A nodes model for creativity in mathematics education

Introduction
This paper sets out a proposition for a model of understanding creativity in the mathematics classroom. It aims to show that creativity in mathematics can be conceptualised as the mental processes involved in ‘meaning making’ in the mathematics classroom. It is currently primarily a conceptual exploration, with only preliminary empirical investigation, however it is hoped that this is to be addressed in the coming academic year. The paper comes from more formal conceptual development of an informal discussion about what creativity in mathematics actually is between the author and a number of researchers at University of Durham. A variety of ideas were put forward, and while there was little consensus on what creativity in mathematics is, there was agreement on what creativity in mathematics is not – creativity in mathematics is not about making classroom mathematics ‘shiny’. While colourful displays and beautifully drawn graphs were useful in themselves, this did not, the group concluded, constitute creativity in mathematics. Similarly, while setting project work could be considered creativity in mathematics it wasn’t necessarily. It was agreed that there was something more fundamental about what creativity in mathematics may be. This nodes model takes one of the ideas discussed and is an attempt to describe what this ‘more fundamental’ aspect may be.

Ontological and epistemological underpinning
During the informal discussion that led to the formulation of this paper, what became apparent were the numerous different contexts that were being discussed – were we discussing the mathematics itself, classroom mathematics (as distinct from, say, research mathematics), mathematics education, mathematics educating, mathematics educators and so on. It is important then to decide from the outset what exactly is being discussed, as each will have its own implications on ontology and epistemology and, as a result, the whole model.

For this model, the discussion will be based around what students do in their mathematics. We start then, ontologically, with an ontological realist approach to mathematics, such as a restricted Platonist (Bernays, 1964), that is, there is an abstract, objective ‘field’ of mathematics, of which we build a picture through our learning mathematics. Regardless of how we may build up this picture (which may be subjective), at the level of the classroom, mathematics is a static, objective body of knowledge. As the focus is very much on the individual child within this objective field of
mathematics, however, we need to focus on an ontological relativist approach to creativity, that is, the ‘truth’ of creativity is entirely subjective and held by the child involved alone. Epistemologically, the restricted Platonist view would suggest that the mathematics learned is either correct or incorrect, whereas creativity is an entirely subjective concept that cannot have any ‘right’ or ‘wrong’ answer or definition.

In this vein, then, learning mathematics in this context can be described as “the process of discovering [the] pre-existing relationships” of mathematics (Ernest, 1991, p. 29) and whilst creativity is notoriously difficult to define (Houtz, 2003), it is tentatively interpreted here as “the novel and personally meaningful interpretations of experiences, actions and events.” (Beghetto & Kaufman, 2007b, p. 73).

Understanding, meaning making and a diagrammatic representation

To start, it is important to establish what is meant here by ‘understanding’. The term ‘understanding’ has numerous definitions and interpretations and is researched from all fields of educational research and beyond. It can be interpreted as a momentary act, as a mental process, as a biological process and so on. An outline of what is interpreted as understanding in the mathematics classroom in this model is required.

Developing understanding is interpreted here as the “making [of] cognitive connections between some new material or some new experience and our existing ideas.” (Haylock & Thangata, 2007, p. 109). This definition is similar to Piaget’s assimilation/accommodation model (e.g. Piaget, 1953), whereby children encounter and take on new experiences and knowledge and relate it to their previous experiences in mathematics. Thus ‘understanding’ is taken to mean children’s ‘meaning making’ in the mathematics classroom. Haylock presents his model for understanding in his books aimed at practitioners (e.g. Haylock, 1978; Haylock & Cockburn, 2008; Haylock & Thangata, 2007). His model attempts to explain not the underlying build up of neuron pathways, but to provide “a simple model that enables us to talk about understanding in mathematics” (Haylock & Cockburn, 2008, p. 9) and as an explanation of what happens in the classroom. It is for this reason that this representation of understanding as ‘making connections’ has been used in the nodes model for creativity in mathematics. As a classroom-based model, the nodes model for creativity aims to explore what creativity in mathematics is, not as an abstract concept but as what happens in the classroom. Haylock’s model for understanding is precisely this. Furthermore, Haylock’s interpretation appears to work well with the original ontological and epistemological premises above of mathematics being a static body of knowledge (in which children
construct networks of connections) and in which there is a correct and incorrect answer. It was for these reasons that Haylock’s interpretation of understanding was used as the basis for this model for creativity in mathematics.

