Open Research Online

The Open University's repository of research publications
and other research outputs

Noticing: roots and branches

Book Section

How to cite:

Mason, John (2011). Noticing: roots and branches. In: Sherin, Miriam Gamoran; Jacobs, Victoria R. and Philipp, Randolph A. eds. Mathematics Teacher Noticing: Seeing Through Teachers' Eyes. Studies in Mathematical Thinking and Learning. New York: Routledge, pp. 35–50.

For guidance on citations see FAQs.

© 2011 Taylor Francis

https://creativecommons.org/licenses/by-nc-nd/4.0/

Version: Accepted Manuscript

Link(s) to article on publisher's website:
http://www.routledge.com/books/details/9780415878623/

Copyright and Moral Rights for the articles on this site are retained by the individual authors and/or other copyright owners. For more information on Open Research Online's data policy on reuse of materials please consult the policies page.

oro.open.ac.uk

Noticing: Roots and Branches

John Mason

Open University & University of Oxford

Starting with roots of the idea of *noticing* as a potentially intentional rather than haphazard act, I first outline aspects of what I call *The Discipline of Noticing* (Mason 1984, Mason 2002). Central to this view is the idea that noticing is a collection of practices designed to sensitize oneself so as to notice opportunities in the future in which to act freshly rather than automatically out of habit.

I next consider ways in which noticing has produced insights and informed action in teaching, learning, and conducting professional development having to do with mathematics. Constructs such as attention and intention, awareness, and consciousness are not only researchable using the discipline of noticing and informative about how noticing actually works but also contribute to our appreciation of intricacies of learning and teaching mathematics.

Roots of the Discipline of Noticing

Noticing is a common enough word in English, with an etymology tracing back to the Latin words *notitia* (being known) and *notus* (known). Clearly one notices all the time: For example, we pin *notices* on a *noticeboard* to bring things to people's attention so that they will *notice* them. In fact, there is a great deal that we do not notice, either because we are not attuned or sensitized or because our attention is directed and occupied elsewhere. Sometimes people do not notice, do not realize that they need to pay attention to some feature in a situation, with the result that things go wrong. For example, mathematics students often ignore structural relationships indicated verbally in a word problem, so they try to manipulate the numbers to get

an answer; older students often forget to check the conditions required by a theorem before trying to apply it to some situation.

My attention was first directed to noticing as an intentional act when I spent a year under the direction of J. G. Bennett during 1973-1974. Some 120 of us ran and maintained an old manor house while learning to observe ourselves and to work with one another. Among the lectures that we attended was one on *noticing* in which Bennett (1976) brought together various strands of the practical work lying at the heart of the program of study. Bennett's lecture was based on what he had gleaned from years of travelling in the Near, Middle and Far East, and especially from being taught by an Armenian, Georgi Gurdjieff. Gurdjieff had brought to the West what Ouspensky (1950) called *fragments of an unknown teaching*, centred on self-observation. Gurdjieff in turn seemed to have been influenced by various Middle Eastern teachers whose roots can probably be traced to the most ancient of writings such as the Rg Veda and the Upanishads, and hence into the mists of time.

My use of *noticing* in mathematics began with working on mathematical animations and posters in the company of a number of colleagues in the Association of Teachers of Mathematics in the United Kingdom, particularly with Dick Tahta (1981). We would look at a poster or watch a short animation and then reconstruct what we had noticed, gradually developing a descriptive *story* or *account-of* what we had seen. Sometimes this story would then be verified or augmented during a second viewing. Only then would we begin to *account-for* what we recalled seeing by explaining the story development mathematically.

In the 1980s, when my colleagues and I at the Open University were asked to prepare videotapes of *best practice* in secondary mathematics classrooms, I transferred this way of working on animation together with insights from noticing to develop ways of working with the

tapes. We eschewed the notion of *best practice*, and we discovered early on that we needed a way to counteract such reactions to the tapes as "my low attainers are lower than those low attainers" and "I wouldn't let that teacher in my classroom." Instead of getting people to analyze practices observed on the tapes, we found it more effective to use the tapes as stimuli to get people to recall and then analyze related incidents from their own teaching. We initiated a practice in which people were asked either to reconstruct collectively what they thought they had seen, incident by incident, or to choose some salient moment and describe it to colleagues while reducing to a minimum all judgments and emotive terms, so that the moment could readily be recognized by everyone. Emphasis was on "what you saw that others may have seen and can recognize," that is, on behavioral rather than affective or emotive aspects. Then we emphasized the importance of people bringing to mind from their own experiences incidents that were similar in some way (Mason, 1988).

