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Dynamic spatial Bayesian models for radioactivity deposition

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Abstract

Dynamic spatial Bayesian (DSB) models are proposed for the analytical modelling of radioactivity deposition after a nuclear accident. The proposed models are extensions of the multivariate time series dynamic linear models of West and Harrison (1997) to Markov random field processes. They combine the outputs from a long-range atmospheric dispersal model with measured data (and prior information) to provide improved deposition prediction in space and time. Two versions of a Gaussian DSB model were applied to the radioactivity deposition in Bavaria over a 15 days period during the Chernobyl nuclear accident. One version had fixed functional forms for its spatial variances and covariances while the other allowed those to adapt and ‘learn’ from data in the conjugate Bayesian paradigm. There were two main sources of information for radioactivity deposition in our application: radioactivity measurements at a sparse set of 13 monitoring stations, and the numerical deposition evaluation of the atmospheric dispersal K-model for the points of a 64×64 regular grid. We have analysed the temporal predictions (one-step-ahead forecasting) of those DSB models to show that the dispersal K-model tended in general to underestimate the deposition levels at all times while the DSB models corrected for that although with different degrees of adjustment.

Key words: Bayesian space-time model; Chernobyl nuclear accident; Environmental statistics; Gaussian Markov random field process; Radioactivity deposition.

1 Introduction

This paper evaluates the forecasting performances of two distinct versions of a class of analytical state-space Gaussian Markov random field models, the *dynamic spatial Bayesian* (DSB) models, developed for the inference of ground deposition of radioactivity after a nuclear accident. They were applied to the Chernobyl's accident contamination of Bavaria in southern Germany.

The DSB models make use of data and information available after a nuclear accident in order to provide real-time approximate estimation of the levels of ground contamination in space and time by the released radioisotopes (together with a measure of their quality) that can be used for decision support purposes. Similarly to Gamerman *et al.* (2007), we adopt a state-space approach based on a spatial extension of the multivariate time series *dynamic linear model* (DLM) developed by West and Harrison (1997). We treat available data as discrete samples from a continuous spatial process taken at discrete time intervals, or equivalently, as realisations of a multivariate time series process in a spatial lattice.

The proposed DSB models assume a Gaussian process for the (transformed) underlying variables and parameters which together with a spatial Markovian covariance structure (see e.g. Besag, 1974) allows the analytical prior-to-posterior updating of means and covariances for fast computing. The assumed Markovian structure in space mimics the similar autoregressive structure in time that is core to the DLM formulation. This way, the temporal conditional independence feature of the DLM that separates past from future (given the present) states is extended to space where, conditionally on the set of its nearest-neighbors, the state of the process at a given location is separated from the state at any other more distant sites not in that set. Therefore, the separability of the spatio-temporal process where the space-time covariance structures can be decomposed into spatial and temporal components separately occurs naturally in our formulation. A pragmatic implication of the Markovian assumption is the lowering of computational costs associated with the inversion of large spatial covariance matrices.

On a related work, Fuentes and Raftery (2005) proposed a Bayesian spatial model for air pollution concentration that combines evaluation from a numerical model with measurements for improved spatial prediction that explicitly accounts for bias on the numerical model. Their approach, although similar to ours in nature, does not account for temporal evolution and rely on computer intensive numerical integration methods. Kennedy and O'Hagan (2001) and Conti *et al.* (2009) amongst others proposed Bayesian statistical methods for uncertainty analysis of a numerical model. Despite accounting for uncertainty associated with a large number of sources including model inadequacy, those

methods are also complex and computationally costly requiring the use of simulation as well as numerical integration methods. The DSB models here take the outputs from a numerical model itself as part of their inputs but can also take the statistical outputs from the Bayesian emulation methods of Kennedy and O’Hagan (2001).

Similarly to Fuentes and Raftery (2005) we evaluate our models by comparing their predictions with the few and sparse measurements available. In our case, spatial interpolations of the models’ forecasts are compared with measured deposition at the monitoring stations sites. Information about the deposition process comes from two main sources: (i) measurements from a sparse network of few monitoring stations and (ii) spatially smooth outputs from a numerical model for at the grid points. It seems clear that combining those two sources of information could lead to improved deposition predictions. In our application, the outputs used by DSB models are the daily averages total ground deposition of radioactivity calculated by the K-model, a Lagrangian model for long-range atmospheric dispersal of radioactivity developed by Lauritzen and Mikkelsen (1999). There are other more sophisticated atmospheric dispersal models, such as the Gaussian plume Rimpuff of Mikkelsen *et al.* (1984), but the K-model was chosen here for its simplicity of implementation and good performance when applied to simulated release exercises as well as to the Chernobyl accident as described in Lauritzen and Mikkelsen (1999) and Lauritzen *et al.* (2006).

The proposed DSB models were applied to real data of the 1986 Chernobyl’s accident contamination of Bavaria in southern Germany. The two models had their forecasting performances analysed and compared. We have tested two different DSB models in regards to the way their spatial covariances are treated. One model, the *exponential DSB* (EDSB) model, assumed the spatial component of its observational covariances to be known and assessed as negative exponential functions. The negative exponential covariance function is a particular form of the Matérn class of valid isotropic correlation functions (see e.g. Matérn, 1986; Cressie, 1993; Schlather, 1999). The second model, the *normal-Wishart DSB* (NWDSB) model, assumes those covariances to be unknown and attributes an inverse Wishart prior distribution to them on a multivariate normal-Wishart conjugate model approach. Notice that because we allow learning (from data) on the covariance structure related to the model, part of our results here are comparable to those of Le and Zidek (1992). However, differently from them, in our case significant temporal structures are included (temporal autocorrelation is allowed) as well as an explicit direct modelling of spatial interpolations. The latter allows for smoothness constraints on the covariance estimates through the specification of a suitable spatial interpolation matrix.

The paper is structured as follows. In Section 2, the general formulation of a DSB

model is introduced together with the definitions of the two applied models. Descriptions of the prior, posterior and forecasting distributions for those models are also presented. Section 3 briefly describes the Chernobyl accident and analyses the measured and the K-model predicted daily radioactivity deposition data in Bavaria. Section 4 shows the main results of the forecasting by the two DSB models together with a comparative analysis of their performances during the initial 15 days of the accident. Section 5 discusses the main issues in the paper and points out further related research.