Haylock provides a diagrammatic representation of his model of understanding (e.g. Haylock, 1997; Haylock & Cockburn, 2008; Haylock & Thangata, 2007), and it is this that is used here as the basis for the nodes model for creativity. He suggests that mathematical understanding in children can be seen as a network of connections between ideas building on Ginsburg’s idea of understanding as the connection between two bits of information (e.g. Ginsburg, 1983). Haylock suggests that, with very young children, for example, their understanding of mathematics increases as they create links between language, pictures, symbols and concrete situations (Figure 1).

![Diagram of Haylock's model of understanding]

(Figure 1, from Haylock & Cockburn, 2008)

Haylock suggests that “our understanding of complex mathematical concepts... can be conceived of gradually constructing networks of connections” (Haylock & Thangata, 2007, p. 109), with the implication that the more connections within the network, the greater understanding a child has of the mathematics and thus it is the role of the teacher to aid the building of these connections in the mathematics classroom.

While Haylock’s interpretation of understanding was chosen due to its simplicity and clarity, to delve deeper into a nodes model for creativity, we must look to develop this, for as will be discussed below, creativity is no simple matter. To further develop Haylock’s simple representation, we can look to psychology of mathematics education work, to the work of, for example, Skemp (e.g. 1987; 1989) who describes the idea of ‘concept mapping’. Skemp originally proposed two uses for concept maps in the classroom. Firstly, he suggested that they have use for teaching planning. He suggested that there is a hierarchical nature to mathematics, that is, some parts of mathematical knowledge that are more important than others (lower order) and that these need to be learnt first, before the more complicated, higher order mathematics. Teachers would therefore be able to use concept maps in order to identify the areas of
lower and higher order mathematics and as such plan their teaching around this. Similarly to Haylock et al., he suggests that the number of connections is important for understanding in mathematics, especially when considering higher order mathematics, which require connections with all of the lower order concepts to be fully understood (Skemp, 1989). Skemp also suggested that concept maps have diagnostic uses for children. By drawing their own concept maps of mathematics, Skemp suggested, it may be possible to identify mathematical misconceptions, which he suggested may “lie further back” in lower order concepts (1987, p. 122).

Here, however, we can identify a third use. Rather than being an explicit, physical tool for planning or diagnostic purposes, we can use it as a conceptual tool for mapping the ontological field of mathematics. From this idea, we can outline a model where nodes represent mathematical concepts and vertices between them represent the connections discussed by Haylock et al. It is the number of connections within the network that shows the strength of understanding, and thus meaning making, as the construction of connection(s) between the nodes on the map, leads to a greater understanding.¹

Looking at these two models then presents us with a starting point for constructing a model to describe creativity in mathematics, as shown in slide 3 (see attachment).

**Creativity**

As was noted above, Haylock suggests that “the development of understanding involves making cognitive connections” (Haylock & Thangata, 2007, p. 109) between new information and our existing mathematical knowledge. What neither Haylock et al. nor Skemp do, however, is discuss how these new connections may be added to the model. This, then, needs to be addressed if we are to use the nodes model as a way of understanding meaning making in mathematics. Firstly, it will be useful to find a description of this process of making new connections, before examining it in further detail.

This ‘making of a connection’ is described by Mason (2000) as “a moment of insight” (p. 2) in which a “connection between previously disparate ideas” in mathematics can be made (p. 7). He describes this ‘moment of insight’ as a “creative moment” (p. 2) for the child. It is proposed in this nodes model then, that it is creativity, in the form of

¹ Barmby et al. (2009) also use a similar ‘nodes’ type approach, referencing Skemp, which they describe as an ‘array representation’. Dr. Patrick Barmby at University of Durham was involved in the initial discussion that led to the formation of this paper, so my thanks go to him for planting an initial seed.
‘creative moments’ that leads to the construction of connections between nodes as part of meaning making. Here we can see ‘something more fundamental’ (as opposed to creativity as ‘shiny’) about creativity in mathematics. The discussion of definition(s) of creativity may be long running, however Mason’s interpretation of creativity as “creative moments” in the classroom appears to be especially useful. Firstly, Mason provides an example of creativity as specific to mathematics – rather than concerned with qualities of people or products, as in other subjects, Mason suggests that creativity in mathematics is distinct as it is concerned with these ‘moments of insight’. Secondly, using Mason’s ‘creative moment’ leaves us with a description of the construction of the new connection between nodes. What is needed now is a more detailed analysis of this ‘creative moment’.