The effect of this practice was that people used what they saw on the video as a combination of metonymic triggers into, and metaphoric resonances with, their own past experiences. By describing their own incidents to others, briefly–but–vividly, and by negotiating the senses in which different incidents were similar or different, the participants developed a collective vocabulary and a rich web of interrelated shared incidents. This experience, in turn, provided a foundation for individuals to recognize in the moment when a similar incident began to emerge, enabling them to avoid the habitual and to act freshly. This is the essence of the *Discipline of Noticing*: arranging to alert oneself in the future so as to act freshly rather than automatically out of habit.

Often, some moments after a habit has been activated, I become aware of that fact: I notice an opportunity retrospectively, too late, rather than in the moment. By making use of this

retrospective noticing to trigger the act of imagining myself noticing an opportunity to act differently (to respond rather than to react) in the future, I *pro-spectively* prepare myself to notice in the future. By continued disciplined use of *re-flection* and *re-construction* in the form of prospective imagining, the moment of noticing moves closer and closer to the moment of instigation of action (*spective*), eventually displacing the habitual reaction with a fresh response.

Methodological Remarks

One unusual feature of the Discipline of Noticing is the form and nature of its results. Because its use to research personal practice is fundamentally experiential, the results of enquiries using the Discipline fully are task-exercises through which others may be sensitized to notice something freshly for themselves, to become aware of useful distinctions and possible actions to be initiated in the future. In papers I usually offer tasks based on workshop tasks, where I tell people that what they get from the workshop will be what they notice happening inside them. Thus, the data offered are immediate experiences and what they trigger or what resonates from past experience, making the Discipline entirely self-consistent. The validity of a *finding* lies not in the verisimilitude of someone's report but in whether others find their future actions informed and their future noticing enriched.

In line with this perspective, the distinctions I offer in this chapter must be treated as conjectures to be tested in experience. It is essential to pause every so often and try to bring to mind either specific instances or at least a general flavour of what is being said, from your own experience. I take the unusual methodological stance that the data I offer are *not* descriptions of my own incidents but rather, what comes to mind for readers from their own experiences when they encounter my descriptions. In parallel with what my colleagues and I learned about using

videotape of classrooms, what is most powerful is not what is presented as stimulus but, rather, what comes to mind from one's own experience, triggered or resonated by what is read.

The Discipline in a Nutshell

The Discipline of Noticing is a collection of techniques for (a) pre-paring to notice in-the-moment, that is, to have come-to-mind appropriately, and (b) post-paring by reflecting on the recent past to select what you want to notice or be sensitized to particularly, in order to *pare*, that is, to notice in-the-moment and so be enabled to act freshly rather than habitually.

In addition to selecting incidents and situations in which you wish you had had and wish in the future to be able to have the mindfulness to act differently (Bateson, 1994; Langer, 1997), you need to accumulate different desirable actions that you prefer to your habitual reactions. In other words, to replace *reaction* by (considered) *response*, you need to have an action come to mind just before your automatic reaction takes over. You need therefore a collection of alternative actions and an awareness of situations in which these actions would be preferable.

Alternative actions are accumulated through noticing other people doing them, reading about them, or noticing yourself doing them. For example, in a pedagogic strategy sometimes referred to as *jigsaw groups* (see Aronson 1978), three roughly equal-sized groups are set to work on three related problems. Each group works initially on its own problem; after a period of work, triples are formed so that each problem has been worked on by someone in each triple. Members of triples can then compare and contrast their tasks and their approaches, thereby enriching the experience of all without each person's having to work separately on each task. Thinking to use this strategy requires careful preparation of the tasks so that when the triples

---

[1] By calling upon etymological roots of words in slightly novel ways, the surprise-novelty can contribute to sensitizing noticing. While writing this I realized that the word *prefer* could be similarly used, to talk about *pre-fering*, *post-fering* (becoming aware afterwards of something you wish in retrospect you had done) and *fering*, meaning 'happening in the moment'. The etymology is based on the Latin *fero* = *to make* or *do*.

form, they learn from the variation in the three problems. One might consider the strategy but then be dissatisfied with the three problems; be overwhelmed by the organisational difficulties; or experience other obstacles, such as concern that students will not react well to a change of ways of working. One may be fully prepared but then literally forget to use it until insufficient time is left in the lesson (possibly a form of displacement activity) or balk at using it at the last moment for other complex self-justifying *reasons.* Even more likely, having experienced or heard about the strategy, one may consider it a good idea yet not have it come to mind while preparing for lessons in which it might be useful, which can happen with any *good idea,* however attractive initially. The Discipline of Noticing can be used to enhance the possibility of *having come to mind.* Indeed, the mark of effective professional development is that participants can imagine themselves in the future acting responsively and freshly rather than habitually. The mark of improving research capacities for individuals lies in their being able to imagine themselves in the future acting (responding) more appropriately than before.

