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Mathematics summer schools for acoustics research training

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Mathematical methods are important for research in many aspects of acoustics. Most researchers in acoustics in the United Kingdom do not have access to master level courses to broaden their postgraduate study, so they advance their fundamental mathematical methodologies taught at undergraduate level through independent learning. They develop their mathematical skills as appropriate rather than being made aware of the potential of advanced mathematical tools at the onset of their research career. Attempts to improve this situation were made through summer schools held in 2003 and 2005 at Southampton University and in 2007 at Salford University. The background to these Summer Schools, their content and structure, recruitment figures and student feedback are reported together with conclusions about their performance and role particularly in respect of PhD completion.

I. BACKGROUND

In the United Kingdom, the arrival of the new millennium was accompanied by considerable concern about the state of the mathematical education of engineers. A survey of the U.K. engineering community in autumn 2001 highlighted the need to increase the general level of mathematical competencies of U.K. postgraduate engineers. This resulted in several strands of funding intended to improve the situation. For example, in 2002 “Funding for the Development of Teaching and Learning (FDTL Phase 4)” supported the “Helping Engineers Learn Mathematics” (HELM) project based at Loughborough University.1 This resulted in 50 workbooks, each approximately 50 pages long, containing learning materials, worked examples and case studies of applications of mathematics in engineering (including a few acoustical ones). Illustrative web-delivered computer aided learning (CAL) courseware on many of the first 20 workbooks are provided to support and enhance the content of the workbooks and either web-delivered or CD-based computer aided assessment.

Also in 2002, the U.K. Engineering and Physical Sciences Research Council initiated a “Maths for Engineers Summer Schools” pilot study. Proposals were sought for summer schools that would give an appreciation of contemporary mathematical techniques to Ph.D. students in core areas of engineering research. Applicants were asked to identify an engineering theme and to develop a summer school program suitable for the purpose and audience.

The mathematical demands of acoustics research are wide: ranging from solution of partial differential equations to the methods of digital signal processing. Some of the mathematical basics are covered by undergraduate mathematics for engineers, but many important topics such as Green’s function methods, asymptotic methods, integral transforms, and variational calculus lie outside the typical engineering mathematics syllabus.

Given the importance of mathematical methods to research in many aspects of acoustics, the opportunity was taken, following a meeting of potential contributors, to submit a proposal for an EPSRC-funded pilot study to address the mathematical needs of U.K. post-graduate researchers in engineering acoustics and physical acoustics. The proposal was for financial support for an intensive but collegial residential week for up to 40 researchers, mainly graduate students drawn from across the U.K., with a high staff/student ratio and with the aim of introducing important mathematical techniques in acoustical contexts.

Funds were requested to cover:

(1) Honoraria for the lecturers and tutors.
(2) The full subsistence and accommodation costs of at least 30 students based at U.K. universities and up to 10 staff (using student accommodation).
(3) Travel costs (organization, lecturers, and tutors).
(4) Hire of summer school venue and technical equipment.
(5) Administration costs including preparation of course material, posters, publicity, and mailing.
The proposal was successful and the first summer school “Support Mathematics for Acoustical Research Training” (SMART-1) was held at the University of Southampton in July 2003. The only other pilot School funded by the EPSRC in 2002 was for process engineering.

Although the planning time available between the funding announcement in March 2003 and the first such school in July 2003 was rather short, considerable interest and enthusiasm was generated by SMART-1. Consequently, in response to further EPSRC calls in 2004 and 2006, second and third versions were proposed, and the proposals were successful. SMART-2 and SMART-3 were held in 2005 and 2007 at Southampton and Salford Universities, respectively. Each School was planned through meetings held earlier in each of these years. The courses were publicized widely. Figure 1 shows the flyer for the 2007 school.

II. SYLLABUS AND STRUCTURE

In the U.K., most students embarking on postgraduate studies in acoustics have attended the mathematics modules commonly found in engineering degree programs and should be familiar with the topics listed in Table I.

Nevertheless it was thought to be likely that many would welcome revision of at least some of these topics including ordinary and partial differential equations, vector