‘Little-c creativity’ has been proposed as the ‘type’ of creativity that might be seen in the classroom (Craft, 2001). It is distinct from ‘Big-C creativity’, the historical or widely recognised feats of creativity as is interpreted as less exclusive. However, Beghetto and Kaufman suggest that, in accounting for ‘everything that is not Big-C creativity’, little-c creativity is too wide a category. They propose a further ‘category’ of creativity in addition to Big-C and little-c creativity, which they call ‘mini-c creativity’ (2007a, 2007b). Beghetto and Kaufman described this mini-c creativity as an “addition unit of analysis” (2007b, p. 78) in the discussion of creativity. It is this ‘unit of analysis’ that will be used to explore Mason’s ‘creative moment’.

The underlying premise of mini-c creativity is that creativity is fundamental to the process of meaning making. It is defined as “the novel and personally meaningful interpretation of experiences, actions and events” (Beghetto & Kaufman, 2007b, p. 73) and it is this interpretation that leads to the construction of knowledge. Just as Mason (2000) suggests creativity in mathematics refers to ‘moments’ rather than properties of people or tasks, Beghetto and Kaufman suggest that mini-c creativity refers not to wider visible outcomes, as might be seen in ‘Big-C’ or ‘little-c’ creativity but to “the individual creative processes involved in student knowledge construction and development” (Beghetto & Kaufman, 2007b, p. 78). It is their suggestion that creativity is at the centre of learning and it this mini-c creativity that is being used at the centre of the nodes model for creativity in mathematics. This mini-c creativity, as ‘meaningful interpretations’ is used in this model as a explanation as to how the connections can be made between nodes on the model, during the ‘creative moment’ that Mason (2000) discusses.
It is important to note that Beghetto and Kaufman do not propose this concept to equate creativity and learning. What they do suggest, however, is that ‘meaning making’ has its “genesis in mini-c creativity interpretations.” (2007b, p. 73). Lipman (2003), too, suggests that “responsible interpretation” is key in “the production of meaning” (p. 211). Thus for Beghetto and Kaufman, mini-c creativity is seen as the root foundations of knowledge construction – transferred to the nodes model, we can say that these ‘mini-c creativity interpretations’ are at the very root of the construction of connections between nodes and can help explain how these connections might be made.

The emphasis in mini-c creativity is very much on personal meaningfulness of the experiences. Interpretations can only be personal, and it is “personal knowledge and understanding” (Beghetto & Kaufman, 2007b, p. 73) that is the focus, while Mason (2000) too, highlights that a ‘creative moment’ can be “a moment of insight which is personally novel and original” (p. 2). This highlights the relativist ontology of the model, that the ‘truth’ of creativity is determined by the child, in contrast to the realist ontology taken to mathematics whereby mathematics is objectively defined. This nodes model suggests that creativity, as part of meaning making, is not an objective notion. Consequently, the implication is that the process of ‘meaning making’ is also not objective. The creative process of meaning making is an individual one through the objective world of mathematics.

**Critical thinking**

A regular feature in definitions of creativity is the necessity of ‘originality’ and of ‘value’ (Sternberg, 2006). What is important in creativity is not solely an outcome but also the qualities of this outcome. It was proposed above that it was mini-c creativity that was involved in the process of establishing connections between mathematical concepts, or ‘meaning making’. However, not all of the connections made are likely to have similar qualities. There therefore needs to be some way of establishing this originality and value for the child. It is proposed, in this model, that it is ‘critical thinking’ that is the tool which provides the evaluative role. While ‘critical thinking’ is a notoriously difficult to define (Kennedy, Fisher, & Ennis, 1991), Dewey’s ‘reflective thinking’ approach or Ennis’ “reflective thinking that is focused on deciding what to believe or do.” (Ennis, 1985, p. 46) and Lipman’s critical thinking as “judgements” (Lipman, 2003, p. 209) are both useful for this approach. The processes involved in mini-c creativity are likely to produce a number of alternative and potential connections for the child. Through ‘reflective thinking’, based on ‘deciding what to believe or do’ a child will be able to judge which of these connections are of ‘value’ and subsequently, which connections are most
useful in his or her own 'meaning making’. This critical thinking then allows for the child to build on those connections ‘of value’ to then reinforce these connections, or, as Bruner’s would put it, make “qualitatively different not quantitatively improved” (2006, p. 99) connections between the nodes. Bruner here was discussing the ‘bonds’ between knowledge, however the analogy seems appropriate here. If we consider Haylock’s example in his model of understanding as the connections between language, pictures, symbols and concrete situations (Figure 1), it is perhaps disingenuous to suggest that the link between language and pictures is quantifiably similar or comparable to the link between, say, pictures and concrete situations. Similarly then, it may be inappropriate to suggest that connections on the nodes model change quantitatively rather than qualitatively.