Branches of Noticing

In the remainder of the chapter, I offer a glimpse into ways in which use of the discipline of noticing has afforded insights not only into the teaching and learning of mathematics but also into the functioning of noticing itself (for what some others have done, see Davis & Lerman, 2009). No natural path through these sections exists because each section draws upon and informs others. Noticing can be used to focus on fine detail while ignoring other aspects, but it can also be used to maintain the complexity of phenomena of teaching and learning. True to the discipline of noticing, what is being offered here is, at best, a collection of signposts indicating that others have been this way before and suggesting foci for further self-study.

Account of and Accounting for

When reporting an incident as part of professional development or research into practice it is usual to intermingle description with explanation, justification, and theorizing. Think, for example, how often people preface a report with some self-deprecating comment or emotive explanation for its inadequacies. This is part of the functioning of self-justification, the construction of narratives for the purpose of self-assertion and self-calming. Think, too, how often data that is offered (especially when derived from video of classrooms) uses theoretical constructs in its very description. For example, reporting two nearly contiguous moments in a classroom video, someone spoke of (a) "the moment the teacher entered and dominated the two children" or (b) "the moment the teacher tried to get the students to present what they had been doing iconically." Identifying precisely which moment is intended is difficult because of the judgemental baggage in the term *dominate*, and no matter how familiar people are with the notion of iconic (re)presentation (Bruner, 1966), the technical term was not what was observed but rather, its use signals an interpretation by means of theory. These statements could be modified to (a) "the moment the teacher entered [the shot] and started talking, standing behind the two children who were slouched on the table" and (b) "the moment the teacher suggested that they use a diagram to record their thinking"; these statements are closer to what could be observed by others, but again the term *slouched* contains evaluative judgement. We still do not know what the teacher said, so we cannot consider what sense the children made of it. A further modification to the first statement became "the moment the teacher entered [the shot] and started talking, standing behind the two children who had their arms on the table, their heads resting on their arms, and who were looking up at the teacher." This more precise description is more easily

identified by someone spinning through the video and more easily recognized, both by people who have seen the video and more generally as an incident within most teachers' experience.

Listening to people's accounts of what they saw in a mathematical animation and to accounts of lesson incidents quickly brought out the distinction between giving an *account-of* and *accounting-for* the account-of. The former must be as free of theorizing, emotional content, justification, and explanation as possible so that others can recognize the incident being described (even if they were not present, they may be able to enter a similar incident of their own). Useful accounts-of provide *brief-but-vivid* descriptions. Only after the incident has been identified, does it make sense to start theorizing, explaining, and *accounting-for* not only what was observed but why it struck the observer sufficiently to be identified or *marked* (see Tripp, 1993). Similar remarks apply to descriptions of moments in a mathematical animation or, indeed, moments during work on tasks.

Labels, Multiple Meanings, and Interpretations

When incidents have been described briefly-but-vividly, listeners can usually recall similar situations from their own experiences. By describing these experiences in turn, the group can negotiate what is similar and what is distinctive so that a rich collection of related incidents become part of the discourse of the group. Finding a descriptive label using words that might occur in similar incidents in the future can help associated actions come to mind when something similar is developing in the future (Mason 1999). Idiosyncratic labels (such as learner's names) are much less effective than descriptive labels in coagulating multiple experiences under one label. Deferring theorizing and accounting-for enriches the collection, for once the situation is *boxed up* and interpreted, it loses a great deal of its force to promote informed non-habitual action in the future.

Part of the practice of self-observation is the search for multiple interpretations. Human beings are complex organisms. Settling quickly on a single interpretation of one's own or someone else's actions promotes simplicity, but in reducing complexity, one lessens the richness and significance of the interpretation. Consequently, a valuable practice when accounting-for incidents captured as data is to seek multiple, preferably conflicting, interpretations. Holding multiplicity opens possibilities whereas classifying and explaining away closes them. Interpretations held in tension have residual energy; when tensions are removed, energy escapes and stasis results. For example, when the learners had their heads down, they might have been off-task, perhaps even dozing, but they might also have been thinking deeply. Their "looking up at the teacher" might be an indication of dread or fear of being caught out, of hope that they would get scaffolding to their thinking, or of concern that their pleasure at struggling might be about to be taken away by a teacher giving them the answer or clues towards an answer. Holding these as possibilities instead of choosing among them invigorates future incidents when learners are acting similarly, whereas otherwise one might be tempted to make an incorrect assumption about what was going on.

Experiencing: Not-Noticing, Barely-Noticing, Marking, and Recording as Energy States

Experience is a curious phenomenon. Of the myriads of sensations with which we are bombarded each day, most are censored out by somatic processing (Norretranders, 1991/1998). The extent of this censoring varies. First, I may think that I am experiencing, yet later, when asked whether I noticed something, I may be entirely oblivious of what the person is referring to—an example of simple *not-noticing*: Nothing about what is described is immediately accessible even though I thought that I was awake and present. Second, I may recognize what is being described, although I had otherwise forgotten it. This is *barely-noticing.* Third, I may think

to make a remark to someone about something I noticed, which I call *marking*. Finally, I may be so struck by the incident that I make some form of *recording* to enable me to re-enter the incident at a future date—the role of brief-but-vivid descriptions in an account-of.