2 The DSB model

We assume a Gaussian random process with *unknown* real value Θ_{st} at a spatial location $\mathbf{s} \in \mathcal{S}$ and time t ($t = 0, 1, \dots, T$), where $\mathcal{S} \in \mathbb{R}^2$ is the geographical region of interest. At a time t , $\mathbf{Y}_{\mathbf{s}_i,t}$ is a sample of $\Theta_{\mathbf{s}_i,t}$ at $q = m + n$ discrete locations $\mathbf{s}_i \in \mathcal{S}$ ($i = 1, \dots, q$). Thus, at a fixed t , the spatial component of a DSB model considers a finite regular grid over $\mathcal{S}$ for which points $\mathbf{s}_{K_i} \in \mathcal{S}$ ($i = 1, \dots, n$), an atmospheric dispersal model calculates the levels of radioactivity ground deposition. Those calculated levels, treated as data $\mathbf{Y}_{Kt} = (Y_{\mathbf{s}_{K_1,t}}, \dots, Y_{\mathbf{s}_{K_n,t}})'_t$, are combined with a joint prior distribution for the state vector $\Theta_t = (\Theta_{Kt}, \Theta_{Mt})'$ and measures $\mathbf{Y}_{Mt} = (Y_{\mathbf{s}_{M_1,t}}, \dots, Y_{\mathbf{s}_{M_m,t}})'$ of ground deposition, coming from (off-grid) fixed monitoring stations located at $\mathbf{s}_{M_i} \in \mathcal{S}$ ($i = 1, \dots, m$), to produce a posterior distribution for Θ_t at those locations.

More specifically, we define $Y_{\mathbf{s}_i,t}$ as the ‘observed’ average total radioactivity log deposition at site $\mathbf{s}_i$ ($i = 1, \dots, m + n$) at a time t . Also, $\mathbf{Y}_t = (\mathbf{Y}_{Kt}, \mathbf{Y}_{Mt})'$ where $\mathbf{Y}_{Kt}$ is the n -vector of log depositions calculated by the K-model at grid points and $\mathbf{Y}_{Mt}$ is the m -vector of log depositions at monitoring sites. The system or state vector Θ_t , associated with the observational vector $\mathbf{Y}_t$, has components Θ_{ct} ($c = K, M$) that are vectors with sub-vectors $\Theta_{\mathbf{s}_{c_i}t} = (\theta_{\mathbf{s}_{c_i}}, \dots, \theta_{\mathbf{s}_{c_i}})'_t$ ($i = 1, \dots, n$ for $c = K$ and $i = 1, \dots, m$ for $c = M$) with elements $\theta_{\mathbf{s}_i,t}$ interpreted as the real *unknown* log deposition at location $\mathbf{s}_i$ at time t . A Gaussian DSB model is defined by the following observation, spatial interpolation and time evolution equations:

$$\mathbf{Y}_t = \Theta_t + \nu_t, \quad \nu_t \sim N[\mathbf{0}, \Sigma_t], \quad (1)$$

$$\Theta_{Mt} = G_t \Theta_{Kt} + \epsilon_t, \quad \epsilon_t \sim N[\mathbf{0}, V_t] \text{ and} \quad (2)$$

$$\Theta_{Kt} = H_t \Theta_{Kt-1} + \omega_t, \quad \omega_t \sim N[\mathbf{0}, W_t] \quad (3)$$

respectively.

The observation equation (1) relates the observational vector $\mathbf{Y}_t$ with the corresponding state vector Θ_{st} where the observational error vector terms $\nu_t = (\nu_{Kt}, \nu_{Mt})'$ have components associated with grid points $\nu_{Kt} = (\nu_{s_{K_1}}, \dots, \nu_{s_{K_n}})'_t$ and with measuring points $\nu_{Mt} = (\nu_{s_{M_1}}, \dots, \nu_{s_{M_m}})'_t$, where $\nu_{s_{c_j}t}$ is the observational deposition error at location s_{c_j} of $c = K, M$ ($j = 1, \dots, n$ for $c = K$ and $j = 1, \dots, m$ for $c = M$). We assume those errors to follow a multivariate normal distribution with zero mean vector and $(m+n) \times (m+n)$ covariance matrix Σ_t . Assuming the observational errors at grid points, ν_{Kt} are independent of those at the measuring sites, ν_{Mt} , then the covariance matrix associated with ν_{st} in equation (1) is diagonal on those elements, i.e. $\Sigma_t = \text{diag}(\Sigma_{Kt}, \Sigma_{Mt})$. Now, the covariance matrix Σ_{Kt} should reflect in the model how we expect the errors of the K-model's outputs to be spatially correlated. Under the Markovian random field assumption we adopt, the covariance between any two of those errors is non-zero only if they are nearest neighbors. The observational errors at measuring sites are assumed independent and Σ_{Mt} is diagonal. Spatial correlations between grid and off-grid points are modelled through the interpolation equation (2). In the time domain, the observational errors are all assumed independent of each other.

Note that those independence assumptions between observational errors do not prevent correlations between the states to follow the Markovian structure we adopt. The spatial Markovian correlations between the real depositions represented by the state vector at grid and off-grid points are modelled through the interpolation equation (2) and the covariance of the prior distribution attributed to those states. Similarly, autocorrelations between states are modelled through the time evolution equation (3). The EDSB and the NWDSB models differ in the way they treat Σ_t as we shall see.

The interpolation equation (2) spatially relates the state vector at grid points, Θ_{Kt} with the state vector at off-grid points, Θ_{Mt} , where G_t is a known $(m \times n)$ spatial interpolation matrix that relates the real unknown log depositions at measuring sites with those at grid points, and $\epsilon_t = (\epsilon_{s_{M_1}}, \dots, \epsilon_{s_{M_m}})'_t$ is the vector of interpolation errors with components $\epsilon_{s_{M_j}t}$ being the interpolation error associated with site s_{M_j} and time t . The interpolation errors are assumed (spatially and temporally) independent and normally distributed with zero mean vector and fixed $(m \times m)$ variance matrix V_t with diagonal elements chosen as $v_{s_{M_i}t}^2 = v_t^2 d_i^*$ ($i = 1, \dots, m$). The variance v_t^2 is a fixed common interpolation variance (estimated from data) and d_i^* is the relative distance between location s_{M_i} and its nearest grid point relative to the largest of such distances amongst all measuring sites.

The spatial interpolation matrix G_t had its non-zero elements g_{ijt} fixed by a functional form that depends on the distance d_{ij} between two locations s_i and s_j where $s_j \in Ne\{s_i\}$,

with $Ne\{\mathbf{s}_i\}$ being the set of locations $\mathbf{s}_j$ that are *nearest-neighbors* to $\mathbf{s}_i$. For $\mathbf{s}_j \notin Ne\{\mathbf{s}_i\}$, $g_{ijt} = 0$. This form for G_t imposes a Markovian structure in the spatial component of the DSB models as mentioned previously. We have chosen the non-zero elements g_{ijt} to decay exponentially with distance such that $g_{ijt} = \alpha_{it} \exp\{-\lambda_{ijt}d_{ij}\}$ where the parameters α_{it} and λ_{ijt} were estimated from data.

