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The mathematical misconceptions of adult distance-learning science students

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Abstract

An analysis of student answers to interactive online assessment questions for the Open University course 'Maths for Science' is providing insight into adult distance-learning science students' mathematical misconceptions. Some of the findings have been unexpected and frequently errors are caused by more basic misunderstandings than lecturers might imagine. The analysis has revealed specific misconceptions relating to units, powers notation, arithmetic fractions and the rules of precedence.

Undergraduate courses in the UK Open University (OU) are completely open entry. One implication of this is that students studying Science Faculty courses have a very wide range of mathematical backgrounds, varying from those who already have a degree in a numerate discipline to those with no previous mathematical qualifications at all. Many OU students have not studied mathematics since they were at school (which, for adult students, might have been many years ago) and they frequently lack confidence in their mathematical abilities. Elementary mathematical skills are embedded in the Science Faculty's interdisciplinary level 1 course Discovering Science, but lack of mathematical ability and confidence remains a problem for many students as they progress to level 2 courses in physics, astronomy, chemistry, Earth science and biology. The 10 point level 1 course Maths for Science [1] was written to meet this need. The course, described in more detail in a paper presented to the Helping Everyone Learn Mathematics Conference in 2005 [2], has now been studied by approximately 7000 students since it was first presented in 2002. It has been both well received by students and instrumental in increasing retention rates on higher level Science Faculty courses.

One of the issues confronting all providers of distance education is the need to provide students with meaningful feedback without necessarily ever meeting them, and the Course Team which produced Maths for Science took the decision to pilot an interactive web-based system for both summative and formative assessment. The system, whose operation and pedagogy is described in more detail in Ross et al. [3], makes minimal use of multiple choice questions and allows students three attempts at each question, with the amount of feedback provided increasing after each attempt. Since the assessment is completed online, feedback is provided in a timely fashion, one of the factors identified by Gibbs and Simpson [4] as important if assessment is to support student learning. The assessment system, currently known as 'OpenMark', is now used by several Open University courses and is being further expanded and integrated into the University's Moodle-based virtual learning environment [5].

In addition to its many benefits for student learning, the OpenMark system has provided a rich source of information about the mistakes made by students. Data from more than 70 Maths for Science assessment questions have been analysed, typically for around 200 students at a time, and this is leading to increased insight into students' mathematical misconceptions. The remainder of this paper will discuss some of these misconceptions, but several points are worth noting at the outset. Although, for reasons of security, the actual questions cited in the paper are taken from Maths for Science's purely formative 'Practice Assessment', most of the reported analysis has been done on questions from the 'End of Course Assessment', which has a summative as
well as a formative function. An implication of this is that students are trying very hard to get the questions ‘right’, so the errors revealed cannot, in the main, be attributed to students guessing the answer. In addition, since so few of the questions are multiple choice, the analysis has been able to go beyond a consideration of commonly selected distractors to look at the actual responses entered by students. Finally, most of the assessment questions exist in several variants, with different questions being presented to different students. This feature, which exists to limit opportunities for plagiarism, has also added to the author’s confidence in the general applicability of some of the findings. For example, for one variant of a question, around 50% of incorrect responses gave the answer 243; for a different variant of the same question a similar percentage of incorrect responses gave the answer 11809.8, and so on. These errors can be explained by an identical misunderstanding, in this case a misunderstanding of the rules of precedence. This is discussed in more detail below.

Common misunderstandings – the physical reality of scientific calculations

Figure 1 shows a question of a type that has been surprisingly badly answered on every Maths for Science assessment to date, with only around 20% of students getting such questions right at the first attempt and 40% still being incorrect after three attempts. Students are provided with an equation and are asked to substitute given values, giving their answer in scientific notation, with correct SI units and to an appropriate precision. Note that no rearrangement of the equation is required.