In order to learn intentionally from experience, we must withdraw from action and reflect on or reconstruct that action and its effects (Simon & Tzur, 2004). But despite the view expressed by William James (1890) about the *flow of consciousness,* observation reveals that experience is recalled in fragments, and the sharper conjecture that 'experience is fragmentary' has considerable justification (Mason, 1988; Tversky, Zacks, & Hard, 2008). Something attracts my attention. I am bright and alert. Then, over time, my alertness fades until there is another sudden attracting of attention. When I recall incidents, I alight on such a fragment, just as I am struck by some fragment of an animation or of a lesson, whether in real time or on video. Being a *narrative animal* (Bruner, 1990), I glue these fragments into a story that helps me make sense of my experience. This scenario applies equally to the carrying out of mathematical techniques; to constructing meaning for mathematical concepts; to learning from experience of doing exercises, working on problems, or exploring; and to creating meta-stories about why I am learning mathematics, my place in the class, and my sense of agency.

When re-calling, re-flecting on, or re-constructing some incident or event, one readily recalls what was marked. What can then be re-constructed through metonymic association and metaphoric resonance (eschewing deductive chains of "I must have …") can then gain in significance or richness so as to be marked or recorded as well. Intentional re-flection and re-construction enhance the possibility of being sufficiently awake at some future moment so as to be able to respond freshly rather than to react habitually to the situation while it develops. An

increasingly popular term for this state, taken from ancient Buddhism, is *mindfulness* (Korthagen & Vasalos, in press; Langer, 1987).

Energy Levels

Examples of noticing, marking, and recording are encountered in work with teachers. Like members of any caring profession, teachers notice all sorts of things while teaching. Some things are sufficiently striking to come to mind later when the teacher is reflecting and reconstructing, perhaps with colleagues, whether formally or informally. But getting teachers to write accounts, to make records of what they notice, is much more difficult, just as sustaining a diary or journal is difficult: Energy is required to overcome bodily, cognitive, and affective resistance ("time is too short," "there are other things that take priority," "I can't be bothered," "I don't really have anything to record," "I can't decide what to record," and so on). The requisite energy can come from commitment to regular exchanges with colleagues; personal discipline developed over time; or in association with some goal, such as pursuit of a higher degree, promotion, and the like. The discipline of noticing provides a structure within which to work on noticing intentionally. The energy required to maintain the discipline comes from the individual's nexus of social relationships, inner resources, and commitment.

Inner Witness / Monitor

Intentional self-observation through disciplined attempts to notice can gradually build what might be referred to as an *inner witness* or *monitor*. This idea can also be traced back to ancient writings such as the Rg Veda:

Two birds, close-yoked companions

Both clasp the self same tree

One eats of the sweet fruit

The other looks on without eating.

(Bennett, 1964, p. 108)

One interpretation is of an inner witness that looks on while the rest of the psyche is involved in action. The monitor does not comment, judge, or evaluate. It simply observes. Its presence gives the person an enriched sense of being present, mindful, and awake to what is happening that goes beyond simplistic notions of consciousness and awareness.

The importance of developing a mathematical inner witness cannot be overstated. To have come to mind such questions as "Why are we doing this … (example, calculation, etc.)?" and "Are you sure that is what you meant to say or do?" in the midst of action is essential when working on mathematical problems (Schoenfeld, 1985) and when making sense of written mathematics. Such a witness can arise spontaneously, but for most people it needs to be nurtured intentionally through disciplined use of reflection on and reconstruction of recent incidents. Awakening the witness-monitor is the central aim of the Discipline of Noticing.

Awareness ≠ *Consciousness* ≠ *Awareness* + *Consciousness*

Gattegno (1987) used the word *awareness* to mean "that which enables action." Thus, not all awareness is conscious, inasmuch as our bodies regularly act to alter breathing, heartbeat, hormone flow, and many other somatic functions. He then suggested that *only awareness is educable*, and this assertion has the property of acting like a protasis (an assertion of generality)

for a syllogism (Mason, 1998b): People naturally draw on and interrogate their own experiences as a particular, which, when juxtaposed with the general protasis, generates a syllogistic action. By holding back from drawing a conclusion, the tension can be put to good effect to stimulate noticing and the growth of an inner monitor.

For example, pointing is an action enabled by an awareness, which when brought to the surface through becoming consciously aware of it, leads to the notion of one-to-one correspondence and counting; labeling is another kind of action enabled by awareness grounded in use of language. These awarenesses are associated with functioning in the worlds in which we act (the material world, the mental world, and the world of symbols; *cf*. Bruner, 1966).