<table>
<thead>
<tr>
<th>TABLE I. Prerequisite topics.</th>
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<tbody>
<tr>
<td>Differentiation and integration</td>
</tr>
<tr>
<td>Basic definition of rate of change and slope of graph</td>
</tr>
<tr>
<td>Derivatives of simple algebraic functions, polynomials, exponentials, logarithms and trigonometric functions</td>
</tr>
<tr>
<td>Rules including product, quotient and function-of-a-function</td>
</tr>
<tr>
<td>Maxima and minima</td>
</tr>
<tr>
<td>Integration as the limit of a sum</td>
</tr>
<tr>
<td>Integration as the reverse of differentiation</td>
</tr>
<tr>
<td>Indefinite and definite integrals</td>
</tr>
<tr>
<td>Numerical integration including rectangular, trapezium and Simpson’s rules</td>
</tr>
<tr>
<td>First order ordinary differential equations and linear second order with constant coefficients, solution methods including integrating factor</td>
</tr>
<tr>
<td>Complex numbers</td>
</tr>
<tr>
<td>Definition of imaginary unit</td>
</tr>
<tr>
<td>Argand diagram</td>
</tr>
<tr>
<td>Complex conjugate and modulus</td>
</tr>
<tr>
<td>Addition and multiplication of complex numbers</td>
</tr>
<tr>
<td>Polar representation including De Moivre’s theorem</td>
</tr>
<tr>
<td>Complex roots and functions</td>
</tr>
<tr>
<td>Matrices, determinants, and vectors</td>
</tr>
<tr>
<td>Addition and multiplication and inverse</td>
</tr>
<tr>
<td>Matrix representation of systems of equations</td>
</tr>
<tr>
<td>Vector algebra including vector addition and scalar and vector products</td>
</tr>
<tr>
<td>Intersection of lines and planes</td>
</tr>
<tr>
<td>Vectors in three dimensions</td>
</tr>
</tbody>
</table>

FIG. 1. (Color online) Publicity flyer for SMART 3.
and matrix algebra, calculus, complex algebra, and Fourier analysis. These topics provided most of the content in the first 2 days. The remaining course content was decided by balancing several factors:

1. Topics considered important for current research in acoustics.
2. Subjects that could be introduced in the time available.
3. Subjects that were appropriate to particular acoustical themes.
4. Availability of tutors.

Three lectures per day were interspersed with appropriate tutorials and opportunities for individual one-to-one "surgeries" with contributing and supporting lecturers. More advanced topics were chosen from the mathematics needed for general acoustics (complex analysis, Green's theorem, asymptotic methods in diffraction theory, waveguides), aero-acoustics (generalized functions, asymptotic methods), and signal processing (integral transforms, iterative methods). In addition to this, students were introduced to the mathematical foundations of widely used numerical techniques such as finite element method, boundary element method, and ray tracing. The purpose of this was to give attendees a clear idea of the capabilities and the limitations of the numerical tools they were likely to use for their research projects. The more advanced sessions were organized by colleagues from the mathematics and acoustics departments at Cambridge, ISVR, Keele, Loughborough, and Salford. Each contributing department was responsible for a specific day or half day.

The summer school week occupied the weekdays from Monday through Friday preceded by registration on Sunday night. Table II shows the structure of the 2007 school week.

Figures 2–4 show examples of tutorial problems for the sessions on vector calculus, complex variables, and waveguides.

Supplementary lectures about general mathematical methods and "guest" lectures on more specialized topics were intended to give attendees an introduction to a wide range of applications of mathematics in acoustics and to the history of mathematical acoustics, one of which has formed the basis for a historical article. Table III lists "general" and "specialist" lecture topics in 2003, 2005, and 2007. Lecturers and their institutions are listed in Table IV.

Attendees in 2005 and 2007 were fortunate to receive guest lectures from prominent U.S. academics. As well as giving guest lectures, Allan Pierce and Peter Monk

<table>
<thead>
<tr>
<th>Day 1</th>
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</thead>
<tbody>
<tr>
<td>Vector calculus</td>
<td>Wave guides</td>
<td>Lecture</td>
<td>Finite elements 1</td>
<td>Aero-acoustics 1</td>
</tr>
<tr>
<td>Complex variables</td>
<td>Mode matching</td>
<td>Signal processing principles</td>
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<tr>
<td>Tutorial</td>
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<tr>
<td>Tutorial</td>
<td>Lecture</td>
<td>Finite elements 2</td>
<td>Tutorial</td>
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<tr>
<td>Tutorial</td>
<td>Lecture</td>
<td>Aero-acoustics 1</td>
<td>Tutorial</td>
<td>Lecture</td>
</tr>
</tbody>
</table>

FIG. 2. Example tutorial problem on vector calculus.