Most students get these questions numerically correct, and the requirement that the answer should be given in scientific notation does not present too many problems. Most students attempt to give appropriate units (so, as discussed above, their errors cannot be attributed to laziness or carelessness) but it is the units of the answer...
and the number of digits quoted (the ‘number of significant figures’) that most frequently cause the response to be incorrect. Sometimes the error can be attributed to a trivial arithmetic mistake, for example, errors in quoting an answer to a particular precision are frequently caused by the fact that students truncate the value rather than rounding it. However it appears that a lack of understanding of the physical reality of the question is at least partly to blame for the large number of errors in questions of this sort – around 50% of all responses have incorrect units. This is in line with the difficulty students have whenever asked to convert the units of an answer from, say, m$^3$ to mm$^3$; despite the fact that this is very carefully taught in the course, many students neglect to cube the conversion factor.

More detailed analysis reveals an interesting pattern in the incorrect units given in answer to questions such as the one shown in Figure 1. In response to this question (where the units of the correct answer are m), the most common incorrect units are m$^2$, m$^{-1}$ and m$^{-2}$, so students are forgetting to find the square root, having difficulty in interpreting negative powers, or both! In a similar question where students were asked to find a value for a time period using the formula $T = 2\pi \sqrt{\frac{L}{g}}$, the commonly incorrect units of s$^2$, s$^{-1}$ and s$^{-2}$ reveal the same misunderstandings. Although, in both of these questions, a handful of students’ numerical answers reveal that they had, for example, neglected to take the square root of their answer, this was done by considerably fewer students than made the equivalent error with the units. One possible explanation of this is that it is possible to get the correct numerical answer by just substituting numbers into a calculator, but it is not so easy to get the units right without writing down any working, and students’ reluctance to write down working has been reported elsewhere [6].

### Common misunderstandings – simplifying, fractions and negative powers

The Course Team’s view has always been that the pivotal section of *Maths for Science* is the one which introduces the rearrangement and combination of algebraic equations. It is therefore pleasing that the questions designed to assess this section are generally well answered. However, students’ ability to rearrange and combine equations is not matched by their ability to simplify them. In a question which requires students to combine two equations and to give the answer in its simplest possible correct form, the correct answer takes the form $A = BC^2$. 19% of incorrect responses (6% of all responses) were equivalent to the correct answer, but not in the simplest form, with virtually all responses in this category being of the form $A = B/C^2$, $A = B/1/C^2$, $A = BD/DC^2$ or $A = B/D/DC^2$.

Student reluctance or inability to simplify algebraic expressions is demonstrated in several other *Maths for Science* questions (for example, many answers to another question are left in the form $\frac{ab}{9a}$) and evidence of similar behaviour has been found elsewhere within the OU Science Faculty. For example, students frequently fail to see that the factor ‘$m$’ is common to the left and right hand side of equations such as $mg\Delta h = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$.

However there is also evidence of a tendency to ‘over-simplify’ on occasions i.e. to attempt to simply an algebraic expression inappropriately, perhaps in an attempt to get to a more meaningful answer, such as would be obtained if dealing with numbers not symbols. In a *Maths for Science* multiple choice question asking students to identify equivalent expressions, the most common incorrect response is to say that $(A + 3)^2$ is equivalent to $A^2 + 9$, and in a second level physics examination, many students simplified $\sqrt{x^2 - y^2}$ to $x - y$. Sawyer [7] reported a similar effect in children’s mathematical development.
The prevalence of answers of the form $A = B/C - 2$ or $A = B/1/C^2$ instead of $A = BC^2$, along with a massive 35% of incorrect responses to the same question (12% of all responses) which were of the form $A = B/C$, illustrates two other areas of student difficulty, both also demonstrated in other Maths for Science questions, and elsewhere.

The first of these difficulties is in understanding the meaning of negative powers i.e. in failing to recognise that

$$\frac{1}{x^{-n}} = x^n$$

The second difficulty is in dividing by a fraction, so students do not recognise that

$$\frac{B}{1/C^2} = BC^2$$

Both of these difficulties persist despite the fact that arithmetic with fractions and the use of powers notation are both taught in the first chapter of *Maths for Science* in a purely numerical context and then applied to symbols and units later in the course.