Gattegno (1987) suggested further that mathematics as a discipline arises or is extended when someone becomes aware of an awareness, often by becoming aware of the action that has been enabled. Lakoff and Nunez (2000) reached similar conclusions that the origins of much of mathematics lies in bodily awareness, though they seem to be less clear about how this idea applies to more advanced concepts. At times, locating a specific bodily awareness underpinning concepts, for example ratio, linear independence, or function, is difficult, but locating the actions and consequently the enabling awarenesses that underpin these and other mathematical concepts is not. To aid clarity, I refer to these awarenesses as *awareness–in–action* because they arise through becoming aware of actions, and so articulating and formalizing them (Mason, 1998a). Examples of awareness-in-action include familiarity with putting things into bags and taking them out as a basis for addition and subtraction as well as for sets (bags within bags) and many-folding as a basis for multiplicative reasoning (I am grateful to Brent Davis, private communication, May 17, 2007 for this etymological insight).

Awareness of awareness arises from noticing; the noticing occurs spontaneously during investigation but usually requires intentional acts as shifts of attention (Mason, 1989; Mason & Davis, 1989) initiated by a teacher. The core awarenesses on which the school mathematics curriculum is built have been elaborated and referred to with different labels by other authors coming from slightly different perspectives, but they all contribute to awareness of the transformations, shifts, or awarenesses that need to be developed in order to make sense of school mathematics. For example, Simon (2006) described *a conceptual advance* as "one that changes students' ability to think about and/or perceive particular mathematical relationships" (p. 362). Similarly, Cuoco, Goldenberg, and Mark (1996) discussed *mathematical propensities* that can be developed.

The study of misconceptions is closely related to awareness of awarenesses because misconceptions often arise from misapplied, incomplete, or inappropriate awarenesses. For example, "more means bigger," "when in doubt, assume linearity," and $0.3 \times 0.3 = 0.9$ are incomplete or inappropriate awarenesses arising from a use of natural powers on incomplete data together with pedagogical lapses (Tirosh & Tsamir, 2004).

Teaching is another matter. Clearly, one can be an expert mathematician without being particularly skilled in teaching mathematics. Different attributes are required of mathematicians and effective teachers. A major thrust in mathematics education currently is to try to articulate precisely the distinction between what effective mathematicians have come to mind and what effective teachers have come to mind in their professional activities. Applying to teaching Gattegno's insight about awareness suggests that in order to become expert, you need to become aware of your awareness-in-action, which I call *awareness-in-discipline*, but which is really

awareness of awareness of awareness.[2] This awareness-in-discipline is a distinct type of awareness, because to function effectively mathematically, you need to have come to mind heuristics such as "try working backwards," powers such as "try specializing in order to re-generalize for yourself," and mathematical themes such as "doing and undoing" and "invariance in the midst of change." An effective teacher needs to become aware of these as awarenesses to be called upon, in order to construct tasks that bring them to learners' awareness and in order to draw attention to them when they are relevant. Whereas to a mathematician these awarenesses are integrated or internalized actions rarely worthy of explicit attention, to a teacher they are important foci of attention as prompts to learners in such a way that they internalize them for themselves. At first they are pointed out directly and explicitly; over time they are referred to less and less explicitly and more and more indirectly until learners integrate them into their own functioning. The labels *directed–prompted–spontaneous* have been used (Floyd, Burton, James, & Mason, 1981) as a reminder to teachers that scaffolding (Wood, Bruner, & Ross, 1976) is accomplished only when the scaffolding has faded away (Brown, Collins, & Duguid, 1989; Love & Mason, 1992). For more examples, see Mason and Johnston-Wilder (2004) and Mason (1999).

Because even more is required of a teacher educator, teaching people to be teachers of mathematics is a discipline in itself, just as teaching mathematics and doing mathematics are disciplines. Applying Gattegno's insight once more, in order to become an effective educator, you need to become aware of your awareness-in-discipline, which, for reference purposes, I call *awareness-in-counsel* (Mason, 1998a). Awareness-in-counsel includes awareness of ways of working with people so that they become aware for themselves of actions they are taking, which

---

[2] I am not claiming that this is sufficient, only necessary. Effective teaching also depends on forming and maintaining appropriate relationships with and sensitivities to students, including patient tolerance of students not understanding new ideas immediately and the effort required to internalize and integrate mathematical ideas and procedures into self-initiated actions.

in turn are designed to prompt learners to learn mathematics effectively. Each level of awareness is built up through noticing, whether spontaneously or intentionally.

In summary then,

- *awareness* is what enables action;
- *awareness of awareness* (*awareness-in-action*) is a formalization and hence institutionalization of awarenesses that enable action;
- *awareness of awareness-in-action* (*awareness-in-discipline*) is what enables articulation and formalization of awarenesses-in-action and so is the basis for and informs teaching;
- *awareness of awareness-in-discipline* (*awareness-in-counsel*) is the self-awareness required in order to be sensitive to what others require in order to build their own awareness-in-action and awareness-in-discipline.