The time evolution equation (3) relates the state vector Θ_{Kt} at time t with the state at $t-1$, Θ_{Kt-1} , where H_t is the $(n \times n)$ diagonal time evolution matrix with known elements h_{it} (for location $\mathbf{s}_{K_i}$ and time t) accounting for radioactivity decay between consecutive time steps $t-1$ and t . Based on the identification of the deposited radioisotopes, we have chosen $h_{it} \propto \sum_j \exp(-\lambda_{ij}t)$ where $\lambda_{ij} = \frac{\ln(2)}{T_{1/2}(j)}$ and $T_{1/2}(j)$ being the half-life of isotope type j , that is, the amount of time required for 50% of the radioactive atoms in a sample to undergo a radioactive (or nuclear) decay.

The time evolution errors $\omega_t = (\omega_{\mathbf{s}_{K_1}}, \dots, \omega_{\mathbf{s}_{K_n}})'_t$ are assumed independent and normally distributed with zero mean vector and variance matrix W_t with diagonal elements $w_{\mathbf{s}_{K_i},t}^2 = \text{var}(\omega_{\mathbf{s}_{K_i},t})$ being the variance of $\omega_{\mathbf{s}_{K_i},t}$ at location $\mathbf{s}_{K_j}$. We have chosen those variances as $w_{\mathbf{s}_{K_i},t}^2 = w_{\mathbf{s}_{K_i}}^2$ ($i = 1, \dots, n$) based on estimations from the available data. Note that the association between Θ_{Kt} and $\Theta_{K,t-1}$ in equation (3) is indirectly carried out to the off-grid measuring sites through the interpolation equation (2).

Further to equations (1), (2) and (3), a DSB model will also need to have the joint prior probability density for the unknown parameters specified at the initial time $t = 0$. Before describing the prior density, which specification will depend on the set of parameters assumed unknown, we define each of the two DSB models we adopt. Both models have the common components G_t, H_t, V_t and W_t described above but differ on the way the observational covariance matrix, Σ_t , is treated. The EDSB model treats Σ_t as known at all times with elements having a negative exponential form while the NWDSB model assumes Σ_t to be unknown and adopts an algorithm that allows it to be sequentially estimated from available data.

2.1 The EDSB model

The EDSB model is such that each element $\sigma_{\mathbf{s}_{K_i}, \mathbf{s}_{K_j}, t} = \text{cov}(\nu_{\mathbf{s}_{K_i}, t}, \nu_{\mathbf{s}_{K_j}, t})$ ($i, j = 1, \dots, n$ and $\mathbf{s}_{K_j} \in Ne\{\mathbf{s}_{K_i}\}$) of Σ_{Kt} is given by

$$\sigma_{\mathbf{s}_{K_i}, \mathbf{s}_{K_j}, t} = \sigma_{\mathbf{s}_{K_i}, t}^2 \exp(-\eta_{\mathbf{s}_{K_i}, \mathbf{s}_{K_j}, t} d_{ij}) \quad (4)$$

where $\sigma_{\mathbf{s}_{K_i}, t}^2$ is the observational error variance at grid point $\mathbf{s}_{K_i}$, $\eta_{\mathbf{s}_{K_i}, \mathbf{s}_{K_j}, t} > 0$ is a fixed scalar reflecting the strength of the spatial decay in the covariance, and d_{ij} is the distance

between points $\mathbf{s}_{K_i}$ and $\mathbf{s}_{K_j}$. For any point $\mathbf{s}_{K_j} \notin Ne\{\mathbf{s}_{K_i}\}$, $\sigma_{\mathbf{s}_{K_i}, \mathbf{s}_{K_j}, t} = 0$.

Note that $\sigma_{\mathbf{s}_{K_i}, t}^2$ and $\eta_{\mathbf{s}_{K_i}, \mathbf{s}_{K_j}, t}$ above could all have been treated as unknowns in the usual Bayesian paradigm, however that would lead to the loss of analytical tractability in obtaining the posterior and the predictive distributions of the parameters and variables. Similarly to Kennedy and O'Hagan (2001), we chose to 'plug-in' estimated values for those in our application at the expense of obtaining understated uncertainties in the prediction and posterior inferences. The extent of such understatement is not believed to be considerable though for reasons similar to those stated by Kennedy and O'Hagan (2001), that is the apparent lack of great pragmatic sensitivity (particularly in kriging) in geostatistical applications.

2.2 The NWDSB model

The NWDSB model, assumes Σ_t to be *unknown* and attributes a normal prior distribution with parameters $\mathbf{a}_t$ and R_t to Θ_t conditional on Σ_t , and on information $\mathbf{D}_{t-1} = \{\mathbf{y}_{t-1}, \mathbf{D}_{t-2}\}$, i.e. $(\Theta_t | \Sigma_t, \mathbf{D}_{t-1}) \sim N[\mathbf{a}_t, R_t]$. It also attributes a prior inverse-Wishart distribution with d_{t-1} degrees of freedom and scale matrix Ψ_{t-1} to the uncertain Σ_t conditional on $\mathbf{D}_{t-1}$, i.e. $(\Sigma_t | \mathbf{D}_{t-1}) \sim IW[\Psi_{t-1}, d_{t-1}]$ where *IW* indicates an inverse-Wishart distribution. The joint distribution of $(\Theta_t, \Sigma_t | \mathbf{D}_{t-1})$ is then a normal-inverse-Wishart model, i.e. $(\Theta_t, \Sigma_t | \mathbf{D}_{t-1}) \sim NIW[\mathbf{a}_t, R_t, \Psi_{t-1}, d_{t-1}]$.

We have adopted an extension of the prior-to-posterior updating approach proposed by Barbosa and Harrison (1992) for the temporal multivariate DLM to include the spatial components of our model. This approach avoids the pragmatic problem usually caused by the numerical computation of posterior covariance matrices in the Bayesian approach where positive semi-definiteness and symmetry must be preserved. The Barbosa and Harrison (1992) approach is an approximate conjugate algorithm that factorises covariance matrices by an eigenvalue decomposition method that (unlike Cholesky's decomposition) is invariant to permutations of the elements of $\mathbf{Y}_t$. It has also shown to improve on other filters such as the Robust filter introduced by Masreliez (1975) and used by West (1982) as an approximation for the marginal posterior distribution of the state parameter vector in a multivariate DLM.

Note that to run a DSB model, apart from specifying the required values for the common components associated with its formulation, the analyst will also need to choose the initial prior hyperparameters accordingly.

2.3 Initial prior densities

After specifying the common components of the DSB models, the analyst will also need to assess for both models an initial Gaussian prior density at $t = 0$ for their unknown parameters, Θ_0 for the EDSB model, and (Θ_0, Σ_0) for the NWDSB model.

Note that because Θ_{Mt} relates to Θ_{Kt} through the spatial interpolation equation (2) we only need to specify a prior density for Θ_{Kt} . Thus, initially we choose for the EDSB model the initial values of the mean n -vector $\mathbf{a}_{K0}$ and the $(n \times n)$ covariance matrix R_{K0} of the normal distribution of Θ_{K0} conditional on any initial information $\mathbf{D}_0$, i.e. $(\Theta_{K0}|\mathbf{D}_0) \sim N[\mathbf{a}_{K0}, R_{K0}]$. Equation (2) implies that $(\Theta_{M0}|\mathbf{D}_0) \sim N[G_0\mathbf{a}_{K0}, G_0R_{K0}G_0' + W]$.