An emphasis on critical thinking again reinforces the relativist ontology when considering creativity. Whilst the mathematics of a primary school child is unlikely to be novel to the teacher (given the stated supposition of relative Platonism,) it may well be to the student, and as it is the child who is the one involved in the personal ‘meaning making’, it is the child who is the one to establish value. It is important to note too, that in taking the evaluative role, it is dependent upon the child as to whether, say, connections that may be reinforcing old connections, are not original, but still of value (as increasing understanding) or that "a new way of thinking about the same thing” is both original and of value to the child. As such, the relativist ontology places the child firmly as the sole arbiter of what constitutes creativity. Indeed, Beghetto and Kaufman state that “we recognize, not all aspects of learning involve creativity” (2007b, p. 76). There is the implication here that there requires some way of distinguishing what aspects do and which do not. Again, the relativist ontology suggests that the child is the arbiter here.

Critical, reflective thinking is essential to evaluate these new potential connections to establish which of these are original and of value, which are useful to the child’s meaning making and to establish what may be considered personal creativity in the mathematics classroom. It is therefore placed as the final stage of the nodes model.

**A nodes model for creativity in mathematics**

We can then bring together each of these ideas to give what is described as a nodes model for describing creativity in mathematics.

Firstly, using the ideas of Haylock et al. (e.g. Haylock, 1978; Haylock & Cockburn, 2008; Haylock & Thangata, 2007) and Skemp (e.g. 1987, 1989), we can present a plan
whereby children’s understanding can be represented diagrammatically. ‘Nodes’ on the diagram represent mathematical concepts or ideas within the ontological realist field of ‘mathematics’, while vertices between these nodes represent the connections discussed by Haylock et al. and Skemp. What this representation does not do (perhaps intentionally) is suggest how children’s understanding grows, or, put diagrammatically, how these vertices are added.

The nodes model for creativity proposes that it is in this ‘adding of vertices’ during what has been described as ‘meaning making’ that creativity occurs in the mathematics classroom. Mason (2000) suggests that creativity in mathematics can be described as ‘creative moments’ or moments of insight, and are central to (and a feature of) creativity in mathematics. These ‘moments of insight’ can be explained in more detail using Beghetto and Kaufman’s ‘mini-c creativity’ (2007a, 2007b). They describe mini-c creativity as “the novel and personally meaningful interpretation of experiences, actions and events” (2007b, p. 73), and it is this that is proposed here as being central to this ‘adding of vertices’ or creation of connections in the nodes model. The processes of mini-c creativity described by Beghetto and Kaufman play a vital role in ‘meaning making’.

Finally, creativity in whatever form needs some form of evaluative process. This process of mini-c creativity – the “process of novel and personally meaningful interpretation of experiences, actions and events” (Beghetto & Kaufman, 2007b, p. 73) – in this situation may well produce many different potential new connections, not all of which may be appropriate. The ‘value’ of each potential new connection needs to be established – without value, a new idea is merely novel. It is proposed therefore that the process of ‘critical thinking’, the “reasonable, reflective thinking that is focused on deciding what to believe or do” (Ennis, 1985, p. 46) focusing on both deciding which of the potential new connections is the most appropriate and how to qualitatively strengthen that connection as part of ‘meaning making’ in mathematics, that is the final part of the nodes model for creativity in mathematics.

Slide 6 (see attachment), pulls together each of the ideas and shows the completed nodes model for creativity in mathematics.

**Empirical work**

The empirical work was undertaken in the summer term of the 2008-09 academic year in a Year 4 class. Empirical work took an interpretive approach to both data collection
and analysis of the qualitative data. On a number of different visits, the whole class was observed, group work and individual work was video recorded. The aim of filming the class from a number of different perspectives (i.e. whole class, group work and individual) was to attempt to gain a view of the classroom environment as a whole in order to contextualise the data set as a whole.