Each type of awareness develops and is internalized through being sensitized to notice, for which the discipline of noticing can provide helpful techniques. Thus an awareness of fractions as actions on sets of objects, and an awareness that different objects or parts of objects can be considered to be *the unit* enables multiplication and addition of fractions to be carried out; awareness of this awareness enables fractions to be considered as objects; awareness of this awareness of awareness is necessary in order to teach others effectively about the arithmetic of fractions; awareness of awareness of awareness of awareness is needed in order to teach others how to teach the arithmetic of fractions.

Role and Structure of Attention

To notice requires attention to something; indeed attention *is* both observation and the medium through which observation takes place. As William James (1950/1890) proposed, this

attending can be either spontaneously reactive or intentionally responsive. An act of attention can be fleeting or sustained. When our attending is sustained, we may be aware, in the sense of being consciously, explicitly aware, and form a sufficiently lasting sense-impression so as to have this incident or something related to it come to mind in the future (*marking*). We may not, however, be consciously aware, yet our bodies may be sufficiently aware subconsciously to incorporate (literally) something, which may then influence future behavior. For example, a new colleague repeatedly used the phrase *bottom line,* and soon other colleagues and I were using it, mostly unwittingly with only the slightest twinge of recognition; another colleague started using "at the end of the day" frequently, with the same effect. The interwoven strands of noticing, attention, awareness, and consciousness form the basis for effective and intentional teaching as well as for socio-psychological analysis of learning (Mason, 2008b).

Noticing is a movement or shift of attention. If I am working in my office and someone passes by in the corridor, my attention is diverted or attracted by my peripheral vision, and I look up; if I am working on a mathematical problem and something else pops into my mind that I am supposed to do, I start doing that; if I am gazing at a mathematical diagram or at some algebraic manipulation and suddenly I notice a recurring expression or something familiar about some part, I start attending to that. Some of these reactions are desirable, whereas some are literally energy leaks, drawing my attention away from a focus and so diminishing concentration.

What is noticed, marked, or recorded is necessarily being attended to. Attention can be seen as the manifestation of will, of intention. However, in a very central sense, 'we are where our attention is,' or even 'we are our attention' (Harding, 1961). Thus, we have habits of speech such as "give me your attention," "thank you for your attention," and the more sarcastic "Are you with us?" The military command "Attention!" is presumably intended to startle people into a

heightened state of wakefulness. These uses signal the centrality of attention in human experience. As James (1950/1890) said,

> Every one knows what attention is. It is the taking possession by the mind, in clear and vivid form, of one out of what seem several simultaneously possible objects or trains of thought. Focalization, concentration, of consciousness are of its essence. (pp. 404–405)

Attention is not a thing to be observed in others, but its influence can be inferred. Even when eye-tracking is used, all we can observe is where the subject's eyes appear to be focused, but not whether the subject is actually attending to that focus or in what manner. This fact raises a question about ways in which we attend to things.

Attention has at least macro-, meso-, and micro-structure, and these can be in rapid flux or relatively stable. In its macro structure, attention can vary (a) in the focus (what is attended to, singular or multiple), (b) in the locus (the source or basis of attention, which can be in various parts of the body or external to it), (c) in the strength or amplitude (from feeble to intense), and (d) in the scope or breadth (broad or narrow) (Mason, 1982, 1998a, 2009; Mason & Davis, 1989).

In its meso-structure, attention can be dominated by a particular collection of beliefs or perspectives. Adolescents, for example, are engaged in an enterprise of discovering themselves as social beings, both dependent upon and independent of surrounding adults. They are concerned primarily about locating themselves within their growing awareness of the social communities of which they are part, so work on getting them to make significant mathematical choices can both resonate with and contribute to that enterprise., whereas imposing tightly structured tasks may not. From quite a young age, children are fascinated by the notion of infinity: As Dick Tahta (private communication, June 12, 1985) pointed out, addressing the

notion of infinity is an opportunity to show how mathematicians work on and control a topic that resonates deeply with children's growing recognition of mortality and yet their youthful sense of immortality. Within mathematics, children may, for example, be dominated by a sense of number as discrete, so that one needs to help them gain confidence with a parallel or encompassing continuous sense of quantity to develop flexibility (Watson, 2008). To them, justifications may have previously been empirical and *ad hoc,* so a shift to deduction from agreed properties needs to be developed, opening the way to axiomatic mathematical reasoning.