For the NWDSB model we choose, at $t = 0$, a normal inverse-Wishart prior for $(\Theta_{K0}, \Sigma_{K0})$ given $\mathbf{D}_0$, that is $(\Theta_{K0}, \Sigma_{K0}|\mathbf{D}_0) \sim NIW[\mathbf{a}_{K0}, R_{K0}, \Phi_{K0}, d_0]$ where Φ_{K0} and d_0 are the initial $n \times n$ covariance matrix and the degrees of freedom (respectively) of the prior inverse-Wishart density for Σ_{K0} .

Those prior parameters should be chosen to reflect the user's degree of information about the deposition process. In case of no prior knowledge about Θ_0 , a non-informative Gaussian prior density can be used. Similarly, for Σ_0 a non-informative inverse Wishart prior can be adopted. There are a number of ways to choose proper non-informative priors for multivariate Gaussian processes and the reader can for example refer to Kass and Wasserman (1996) and Liechty *et al.* (2004) for general reviews. In our case, we have chosen prior mean values to reflect the normal radioactivity levels before the nuclear accident. We have also imposed a structure to the initial inverse Wishart prior specification of the NWDSB model with elements having the functional spatial exponential decay form similar to the EDSB model in equation (4) above. The prior variances for the grid points relative to the K-model's depositions were deliberately chosen large comparatively to those at monitoring stations to reflect our uncertainty about the K-model's outputs.

After those values are chosen, data assimilation and the Bayesian sequential prior-to-posterior updating for those models can be performed by the algorithms described in the following subsection. Those algorithms require the inversion of large but typically sparse covariance matrices. In their implementation we have adopted both the Cholesky and the eigenvalue decomposition methods (similarly to Barbosa and Harrison, 1992) to guarantee positive semi-definiteness and facilitate the inversion of the covariance matrices in our application.

2.4 Posterior and forecasting densities

In general the sequential approach for prior-to-posterior density updating starts from the posterior density for the unknown parameters at a time $t - 1$, before observing $\mathbf{Y}_t$, from which the prior density at time t is obtained together with the forecasting density for $\mathbf{Y}_t$ given $\mathbf{D}_{t-1}$. After $\mathbf{Y}_t$ is observed, the forecasting error is used to determine the posterior density at t as follows.

2.4.1 The Normal-normal DSB model

For the normal-normal EDSB model, at time $t - 1$, the posterior density $(\boldsymbol{\Theta}_{t-1}|\mathbf{D}_{t-1}) \sim N[\mathbf{m}_{t-1}, C_{t-1}]$. At time t , before observing $\mathbf{Y}_t$, the prior density $(\boldsymbol{\Theta}_t|\mathbf{D}_{t-1}) \sim N[\mathbf{a}_t, R_t]$ where $\mathbf{a}_t = (\mathbf{a}_{Kt}, \mathbf{a}_{Mt})'$ with $\mathbf{a}_{Kt} = H_t \mathbf{m}_{K,t-1}$ and $\mathbf{a}_{Mt} = G_t H_t \mathbf{m}_{K,t-1}$, and

$$R_t = \begin{pmatrix} H_t C_{K,t-1} H_t' + W_t & G_t H_t C_{K,t-1} H_t' + G_t W_t \\ H_t C_{K,t-1} H_t' G_t' + W_t G_t' & G_t H_t C_{K,t-1} H_t' G_t' + G_t W_t G_t' + V_t \end{pmatrix},$$

where $C_{K,t-1}$ is the submatrix component of C_{t-1} associated with the grid points only.

The one-step-ahead forecasting density is

$$(\mathbf{Y}_t|\mathbf{D}_{t-1}) \sim N[\mathbf{f}_t, Q_t] \quad (5)$$

where $\mathbf{f}_t = \mathbf{a}_t$ and $Q_t = R_t + \Sigma_t$.

After observing $\mathbf{Y}_t = \mathbf{y}_t$, the posterior density is $(\boldsymbol{\Theta}_t|\mathbf{D}_t) \sim N[\mathbf{m}_t, C_t]$ where $\mathbf{m}_t = \mathbf{a}_t + A_t \mathbf{e}_t$ and $C_t = R_t - A_t Q_t A_t'$ with $A_t = R_t Q_t^{-1}$ and $\mathbf{e}_t = \mathbf{y}_t - \mathbf{f}_t$ is the one-step-ahead forecasting error.

All the above updating equations are straightforward to obtain from the standard multivariate Gaussian distribution theory or (more generally) via Bayes' procedure as in Theorem 4.1 of West and Harrison (1997). They can be seen as extensions of the Kalman filter to include the spatial component.

2.4.2 The normal-inverse-Wishart DSB model

For the NWDSB model, we have applied the standard normal-inverse-Wishart distribution theory as in Le and Zidek (1992) together with an algorithm similar to Barbosa and Harrison (1992). This algorithm is based on the eigenvalue decomposition of the covariance matrices that preserve their positive semi-definiteness after inversion. That is,

considering the covariance matrix Σ with eigenvalue matrix Λ and eigenvector matrix L , $\Sigma = L\Lambda L' = S^2$ where $S = L\Lambda^{1/2}L'$ is unique.

At time $t - 1$, the posterior density $(\Theta_{t-1}, \Sigma_{t-1} | \mathbf{D}_{t-1}) \sim NIW[\mathbf{m}_{t-1}, S_{t-1} C_{t-1} S_{t-1}, \Phi_{t-1}, d_{t-1}]$ where $NIW[\cdot]$ represents a normal-inverse-Wishart density. For clarity, we can factorize the joint density for $(\Theta_{t-1}, \Sigma_{t-1} | \mathbf{D}_{t-1})$ into the product of two conditional densities for $(\Theta_{t-1} | \Sigma_{t-1}, \mathbf{D}_{t-1})$ and $(\Sigma_{t-1} | \mathbf{D}_{t-1})$, where $(\Theta_{t-1} | \Sigma_{t-1}, \mathbf{D}_{t-1}) \sim N[\mathbf{m}_{t-1}, S_{t-1} C_{t-1} S_{t-1}]$ and $(\Sigma_{t-1} | \mathbf{D}_{t-1}) \sim IW[\Phi_{t-1}, d_{t-1}]$.

Thus, the prior densities at time t , before observing $\mathbf{Y}_t$, are $(\Theta_t | \Sigma_{t-1}, \mathbf{D}_{t-1}) \sim N[\mathbf{a}_t, S_{t-1} R_t S_{t-1}]$ and $(\Sigma_t | \mathbf{D}_{t-1}) \sim IW[\Phi_{t-1}, d_{t-1}]$, where $\mathbf{a}_t = H_t \mathbf{m}_{t-1}$, $R_t = H_t C_{t-1} H_t' + W_t$ and $S_{t-1}^2 = \frac{1}{d_{t-1}} \Phi_{t-1}$.

