Specific and general underpinnings to number; parallel development

Journal Item

<CT>**Specific and general underpinnings to number; parallel development**

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<C-AB>**Abstract:** In this commentary, we outline an epistemological continuum between earlier and later number concepts, showing how empirical findings support the view that specific and general underpinnings to number develop in parallel in children, and raise the question, based on cross-syndrome comparisons in infancy, as to whether exact or approximate number abilities underlie these later skills.

<C-Text begins>Post-Piagetian "top-down" approaches to number development require the definition of an epistemological continuum that relates, within the same framework, simpler to more complex mathematical schemas and concepts. The word continuum is key, as it brings together both "bottom-up" and top-down strands of research and enables the empirical testing of domain-specific and domain-general hypotheses about number development in the same groups of children. The authors have not yet outlined a framework to relate their "simple" and "advanced" counting schemes, which limits their approach, but such an outline is possible using our epistemological continua framework.

Finding a conceptual continuum between Gelman and Gallistel's (1978) domain-specific number tasks, such as counting or cardinality on the one hand and the Piagetian (1952) domain-general logical requirements on the other, is not an easy task considering that their definitions of cardinality, for example, are different and conceptually asymmetric (Bryant, 1994; Karmiloff-Smith, 1992). Alternatively, however, it is possible to find conceptual compatibility between counting and the classic Piagetian (1952) task of class inclusion, provided that the latter is reinterpreted within a numerical context such as children's understanding of the structure of the numeration system (SNS, or the natural numbers). This is viable considering that the idea that any group of 10 units may be regrouped as one of the next unit (i.e., 10-ones become 1-ten, 10-tens become 1-hundred, and so on) is quite similar to the Piagetian schema of a hierarchical classification system of inclusive relations (Resnick, 1983), in which the class containing only one element is included in the class containing two elements, which in turn is included in the class containing three, and so on (Piaget and Szeminska, 1952).

The continuum between counting and knowledge of the SNS can also be analyzed in terms of their logical invariants' similarities and differences. Whereas counting implies the use of units of the same denomination (or size; i.e., ones), mastery of the SNS implies the ability to count and combine units of the same and different denominations; that is, ones, tens, hundreds, and so on (Martins-Mourao and Cowan, 1998; Nunes and Bryant, 1996). Second, whereas counting only allows children to relate numbers sequentially, either as larger or smaller in terms of their immediate position in the number line, knowledge of the SNS empowers them to interpret numerals as a composition of other numbers or as a composition of units of different denominations, long before any knowledge of written numbers (Martins-Mourao, 2000; Nunes and Bryant, 1996). Third, the idea that any numeral can be composed by the addition of any smaller number that came before it in the number system (e.g., "125" = 100 + 10 + 10 + 1 + 1 + 1 + 1 + 1) defines, from the child's point of view, a break with simpler concepts of the past and a reconceptualization of number itself (Hiebert and Behr, 1988). It is this notion, which is also equivalent to the part-whole schema specifying that any quantity can be divided into parts as long as the combined parts neither exceed nor fall short of the whole (Resnick, 1983), that enables children to generate any number in the system without having to memorize the number line in its entirety (i.e., Rips et al.'s advanced counting). Hence, the definition of an epistemological continuum between domain-specific skills such as counting and domain-general knowledge such as the part-whole schema is possible provided that it is framed within children's natural progression from counting small sets of objects (simple counting) to being able to generate any number in the system (advanced counting).

Our research suggests that both domain-specific and domain-general knowledge may develop in parallel almost simultaneously – instead of sequentially – between the ages of 3 and 5 years, although the latter may only be explicit to children much later (Karmiloff-Smith, 1992). Evidence to support this argument must show that children have developed at least a simple version of the part-whole schema at a quite young age but not yet have learned all of the situations to which they may apply it successfully (Resnick, 1983).

Instead of looking at knowledge of Gelman and Gallistel's (1978) counting principles, Martins-Mourao and Cowan (1998) examined children's developmental stages in counting ability and found important conceptual changes defined by the progression from the unbreakable chain level to the breakable chain level between the ages of 3 and 5 years (Fuson, 1988). When they asked 4- to 6-year-olds "what numbers come after 10?" those at the unbreakable chain level were unable to interrupt the number line and had to count up from one (i.e., "1, …, 10, **11**, **12, 13**!"), whereas those at the breakable chain level said "10, 11, 12, 13, …" therefore showing the ability to manipulate the number line and judge 10 as a unit of different size, composed of 10 ones. The researchers then asked the same group to provide an answer to an additive composition task that required them to pay for items in a shop. In a typical item, the child was given three 10p coins and six 1p coins to pay for an item costing 14p. Success in this task required decomposing the total amount to be paid into one unit of 10 and several ones and then composing the quantity from units of different denominations (i.e., ones, tens, and hundreds). Supporting our argument, results showed that no child at any point passed the additive composition task without also being able to continue counting up from an arbitrary number in the list, which suggests that continuation of counting allows the child to establish simpler part-whole relations much earlier than previously thought.

Comment [SCR1]: Au: Are you sure that you're referring to the unbreakable level here? Shouldn't this go with the discussion of the breakable chain level?

Comment [A2]: Agreed with SCR1 and made corrections.

How can researchers trace the developmental pathways underpinning number development? Our cross-syndrome comparison of Down syndrome and Williams syndrome yielded interesting results about such pathways. Older children and adults with Down syndrome outstrip those with Williams syndrome in all number tasks (Paterson et al., 2006). Although children with Williams syndrome learn to count fluently, they have serious problems understanding cardinality, for instance (Ansari et al., 2003). Yet, in infancy, children with Williams syndrome are as successful as healthy controls in discriminating changes in exact small-number displays, whereas infants with Down syndrome fail (Paterson et al., 1999). However, the same Williams syndrome infants fail to discriminate approximate large-number displays differing in ratio (1:2/2:3; Van Herwegen et al., in press). Given all the subsequent number problems experienced by older children with Williams syndrome, this suggests that approximate number abilities may be a more important underpinning to subsequent conceptual number development than early discreet number abilities.

In sum, to understand the development of number abilities, we need to consider full developmental trajectories of both domain-specific and domain-general abilities that develop in parallel.<C-Text ends>

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Comment [SCR3]: Au: Please update this reference.

Comment [A4]: Out in 2008, but no Vol. or page numbers yet.