In terms of micro-structure, people can attend differently at different times (Mason-1998a, 2009) in the following ways:

1. *Holding Wholes* is attending by gazing at something without particularly discerning details. Examples include gazing at a diagram, at the ceiling, or at algebraic calculations, as well as holding a problem in mind and allowing the subconscious to work away at it (Hadamard, 1945).
2. *Discerning Details* is picking out bits, discriminating this from that, decomposing or subdividing and so distinguishing and, hence, creating *things.*
3. *Recognizing Relationships* is becoming aware of sameness and difference, other relationships among the discerned details in the situation. In mathematics this awareness includes functional relationships; operational relationships such as additive, multiplicative or exponential; geometrical relations such as similarity, congruence and symmetry, and more general relations.
4. *Perceiving Properties* is becoming aware of particular relationships as instances of properties that could hold in other situations.

5. *Reasoning on the Basis of Agreed Properties* is going beyond the assembling of things you think you know, intuit, or induce must be true in order to use previously justified properties as the basis for convincing yourself and others, leading to reasoning from definitions and axioms.

In mathematics, the shift from recognizing relationships to perceiving properties is often subtle but immediate for experts and yet an obstacle for students. When teacher and students are attending to different things, communication is unlikely to be efficient. Even when teacher and students are attending to the same things, they may be attending differently, and so communication may be, at best, restricted and incomplete, if it does not break down altogether. Teaching people to reason mathematically is well known as a pedagogical challenge, despite its apparently deeply rational nature. Those who make the transition easily are challenged to see any difficulty. For example Henri Poincaré (1956) reported being astonished that most people find mathematics difficult, despite its being the most rational of disciplines. However, as Swift (1726/1941) noted, human beings are not so much rational animals as animals capable of reason. Most of us need help in making the requisite shifts of attention.

These distinctions in the micro-structure of attention arose for me from considering neo-Pythagorean studies of number (Bennett, 1956-1966) but match well with studies by Pierre and Dina van Hiele in geometry (Usiskin, 1982), with one notable difference: Shifts among ways of attending are usually frequent, and not confined to *levels of understanding*.

Summary

The core of the Discipline of Noticing is a collection of techniques for (a) pre-paring to notice in-the-moment, that is, to have come-to-mind appropriately; (b) post-paring by reflecting on the recent past to select what one wants to notice or be sensitized to particularly; in order (c)

to *pare*, that is, notice in-the-moment and so be enabled to act freshly rather than habitually. This core applies to both personal and professional development. In this chapter, I have presented a brief version of the Discipline of Noticing and how it arose for me and came to be articulated. I am aware of some of its roots but, of course, not all: Ideas travel quickly, subtly, and often below the visible surface both across and within cultures. I have also indicated ways in which noticing can be and has been used and how it interweaves with attention, awareness, and consciousness.

In addition, I have tried, despite writing *about* noticing, to insert examples and descriptions of observations and to write in such a way that might resonate with or trigger associations with readers' experiences so as to initiate actions that might lead to informed choices in the future, whether those choices are concerned with professional development, research, or personal development. Validity, for you the reader, lies in what you find does or does not inform your future practice.

References

Aronson, E. (1978). *The jigsaw classroom*. Beverly Hills, CA: Sage.

Bateson, M. (1994). *Peripheral visions: Learning along the way*. New York: HarperCollins.

Bennett, J. G. (1956–1966). *The dramatic universe* (Vols. 1–4). London: Hodder & Stoughton.

Bennett (1964). *Energies; material, vital, cosmic.* Coombe Springs: Coombe Springs Press.

Bennett, J. G. (1976). *Noticing*. The Sherborne Theme Talks, Vol. 2. Sherborne, UK: Coombe Springs Press.

Brown, S., Collins, A., & Duguid P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32–41.

Bruner, J. (1966). *Towards a theory of instruction.* Cambridge, MA: Harvard University Press.

Bruner, J. (1990). *Acts of meaning*. Cambridge, MA: Harvard University Press.

Cuoco, A., Goldenberg, P., & Mark, J. (1996). Habits of mind: An organizing principle for mathematics curricula. *Journal of Mathematical Behavior, 15,* 375–402.

Davis, B., & Lerman, S. (2009). *Mathematical Actions and Structures of Noticing*: *studies on John Mason's contribution to Mathematics Education*. Rotterdam, The Netherlands: Sense Publications.

Floyd, A., Burton, L., James, N., Mason, J. (1981). *EM235: Developing mathematical thinking*. Milton Keynes, UK: Open University.

Gattegno, C. (1987). *The science of education, Part I: Theoretical considerations*. New York: Educational Solutions.

Hadamard, J. (1945). *An essay on the psychology of invention in the mathematical field.* Princeton, NJ: Princeton University Press.

Harding, D. (1961). *On Having No Head: Zen and the re-discovery of the obvious*. London: Arkana (Penguin).