The one-step-ahead forecasting density is

$$(\mathbf{Y}_t | \mathbf{D}_{t-1}, \mathbf{a}_t, R_t, \Phi_{t-1}, d_{t-1}) \sim St[\mathbf{f}_t, S_{t-1} Q_t S_{t-1}, d_{t-1}] \quad (6)$$

where $St[\mathbf{f}_t, S_{t-1} Q_t S_{t-1}, d_{t-1}]$ is a multivariate Student-t density with mean vector $\mathbf{f}_t = \mathbf{a}_t$, covariance matrix $S_{t-1} Q_t S_{t-1}$ with $Q_t = I + R_t$, I an $(m + n) \times (m + n)$ identity matrix, and d_{t-1} degrees of freedom.

The posterior densities, after observing $\mathbf{Y}_t = \mathbf{y}_t$, are $(\Theta_t | \mathbf{D}_t) \sim N[\mathbf{m}_t, S_t C_t S_t]$ and $(\Sigma_t | \mathbf{D}_t) \sim IW[\Phi_t, d_t]$ where $\mathbf{m}_t = \mathbf{a}_t + A_t \mathbf{e}_t$, $C_t = S_t(R_t - B_t Q_t B_t') S_t$, $S_t^2 = \frac{1}{d_t} \Phi_t$, $d_t = d_{t-1} + 1$, and $\Phi_t = \Phi_{t-1} + S_{t-1} U^{-1} \mathbf{e}_t \mathbf{e}_t' U^{-1} S_{t-1}'$. Here, $U_t^2 = Q_t$, $B_t = R_t Q_t^{-1}$, $A_t = S_t B_t S_t^{-1}$ and $\mathbf{e}_t = (\mathbf{e}_{Kt}, \mathbf{e}_{Mt})' = (\mathbf{y}_{Kt} - H_t \mathbf{m}_{K,t-1}, \mathbf{y}_{Mt} - G_t H_t \mathbf{m}_{K,t-1})'$.

Now that the DSB models we adopt in the application section of this paper are defined, we proceed to describe the data used for assimilation and the results of the space-time forecasting exercise.

3 Chernobyl's radioactivity deposition in Bavaria

We have applied the three DSB models to data of daily average radioactivity deposition in the German Bavarian region over a period of 15 days at the time of the Chernobyl nuclear accident. The measured deposition data was obtained from time series of daily near-ground radioactivity air concentrations in 13 fixed monitoring stations provided by the Bavarian office for the environment.

The atmospheric release from the Chernobyl nuclear accident started on the 26 April 1986 (considered as day 1 here) and lasted for about 10 days. Most of the released material was deposited as dust and debris near Chernobyl, but lighter materials, like the isotopes

described below, were carried by wind over Ukraine, Belarus, Russia, Scandinavia and Europe, including Bavaria, in southern Germany.

Figure 1: (a) The estimated daily total radioactive releases from the Chernobyl nuclear accident between 26 April (day 1) and 10 May 1986 (day 15); (b) The 13 fixed radioactivity monitoring stations in Bavaria and the 4096 points of the 64×64 regular grid over the region. The distance between consecutive grid points is $8Km$.

Figure 1(a) shows the time series of the estimated daily atmospheric release from Chernobyl. Notice that, after the initial peak on day 1, the release practically halved in day 2. From days 3 to 6 the release slightly decreased further, after which it started to grow again to reach a second peak on day 10. The release stopped on day 10 when the reactor's core was finally isolated. The values in Figure 1(a) were based on the OECD's Nuclear Energy Agency (NEA, 2002) initial assessment of releases made by scientists from sampled material deposited in the former Soviet Union.

The meteorological conditions at Chernobyl during the release phase of the accident together with the varying characteristics of the release led to a complex pattern of atmospheric transport and ground deposition in space and time. The general rise in measured radioactivity deposition levels in Bavaria on the fourth and fifth days after the accident seemed consistent with the fact, reported by NEA (2002, page 44) that “initially the wind was blowing in a Northwesterly direction and was responsible for much of the deposition in Scandinavia, the Netherlands and Belgium and Great Britain. Later the plume shifted to the South and much of Central Europe, as well as the Northern Mediterranean and the Balkans, received some deposition, the actual severity of which depended on the height of the plume, wind speed and direction, terrain features and the amount of rainfall that

occurred during the passage of the plume.”

The measured data in Bavaria, $\mathbf{Y}_{Mt}$, were taken from $m = 13$ fixed monitoring stations while the predicted deposition data $\mathbf{Y}_{Kt}$ from the K-model were determined for the $n = 4,096$ points of a fixed 64×64 regular grid over the Bavarian region as can be seen in Figure 1(b). The distance between consecutive grid points were $8Km$. The data consisted of daily average of total radioactivity near-ground concentrations (in Bq/m^3).

3.1 The measured data

The 13 monitoring stations were rather spatially sparse and few in numbers to cover the region of approximately $70,550Km^2$ as can be seen in Figure 1(b). Most of them were located in the south of the region with the majority of those situated towards the south-east. There was a single station in the center-east which was quite isolated distancing by $120Km$ from its nearest station. The north had only three stations which were also located far apart from each other. A large area in the center-west had no monitoring stations whatsoever. Further to those, there are about three clusters of more closely located stations near Munich (stations 1,2 and 3), Essenbach (stations 7 and 8) and Gundremmingen (stations 5 and 6), all in the south. The two closest of those stations (7. Essenbach and 8. Niederaichach) were in the south east and separated by a distance of approximately $10 Km$.

Figure 2 shows the time series plots of measured (solid lines) daily average depositions at the 8 monitoring stations where depositions were significantly larger than the normal levels in the period from the 26 April (day 1) to 10 May 1986 (day 15). All but one of those stations were in southern Bavaria, the only exception being Schwandorf located in the center-east. The remaining stations omitted here (1. Egling and 4. Garching in the south, and 11. Kahl, 12. Grafenrheinfeld and 13. Hof in the north) showed relatively low depositions in the period. We can notice from those plots that in general the measured radioactivity concentrations (solid lines) were at the normal background levels of $0.012Bq/m^3$ on average from days 1 to 4. After that period, there were sudden changes in the measured deposition levels at different periods of time. Most stations showed peaks on days 5 and 6 with the measurements changing to much higher levels in the range from 10 to about $120Bq/m^3$. The only exception being station 5 (Wasserburg) which only showed a single significant peak of about $60Bq/m^3$ on day 11. Those higher depositions then dropped to lower levels after day 7 with approximately negative exponential trends with differing decrease rates. Exceptions to that behavior were stations 5 and 8 (Niederaichach) with station 8 showing an approximately constant level around $30Bq/m^3$ from day

Figure 2: Measured (solid line) and K-model predicted (dashed line) daily average total radioactivity deposition (solid line), from 26 April ($t = 1$) to 10 May 1986 ($t = 15$), at (a) 2. München, (b) 3. München-Ebersberg, (c) 5. Wasserburg, (d) 6. Gundremmingen, (e) 7. Essenbach, (f) 8. Niederaichbach, (g) 9. Passau, and (h) 10. Schwarzdorf stations in Bavaria.