Question 1.1 Suppose \( r = x\hat{i} + y\hat{j} + z\hat{k} = \overrightarrow{OP} \) is the position vector of a general point \( P \) relative to a given origin \( O \). If \( r = |r| = \sqrt{x^2 + y^2 + z^2} \) confirm that

\[
\nabla \cdot r = 3, \quad \nabla \times r = 0,
\]

Use the above results, together with particular vector identities, to evaluate

\[
(i) \quad \nabla \cdot [f(r)], \quad (ii) \quad \nabla \cdot [f(r)r],
\]

\[
(iii) \quad \nabla \times [f(r)r], \quad (iv) \quad \nabla^2 [f(r)],
\]

where \( f \) is an arbitrary function of \( r \). Hence, for \( n \) integer, obtain

\[
\nabla [r^n], \quad \nabla [\ln r], \quad \nabla \cdot [r^n r], \quad \nabla^2 [r^n], \quad \nabla^2 [\ln r].
\]

Question 1.2 Using cylindrical polar coordinates, namely

\[
x = R\cos \phi, \quad y = R\sin \phi, \quad z = z,
\]

find, starting from their (multiple) integral definitions,

(i) the curved surface area \( A \) of the cylinder \( S \): \( x^2 + y^2 = a^2, \quad 0 \leq z \leq H; \)

(ii) the volume \( V \) of the region \( \Sigma \): \( x^2 + y^2 \leq a^2, \quad 0 \leq z \leq H \) within the cylinder.

Confirm that the evaluations of the double and triple integrals match the expected results.

Question 2.1 By using Cauchy’s residue theorem, evaluate
\[ I = \oint_{C:|z|=1} z^n e^z \, dz, \]
for integer \( n > 0 \).

Question 2.2 By using Cauchy’s residue theorem, evaluate
\( a \quad I_1 = \oint_{C:|z|=\frac{1}{2}} \frac{z + 2}{z(z + 1)^2} \, dz, \quad a \quad I_2 = \oint_{C:|z|=2} \frac{z + 2}{z(z + 1)^2} \, dz. \)

Question 2.3 Suppose that rational function \( f(z) \) has a simple pole at \( z = z_0 \), such that
\[ f(z) = \frac{g(z)}{h(z)} \quad \text{where} \quad h(z_0) = 0, \quad g(z_0) \neq 0, \]
where \( g \) and \( h \) are polynomial functions. By writing
\[ h(z) = (z - z_0)R(z) \quad \text{where} \quad R(z_0) \neq 0, \]
show that the residue of \( f \) at \( z = z_0 \) is given by
\[ \text{Res} \{f(z)\}_{z=z_0} = \frac{g(z_0)}{h'(z_0)}. \]

Find the four simple poles \( z = z_n \) for \( n = 1, 2, 3, 4 \) of the rational function
\[ f(z) = \frac{z^2}{z^4 + 1} \]
and hence show that
\[ \text{Res} \{f(z)\}_{z=z_n} = \frac{1}{4z_n}. \]


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FIG. 3. Example tutorial problems on complex variables.

FIG. 4. Example tutorial problem on waveguides.

Q1. Calculate the modes for a rigid rectangular guide with cross-section
\[-a < z < a, \quad -b < y < b,\]
the guide being aligned with the \( x \)-axis.

- Write down the Helmholtz equation in 3D Cartesian coordinates.
- Let \( \Phi(x, y, z) = X(x)Y(y)Z(z) \).
- Substitute this into the Helmholtz equation and rearrange so as to introduce two separation constants.
- Apply the boundary conditions on the guide walls.
- Calculate the expression for a general mode in the guide.
- What are the cut-off frequencies?
III. RECRUITMENT

The numbers of registrants and their institutions are listed in Table VI. Attendance at the 2003 and 2005 schools was bolstered by the large cohorts of graduate students and young researchers in Southampton. Relatively few researchers from Southampton attended the 2007 school in Salford. On the other hand, there were 11 attendees from Salford at this School. The institutions described as “other” in Table VI include the Universities of York, Hull, Bradford, Sheffield, Reading, Brighton, Surrey, and London and research organizations such as the National Physical Laboratory.

IV. FEEDBACK

Feedback questionnaires were distributed and collected among the attendees in 2003 and 2005. The questions related to the course outcomes, the level of material, course content, and lecture quality. Table VII lists some responses to a question concerning the appropriateness of the summer school.

The 2003 school included a focus group discussion conducted as part of a research project into mathematical communications. Responses to the question “What one thing would you pick out as the best thing at this event?” are summarized under the various components as follows.

1. Surgeries and tutorials.
   (a) The tutorials and coffee time talks with staff and other students, give good insights.
   (b) The surgeries.
   (c) Good tutorial staff who go over the problems in small groups.
   (d) The surgeries and tutorials are an opportunity to go over points that you did not understand with other staff.

   (a) MATLAB functions and scripts that can be taken away and used.
   (b) Information and material to take away.
   (c) The course material is most valuable resource.
   (d) References given on the handouts/slides.

3. Lectures and lecturers.
   (a) Lectures motivated you to investigate more difficult areas that you would not have been able to teach yourself.
   (b) Lecturers coming from the different views and approaches.

4. Networking and sharing.
   (a) Sharing problems with so many individuals from the same field.
   (b) Opportunity to do networking with 40+ people in the same field.