James, W. (1950). *Principles of psychology, Vol. 1.* New York: Dover. (Reprinted from Title, 1890, City: Publisher)

Korthagen, F., & Vasalos, A. (in press). From reflection to presence and mindfulness: 30 years of developments concerning the concept of reflection in teacher education. In N. Lyons (Ed.), *Handbook of reflection and reflective inquiry*. New York: Springer.

Lakoff, G., & Nunez, R. (2000). *Where mathematics comes from: how the embodied mind brings mathematics into being*. New York: Basic Books.

Langer, E. (1997). *The power of mindful learning*. Reading, MA: Addison Wesley.

Love, E., & Mason, J. (1992). *Teaching mathematics: Action and awareness*. Milton Keynes, UK: Open University.

Mason, J. (1982). Attention. *For the Learning of Mathematics, 2*(3), 21–23.

Mason, J. (1984). Towards one possible discipline for mathematics education. In H. Steiner (Ed.), *Theory of mathematics education* (pp. 42–55). Bielefeld, Germany: Institute for the Didactics of Mathematics, Bielefeld University.

Mason, J. (1988). Fragments: The implications for teachers, learners and media users/researchers of personal construal and fragmentary recollection of aural and visual messages. *Instructional Science,* 17, 195–218.

Mason, J. (1989). Mathematical abstraction seen as a delicate shift of attention. *For the Learning of Mathematics,* 9(2), 2–8.

Mason, J. (1998a). Enabling teachers to be real teachers: Necessary levels of awareness and structure of attention. *Journal of Mathematics Teacher Education, 1,* 243–267.

Mason, J. (1998b). Protasis: A technique for promoting professional development. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times*: *Proceedings of the Twenty-first annual conference of the Mathematics Education Research Group of Australasia* (MERGA-21) (Vol. 1, pp. 334–341). Gold Coast, Queensland: Mathematics Education Research Group of Australasia.

Mason, J. (1999). The role of labels for experience in promoting learning from experience among teachers and students. In L. Burton (Ed.), *Learning mathematics: From hierarchies to networks* (pp.187–208). London: Falmer.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. London: RoutledgeFalmer.

Mason, J. (2008). Being mathematical with & in front of learners: Attention, awareness, and attitude as sources of differences between teacher educators, teachers & learners. In T. Wood (Series Ed.) & B. Jaworski (Vol. Ed.), *International handbook of mathematics teacher education: Vol.4. The mathematics teacher educator as a developing professional* (pp. 31–56). Rotterdam, the Netherlands: Sense Publishers.

Mason, J. (2009). Teaching as disciplined enquiry. *Teachers and teaching: Theory and practice,15*(2–3), 205–223.

Mason, J., & Davis, J. (1989). The inner teacher, the didactic tension, and shifts of attention. In G. Vergnaud, J. Rogalski, & M. Artigue, (Eds.), *Proceedings of the 13th Psychology of Mathematics Education international conference* (Vol. 2, pp. 274–281).

Mason, J., & Johnston-Wilder, S. (2004). *Designing and using mathematical tasks*. Milton Keynes, UK: The Open University.

Norretranders, T. (1998). *The user illusion: Cutting consciousness down to size* (J. Sydenham Trans.). London: Allen Lane. (Original work published 1991)

Ouspensky, P. (1950). *In Search of the Miraculous*. London: Routledge & Kegan Paul.

Poincaré, H. (1956 reprinted 1960). *Mathematical Creation: lecture to the Psychology Society of Paris*, in J. Newman. The World of Mathematics. London: George Allen & Unwin. pp.2041-2050.

Schoenfeld, A. (1985). *Mathematical problem solving*. New York: Academic Press.

Simon, M. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking and Learning, 8,* 359–371.

Simon, M., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning, 6,* 91–104.

Swift, J. (1941). Gulliver's travels. In H. J. Davis (Ed.), *The prose works of Jonathan Swift. Gulliver's Travels* (Vol. 11). Oxford, UK: Blackwell. (Original work published 1726)

Tahta, D. (1981). Some thoughts arising from the new Nicolet films. *Mathematics Teaching, 94,* 25–29.

Tirosh, D., & Tsamir, P. (2004). Commentary: What can mathematics education gain from the conceptual change approach? And what can the conceptual change approach gain from its application to mathematics education? *Learning and Instruction, 14,* 535–540.

Tripp, D. (1993). *Critical Incidents in teaching: Developing professional judgement*. London: Routledge.

Tversky, B., Zacks, J., & Hard, B. (2008). The structure of experience. In T. Shipley & J. Zacks (Eds.*), Understanding events* (pp. 436–464). Oxford, UK: Oxford University.

Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry*. Chicago: University of Chicago.

Watson, A. (2008). Necessary shifts in secondary mathematics. Retrieved July 14, 2009, from http://www.education.ox.ac.uk/uploaded/Mathematical%20thinking%20in%20adolescence.ppt

Wood, D., Bruner, J., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, *17,* 89–100.