6 to day 11 after which it showed an approximately linear negative trend to a level around $10Bq/m^3$. Station 9 showed a second peak (much smaller than the initial one) around day 10, and only three stations (6, 7 and 8) showed levels (all lower than $10Bq/m^3$) larger than background at day 15.

The general measured deposition behavior described above seems consistent with both the releases at source and the weather conditions in Europe at the time. The second peak in the source term release on day 10 seems to have had not much effect on the measured depositions in Bavaria after that day.

3.2 The depositions by the K-model

The K-model is a non-linear first-order approximation of a two-dimensional long-range advection-diffusion process that projects in time and space expected radioactivity concentrations. It accounts for the total radioactivity release at source as well as wind speed amongst others. In our application, it has taken the estimated average daily source term releases from Chernobyl shown in Figure 1(a) and projections (towards Germany) of the prevailing wind speed vector as inputs to calculate its predicted radioactivity depositions at the 4096 points of the grid in Bavaria. Those predicted values were also adjusted to account for local rainfall measures in the region at the time as described in De (2009). Because of the distance (of approximately $1900Km$) as well the prevailing weather conditions there was an average time delay of 3 days between radioactivity release in Chernobyl and deposition in Bavaria. So, the K-model predicted deposition values $\mathbf{Y}_{Kt}$ considered both source term releases and wind speeds in Chernobyl at day $t - 3$.

The K-model depositions for the initial 3 days were low (at natural levels) for the whole region. On day 4 there were relatively small increases in the predicted depositions over the region (in the range $2.9 - 3.7Bq/m^3$). Figure 3 shows the contour plots of the calculated depositions for (a) day 5 (30 April) and (b) day 6 (1 May 1986). For day 5, they increased considerably almost everywhere in the region to the range $30 - 40 Bq/m^3$ followed by $35 - 43Bq/m^3$ on day 6 with the western and southern Bavaria points showing the largest predicted values (in the range $40 - 43Bq/m^3$). In general, the contour plots of the K-model outputs showed smooth decreases in the predicted deposition in the southwestern direction. Larger localised depositions occurred at places (in the south) where rainfall of some intensity occurred those days. Rainfall intensity measures from 24 fixed stations fairly evenly scattered through the whole Bavarian region were considered in the K-model deposition values at grid points. On day 7, the K-model deposition levels dropped to the range $13 - 14Bq/m^3$ almost everywhere followed by $8 - 12Bq/m^3$ from

Figure 3: Contour plots of the K-model's calculated total radioactivity deposition in Bavaria for days (a) 5 (30 April) and (b) 6 (1 May) after Chernobyl's initial release on 26 April 1986.

days 8 to 12. From days 13 to 15, the predicted levels were even lower varying in the range $1.5 - 4Bq/m^3$ almost everywhere. Those deposition levels were consistent with the general rainfall pattern over Bavaria over the period, as measured by the 24 rainfall gauging stations, where no rainfall was measured anywhere on day 7. From days 8 to 10, no rainfall intensities were measured in the central and southern regions, and only a little in the northern. On day 11, the average rainfall increased slightly reaching a peak on day 13 and dropping again to lower levels after that.

Figure 2 also shows the time series plots of the K-model depositions predicted at 8 monitoring sites (dashed lines). Notice that in general the K-model predictions followed reasonably well the behavior of the actual measured depositions (solid lines) in time however with usually lower values. The only exceptions were at the Wasserburg station, Figure 2(c), and the other 5 stations (not showed here) where not only the K-model depositions were larger but also the general pattern, of sudden increases with peaks around days 5 and 6 followed by approximately exponential decreases after that, was not matched by the measurements. Despite the tendency to underestimate the measurements on the days of largest depositions, the K-model showed a reasonably good overall performance with a root mean square error of $16.24Bq/m^3$ on average over the period.

4 Deposition forecasting by the DSB models

Once the values of the common components G_t, H_t, V_t and W_t (for all t) as well as the initial hyperparameters of each prior density were chosen, the three DSB models were sequentially applied to assimilate the deposition data $\mathbf{Y}_t$ described in the previous section. This sequential approach, produced at each time $t = 1, \dots, 15$ the one-step-ahead forecasting densities of equations (5) and (6) accordingly for each model.

Figure 4: Contour plots of the one-day-ahead forecasting means of total deposition in Bavaria by the EDSB and the NWDSB models for days 5 and 6.

In general, both DSB models produced forecasts which seem to adapt fast to changes in the 'observed' (measured and K-model predicted) data. For example, the one-step-ahead

forecasts for day 5 had mean values varying in the range $30 - 160$ and $0 - 25Bq/m^3$, as can be seen in plots (a) and (b) of Figure 4, for the EDSB and the NWDSB models respectively. Those forecasts were made on day 4 after a change from natural levels to higher levels in the 'observed' depositions as described in the previous subsection. For day 6, the forecasts were even larger in general as the ranges of values in the plots (c) and (d) in Figure 4 indicate. The mean forecasts varied in the range $150 - 450Bq/m^3$ for the EDSB model and in the range $10 - 200Bq/m^3$ for the NWDSB model.

We can also note from the plots of Figure 4 the lower range forecasts were in general for the north-western Bavaria. Further to those, the models forecasts showed further increases for day 7 after which (when the 'observed' values fell considerably almost everywhere) they decreased day after day in the period. In general, they usually displayed for all times (similarly to the K-model) their spatial smoothness broken down by local inhomogeneities due to rainfall and assimilation of measured data. Despite this general behavior, there were clear differences in the actual forecasts produced by the two models. Both models showed remarkable similarity in the shapes of their contours of forecasting means but not in their actual forecasted levels with the EDSB model consistently producing larger means.

Figure 5: Contour plots of the one-day-ahead forecasting standard deviations of total deposition in Bavaria by the EDSB and the NWDSB models for day 5.