5. Inspiration.
   (a) Inspiration to take back to your research work.
   (b) Finding out about stuff you did not know existed, at a level beyond undergraduate knowledge especially.

Comments were gathered from the teaching staff also. A common sentiment was that it was enjoyable to teach students who were all interested in the material and wanted to learn it. Although most of the material was considered appropriate, it was felt after the attempt in SMART-1 that the time taken to explain the Wiener–Hopf method adequately would have led to an imbalance in the material.

V. INFLUENCE ON Ph.D. COMPLETION

A report of the most recent broad study of the progress of Ph.D. students in the U.K. over seven academic years, from the start of their studies in 1996–1997 through 2002–2003
(Ref. 4) found that approximately 80% of students of those initially registered for Ph.D.s completed them in this period. Significant and material influences in the rate of Ph.D. completion were found to include differences in financial backing whether they were from the U.K. or from overseas, age on entry, previous qualifications, and subject. Higher rates of completion were found for (full time) students with secure financial backing, students from overseas, younger students, and students following programs in the natural sciences.

There are several difficulties with making an assessment of the influence of the SMART schools on Ph.D. completion:

(1) Students who are motivated enough to give up a week of their summer to do mathematics are likely to be more than typically conscientious (i.e., they are self-selecting).
(2) Completion rates are not necessarily a measure of “quality” because better students might explore issues in more depth than their fellows and therefore take longer.
(3) Information about students after their SMART school attendance is hard to find.

Nevertheless from information made available it can be reported that six of eight ISVR students and 10 of 11 Salford students who attended SMART schools have subsequently completed their Ph.D. theses, and all of these students will have graduated by July 2011.

Some of the comments made by ISVR supervisors about Ph.D. completion and the influence of the SMART Schools are as follows:

(1) “ZH completed in 3.5 yr; I would say was the course had a positive benefit though hard to measure.”
(2) “JR completed in 3 yr; probably would have done [so] anyway but was helped by the course.”
(3) “His rate of work was excellent, and he submitted his EngD thesis on time just before the end of his studentship period. As most of our Ph.D./EngD students do not finish writing their thesis by the time their funding runs out,... I would say that CB did finish more promptly than most.”

(4) “FF ... a very competent theoretician who will almost certainly have derived benefit.”
(5) “TP was excellent, and I would have expected him to have benefited from the program .... He did ... quite a lot of analytical work, which he undertook very competently.”

VI. CONCLUDING REMARKS

Three summer schools in mathematics for acoustics have attempted to address an important need in education for postgraduate and postdoctoral acoustics researchers in the U.K. The primary aim of the summer schools was to convey awareness of the essential mathematical tools and concepts that could enable the participants, typically first year graduate researchers in physical and engineering acoustics, make a rapid impact in their research. However the schools have also fulfilled a secondary aim, which was to encourage a sense of community and offer opportunities for networking. This has proved particularly important for research students working in acoustically related topics who were relatively isolated or in small groups.

One of the (post-doctoral) attendees in 2003 subsequently acted as a tutor in 2005 and 2007. Another student who gained his doctorate after SMART-1 was a tutor at SMART-3.

TABLE VII. Example responses to “How well did the summer school meet your needs?”

| SMART-1 | \ldots now aware of maths in acoustics in a general sense | to a great extent | helped quite a lot | needs met to a highly satisfactory level | I have lots of new ideas on my project now | has given a broad idea of what kind of maths might be required in the future |
| SMART-2 | The SMART school was very helpful to me, especially because it occurs during my problem formulation for Ph.D. I have gained a good background for the mathematical methods I was planning to use and some good ideas for alternative methods | \ldots even though some of the material was way too hard to grasp over the course of five days, I know that if I ever come across any of the material in the future I will be able to have a better understanding of how to tackle the problems posed | \ldots the material (book and handouts) is very useful as it gives a deep insight of the maths for acoustics research, being also well presented and easy to follow |
Despite its rather specialized nature, support for acoustically related research has consistently accounted for about 20% of the grant expenditure in the EPSRC Engineering Program during each of the last 5 yr. There is a continuing need in the U.K. for courses such as the SMART schools, so it is to be hoped that a similar provision will be encouraged and supported in the future.

ACKNOWLEDGMENTS

The SMART Summer Schools were supported by the Engineering and Physical Sciences Research Council (UK) through Grant Nos. GR/S47021/01, EP/C547349/1, and EP/D507820/1 awarded while K.A. was Head of Department and Research Professor in Engineering at the University of Hull.

1For more information on the Helping Engineers Learn Mathematics (HELM) project, see http://www.lboro.ac.uk/research/helm/ (Last viewed March 24, 2011).