.

Finally, although this study provides a theoretical basis for the connection between these three factors and outcomes that are indicative of mathematical resilience, no empirical evidence representing outcomes was presented. Future research in this area could lead to improvements in our understanding of why some students continue to study mathematics while others do not, a key component towards the goal of increasing representation and participation in STEM majors.

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