In general, the uncertainties associated with the one-step-ahead forecasting means, represented by the standard deviations of the forecasting densities, for both models were rather spatially homogeneous at every day with small differences between the largest and

the smallest values. In time, however, both models showed considerable increases in their standard deviations at times when there were large overall changes in the data. In fact, for day 5, the forecasting standard deviations of the EDSB model showed an increase from the range $0.2 - 0.9Bq/m^3$ (on day 4) to $5 - 5.6Bq/m^3$ as can be seen in Figure 5 (a). After this initial increase, the forecasting variances of the EDSB model only decreased slightly in time. The NWDSB model on its turn showed considerably smaller forecasting variances compared to the EDSB model. It also seemed more robust to changes after changes in the data. For example, for day 5, its range of forecasting standard deviations increased from $0.45 - 0.75Bq/m^3$ (on day 4) to $0.5 - 0.8Bq/m^3$ as can be seen in Figure 5 (b). The largest change occurred on day 8 when the range increased from $0.45 - 0.8Bq/m^3$ (on day 7) to $1.0214 - 1.0221Bq/m^3$. From day 8, the NWDSB forecasting variances did not change much in time. Spatially the uncertainties associated with the forecasts by NWDSB model were considerably smoother than those associated with the EDSB model. An example of this can be seen in the uncertainty contour plots Figure 5. It worth recalling that, due to the adopted plug-in values for the models hyperparameters together with the K-model outputs being smoother than the real data, those variances are in fact deflated approximations of the real values. In particular the NWDSB model, comparatively to the EDSB model, seems to display rather low uncertainties.

Formal assessment of forecasting performance of models using the cumulative product of given one-step-ahead forecasting densities can be applied to the entire multivariate models. For simplicity and economy (having in mind that the differences in performances are very clear), however, we have assessed each model forecasting performance by comparing the means of its forecasting densities interpolated to each of the 13 monitoring sites with the actual measurements taken at those sites. We have compared each model's *root average mean squared errors* (RAMSE) as well as analysed their plots of interpolated forecasts against measurements.

Comparing the interpolated means of the forecasting densities with the measured depositions at the monitoring stations, we notice in Figure 6 that the NWDSB model (solid line) produced the best results (closest to the measured values) at all times from day 5 to day 15. In fact, the NWDSB model produced by far the best forecasts (closest to the measured values) in terms of the daily RAMSE as well as over the whole period showing an average gain of 30.6% over the EDSB model. As we have seen above, the one-day-ahead forecasted depositions increased from practically natural levels from days 1 to 4 to much higher levels in the period from days 5 to 7. Both models' performances decreased during this later period with the RAMSE of the NWDSB model going from 1.63 on day 5 to a peak at about $15Bq/m^3$ on day 7. Similarly, the EDSB model's RAMSE

Figure 6: The RAMSE for the one-day ahead forecasts for the EDSB (dashed line) and the NWDSB (solid line) models.

went from about 10 on day 5 to about 18 Bq/m^3 on day 7. After day 7 both models performances increased with the almost linear negative trend decrease in their RAMSEs. In a similar exercise, the NWDSB model also produced the best performances for three-days-ahead forecasts showing an average improvement of 18.5% over the EDSB model (see De, 2009; De and Faria, 2009). On the basis of the RAMSE alone, the NWDSB model with learning from data on the spatial variance-covariance matrix clearly outperformed the EDSB model. Further to those, interpolations of the one-day-ahead forecasts by the NWDSB model to the largest cities in Bavaria (Augsburg, München, Ingolstadt, Nürnberg and Würzburg) compared well in general with interpolations of measurements only from the nearest stations as described in De (2009).

Figure 7 illustrate the performances of the DSB models. That figure shows the plots of the interpolated deposition forecasts from the (a) EDSB and (b) the NWDSB models versus the measured depositions at the 13 monitoring sites during the 15 days. Matches between interpolated forecasts and measurements would fall on the solid line of slope 1 (forecasts = measurements) in those plots. It is clear from Figure 7 that both DSB models in general produced interpolated forecasts which were larger than the measurements. The interpolated EDSB forecasts were usually larger than those of the NWDSB model. In fact, only 2.56% of the EDSB model forecasts were smaller than the measured values while that figure raises to 6.41% for the NWDSB model. Also, the residual means and standard deviations were (104.69, 109.86) and (38.60, 46.14) for the EDSB and the NWDSB models respectively, showing the much larger variability in the forecasts of the former model. Despite the seemingly excessive over-forecasting by the EDSB model, it is worth pointing

(a)

(b)

(c)

(d)

Figure 7: The interpolated forecasting depositions versus measurements, at all 13 sites during the 15 days period, for the (a) EDSB and (b) NWDSB models. Quantile-quantile plots for the interpolated forecasting means of the (c) EDSB and (d) NWDSB models.

out that on the days and sites where the depositions were largest its forecasts were much closer to the measurements.

An analysis of the residual quantile-quantile plots of those models in Figure 7(c) and (d) shows a slightly better fit of the data to the forecasting distribution of the NWDSB model than the EDSB model. However, both plots show heavy-tailed characteristics partly due to the local features of the process that the DSB forecasting models are failing to predict but also due to those models forecasting distributions having tails that are too thin.

The DSB models forecasts (and their performances) could have been improved had more measured data at extra monitoring stations been available for assimilation and evaluation. Naturally, better estimation of deposition levels at grid points by the K-model (or another atmospheric dispersal model) that accounted for local weather patterns and forecasts could crucially help improve the quality of predictions by a DSB model. Despite that, the NWDSB model in particular still seemed to have produced relatively good forecasting under the restrictions we faced in this application. Despite not presented in this article, the results for those models posterior densities of the true depositions follow in general similar behavior to their forecasting densities described above albeit with comparatively smaller posterior means and variances (see De, 2009, for details).

5 Discussion

A Gaussian DSB model combines, in analytical closed form, the spatio-temporal outputs from a complex numerical model with available measured data (and prior information) to provide statistical inputs that can be used by another model in a chain. It can also provide statistical predictions in real-time useful to support decision making. In this paper we have investigated the forecasting performances of two variations of a DSB model in an application to the radioactivity deposition from the Chernobyl accident in Bavaria, southern Germany. Despite spatial kriging for a fixed time being straightforward to obtain by augmentation of vectors and matrices, we have focused on the temporal predictions of the space-time process. This application used the outputs of a well-known long range atmospheric dispersal model, the K-model, as surrogate for radioactivity deposition data at the points of a regular grid over the region of interest. Those outputs, treated as data, were combined with off-grid in-situ measurements of deposition and prior information (in the form of probability density functions) about the real (but unknown) deposition process to obtain space-time posterior and forecasting probability densities for that process.

Because there is far more grid points than measuring stations, the quality of the statistical inferences obtained depends in great part on the quality of the K-model's outputs.

The obvious difficulty in evaluating the K-model (as well as the DSB models) in our application was the lack of real measured data at grid point locations. However, comparisons with the estimated source term releases as well as with the few available in-situ deposition measurements, have shown those outputs to follow the general shape of the measured depositions in time albeit with usually lower assessment of their levels at times when those were large. Spatially, those outputs coming from a long range atmospheric dispersion model were rather smooth even after accounting for some localised rainfall measurements. This smoothness had an undesired effect on the NWDSB model in particular which tended to obtain quite small posterior and predictive spatial variances. Despite the tendency to underestimate the measurements on the days of largest depositions, the K-model showed an acceptable overall performance as we have seen in Section 3. An extension of the DSB model to include the estimation and removal of the K-model bias should help improve its outputs. Further to that, more sophisticated atmospheric dispersal models, such as the Gaussian plume RIMPUFF model (Mikkelsen *et al.*, 1984), could be adopted to produce better quality outputs with consequent improvement of the DSB models.