Data analysis of the video data appeared, unfortunately, to offer little toward the model, possibly due to the activities videoed. A short one-to-one interview was also carried out with one child. This was far more productive, as it allowed for a more direct interaction with the individual child, which the videos appeared unable to do, and allowed the researcher to follow particular routes and interests. This interview followed up a discussion during class time between the child and the teacher about how the child ‘came to know’ a particular topic in mathematics earlier in the academic term. The classroom discussion revolved around the teacher eliciting an explanation from the child about how he learnt the process of multiplication as a possible way of helping other children in the class.

In the interview, the child suggested that he “stumbled along” before he “just suddenly... grasped [multiplication]” without quite knowing how. The child suggested that practise and repetition was involved, and that after this “grasping” moment, it went from “just, like, too hard” to a situation where “I can't forget it.” Analysing this with the nodes model in mind, we can suggest that this ‘grasping’ moment is where the child makes the connection between new and existing knowledge (as discussed by Skemp.) This example then needs to be examined with mini-c creativity in mind as to whether this ‘grasping’ moment is equivalent to a moment of mini-c creativity, and what, specifically, “the creative processes involved in the construction of personal knowledge and understanding” (Beghetto & Kaufman, 2007b, p. 73) are and whether they can indeed be observed. It may be appropriate to discuss this as a ‘moment of insight’ or a ‘creative moment’ as described by Mason (2000), however, it is difficult from this particular example to identify what these ‘creative processes’ may be in this situation and consequently what role mini-c creativity may or may not have had in this situation.

Due to time constraints the data set was small and therefore it is hard to draw any wide-reaching conclusions. This was, of course, anticipated, and thus the aim was therefore not to attempt to empirically validate the model (or otherwise) but to explore what empirical work could tell us about the model, or indeed, what the model could tell is about the empirical observations. The empirical work would also identify what may be needed for the future. The empirical work did provide some interesting routes to follow.
up in further empirical work. Firstly, this unknowing, ‘grasping’ notion described by the child appears similar to that of the ‘aha!’ experience, or “effective surprise” described by (amongst others) Bruner (1962), so the role that this plays in the model may be worth investigating, both from a literary, conceptual point and from an empirical point of view. As noted above, it was difficult to ascertain what mini-c creativity processes might have occurred in this ‘stumbling and grasping’ process so a more thorough, perhaps longitudinal study may be appropriate in investigating this. The role of what Beghetto (2009) describes as ‘micromoments’ in the classroom – “fleeting, easy-to-miss classroom interactions and experiences” (p. 2) between students and teachers – may also be worth considering. How these interactions effect (and affect) creativity and meaning making in the (mathematics) classroom and the role of these in this nodes model is certainly something to be investigated in future empirical study.

Conclusion

This nodes model, then, pulls together a number of different concepts from the literature in an attempt to establish an understanding of what creativity in the mathematics classroom may be. It proposes that not only is creativity not something ‘shiny’ and purely aesthetics-based, but it is in fact something far more fundamental to children’s meaning making in mathematics. The model does not intend to re-brand ‘meaning making’ as ‘creativity in mathematics’, nor does it propose an entirely new way of child learning. What it does propose is that creativity is involved in the process of meaning making, and that this should be recognised.

As such, fostering creativity is the mathematics classroom is something that should not only be encouraged but is in fact essential to meaning making in mathematics. Of course, Sternberg and Lubart’s (1999) argument of “view[ing] a part of the whole as the whole” (p. 9) in creativity research is noted and it is acknowledged that there are many other possible interpretations of creativity in the mathematics classroom, and indeed this may be only one of a number of different ‘types’ of creativity (as with ‘Little C Creativity’ and ‘mini c creativity’) that may be present. This ‘meaning making’ approach to creativity may be especially useful for classroom-based educational research as it may allow us to provide an initial conceptualisation of the potential processes involved in creativity in classroom mathematics.

What this model does not do, nor does it intend to do, is provide a map or a ‘how to’ for creativity in mathematics pedagogy or teaching. This is must be the topic of much further debate, hopefully a debate to which this paper can contribute.
A number of questions surfaced from this research. Perhaps two of the most pertinent might be;

- What role does creativity play in meaning making? Is this different in mathematics than in other subjects? Is this model domain specific (especially with regards to ontology)?
- Is this model a useful way of conceptualising creativity in mathematics? If so, how can it be developed, if not, what might be a better approach?
Bibliography