Certainly, a fully Bayesian approach that explicitly accounts for uncertainties about the K-model's outputs can be adopted. This would imply in the loss of the analytical closedness obtained in this paper (and useful in real time applications) as numerical integration approaches are required. Alternatively, an approach such as in Kennedy and O'Hagan (2001) could be employed where emulations of outputs from the K-model are obtained for suitable variations of its inputs and measures of uncertainty about them constructed. However, this approach is also computer intensive and not practical for real time applications though.

The analysis of the one-step-ahead forecasts of the EDSB and the NWDSB models, showed the later to produce a far better overall forecasting performance. It produced in average an improvement of 30.6% in the average root MSE statistics. The DSB models which assimilated both the K-model outputs as well as the available measured data, tended to produce forecasts which were generally larger than the next day's measurements. In particular, the EDSB model consistently produced higher forecast levels when compared with the NWDSB model. Despite that, on days and locations where measurements were largest, the EDSB model forecasts practically matched those measurements. This general behavior by both models can be partly explained by the use of prior densities with larger variances for the K-model's outputs comparatively with those for the process at measuring

sites. However, at sites where measured depositions were at natural levels (mainly in northern Bavaria) the comparatively very large forecasts implied in rather large errors in the computation of the average MSE. The local dependency of the deposition process on weather conditions at every grid point location makes it hard to obtain very accurate estimates (and predictions) by the DSB or any other statistical models when information (and predictions) about those are scarce or unavailable.

A natural extension of the DSB models, and subject of a forthcoming paper, to allow the inclusion of covariates (such as measures of precipitation and bias in the K-model) involves a matrix-variate formulation with the use of the family of hyper-inverse Wishart and matrix-variate normal distributions. Further to that, and to help with the validation of the DSB models in radioactivity deposition applications, a non-analytical Bayesian formulation with the use of MCMC methods for obtaining posterior and predictive distributions which account for uncertainties on the hyperparameters of the DSB models is also envisaged for future research. An expected pragmatic advantage of that formulation would be a better quality estimation of the posterior densities achieved by the inclusion of the process non-linearities in the methodology. However, in this non-linear setting, complications associated with unidentifiability and convergence are likely to be significant.

Finally, the DSB models proposed in this paper provide a flexible framework for modelling radioactive deposition in time and space. A more general formulation shown in De (2009) and De and Faria (2009) allows the modelling of nuclide specific radioactivity deposition by including three components: total ground radioactivity deposition, proportions of radionuclides in the total deposition, and fraction of wet deposition from the total. They can be adapted for use in different countries with different measuring resources. Further to that, they should in principle be applicable in the modelling of air pollution or similar processes in general where available data sets are discrete in time and continuous in space, including data observed at irregular grids.

References

- Barbosa, E. and Harrison, J. (1992). Variance estimation for multivariate dynamic linear models. *Journal of Forecasting*, **11**, 621–628.
- Besag, J. E. (1974). Spatial interaction and the statistical analysis of lattice systems. *Journal of the Royal Statistical Society B*, **36**, 192–225.

- Conti, S., Gosling, J., Oakley, J., and O’Hagan, A. (2009). Gaussian process emulation of dynamic computer codes. *Biometrika*, **96**, 663–676.
- Cressie, N. A. C. (1993). *Statistics for Spatial Data*. John Wiley and Sons, NY, revised edition edition.
- De, S. (2009). A bayesian space-time dynamic linear model for radioactivity deposition after a nuclear accident. Unpublished PhD Thesis, The Open University.
- De, S. and Faria, A. E. (2009). A hierarchical bayesian space-time model for radioactivity deposition. Technical Report 09/06, Department of Statistics, The Open University, UK.
- Fuentes, M. and Raftery, A. E. (2005). Model evaluation and spatial interpolation by bayesian combination of observations with outputs from numerical models. *Biometrics*, **61**, 36–45.
- Gamerman, D., Salazar, E., and Reis, E. A. (2007). Dynamic gaussian process priors with applications to the analysis of space-time data. In J. M. Bernardo, M. Bayarri, J. Berger, A. Dawid, D. Heckerman, A. Smith, and M. West, editors, *Bayesian Statistics*, number 8. Oxford University Press.
- Kass, R. E. and Wasserman, L. (1996). The selection of prior distributions by formal rules. *Journal of the American Statistical Association*, **91**(435), 1343–1370.
- Kennedy, M. C. and O’Hagan, A. (2001). Bayesian calibration of computer codes. *Journal of the Royal Statistical Society, Series B*, **63**, 425–464. (with discussion).
- Lauritzen, B. and Mikkelsen, T. (1999). A probabilistic dispersion model applied to the long-range transport of radionuclides from the chernobyl accident. *Atmospheric Environment*, **33**, 3271–3279.
- Lauritzen, B., Baklanov, A., Mahura, A., Mikkelsen, T., and Sorensen, J. H. (2006). K-model description of probabilistic long-range atmospheric transport in the northern hemisphere. *Atmospheric Environment*, **40**, 4352–4369.
- Le, N. D. and Zidek, J. V. (1992). Interpolation with uncertain spatial covariances : A bayesian alternative to kriging. *Journal of Multivariate Analysis*, **43**(2), 351–374.
- Liechty, J. C., Liechty, M. W., and Müller, P. (2004). Bayesian correlation estimation. *Biometrika*, **91**(1), 1–14.

- Masreliez, C. J. (1975). Approximate non-gaussian filtering. *IEEE Trans. Automat. Control*, **20**, 107–110.
- Matérn, B. (1986). *Spatial Variation*. Berlin: Springer-Verlag, 2nd edition.
- Mikkelsen, T., Larsen, S. E., and Thykier-Nielsen, S. (1984). Description of the risø puff diffusion model. *Nuclear Safety*, **67**, 56–65.
- NEA (2002). Nuclear energy agency. Chernobyl: Assessment of radiological and health impacts. (Available from <http://www.nea.fr/html/rp/chernobyl/chernobyl.html>).
- Schlather, M. (1999). Introduction to positive definite function and to unconditional simulation of random fields. Technical report, Dept. of Mathematics and Statistics, Lancaster University, UK.
- West, M. (1982). Aspects of recursive bayesian estimation. *Unpublished PhD thesis, University of Nottingham*.
- West, M. and Harrison, P. J. (1997). *Bayesian Forecasting and Dynamic Models*. Springer-Verlag, New York, 2nd edition.