**Common misunderstandings – precedence**

Although most students obtain the correct numerical answer to the question illustrated in Figure 1, those who do not are frequently wrong because of an incorrect understanding of precedence (or perhaps an over-reliance on their calculator!). So instead of calculating $\sqrt{\frac{L}{4\pi F}}$ they find $\sqrt{L + 4 \times \pi \times F}$, which in the case of the variant of the question illustrated in Figure 1, leads to an incorrect answer of $4.6 \times 10^{10}$m. Figure 2 shows the result of a similar error. Here the student has found $\frac{27^4}{3}$ instead of $27^{4/3}$, in a similar misunderstanding to the one which leads many students to evaluate $3^6$ instead of $3^{6/3}$ (and so to obtain an answer of 243) when asked to find $(3^6)^{1/3}$.

![Figure 2](https://students.open.ac.uk/openmark/s151.pdf)

Figure 2. A ‘Maths for Science’ assessment question, showing a student response that is incorrect because of misunderstanding of the rules of precedence.
Common misunderstandings – graphs, gradient and the basis of calculus

The concept of gradient (and the method for calculating the gradient of a straight line) is taught about half-way through the course and then this is developed into a discussion of elementary differential calculus right at the end of the course. The assessment questions on calculus, along with those on angular measure, trigonometry, logarithms, probability and statistics, have yet to be analysed. However, early analysis of questions designed to assess students’ understanding of the gradient of a straight line indicates that, in much the same way as many problems in algebra can be attributed to ‘lower-level’ misunderstandings in arithmetic (for example with fractions), some students’ difficulty with differentiation may stem from their poor understanding of gradient.

Discussion

The increased insight gained into Maths for Science students’ mathematical misconceptions is being used in several ways. The assessment questions themselves have been improved, so that targeted feedback is provided in response to commonly incorrect responses, as shown in Figure 1 and Figure 2. In some cases, the analysis has revealed that different variants of the ‘same’ question are in fact of different difficulty. For formative only assessment this is not a major consideration, but for summative assessment it is considered important that each student should receive questions of comparable difficulty, so some variants have been altered or removed. In addition to improving Maths for Science’s assessment, some changes have been made to the course itself, in particular, additional practice questions have been provided for areas of common student difficulty. Future OU Science Faculty courses are also benefiting from our increased understanding of what students do wrong, and why. For example, the teaching of arithmetic with fractions is being incorporated into the new introductory course ‘Science Starts Here’.

The evidence presented in this paper relates to students who may be considered to be atypical in three respects: they are adult students, sometimes returning to study after a considerable length of time; they are, in the main, studying towards a qualification in science not mathematics; and they are studying at a distance. However, the author has no reason to suppose that younger students studying science in conventional universities, especially those who have not come from particularly numerate backgrounds, do not have similar mathematical misconceptions. In addition, there is no evidence to support the notion that students’ mathematical misunderstandings result from bad teaching at school. Taken as a whole, Open University students have a widely varied experience of the primary and secondary education system both in the UK and elsewhere and, as soon as they are reminded, most remember what they learnt at school about, for example, fractions. However it is perhaps unreasonable to assume that this knowledge will remain at the forefront of students’ memories over a period of several years or decades, especially since the skills of arithmetic with fractions are not much practised in everyday life.

The ‘Maths problem’ is sometimes thought of as relatively recent, and specific to particular cultures. However, as early as 1939 and 1959, the Russian authors Bradis, Minkovskii and Kharcheva [8] recognised difficulties in algebra caused by misunderstandings of fractions and square roots. Nevertheless, the implications of a lack of basic understanding of arithmetic etc. on study at a higher level remain serious. For example, when students are asked to rearrange \( E_k = \frac{1}{2}mv^2 \) to make \( v \) the subject, it is frequently not the squared term, \( v^2 \), that causes the difficulty, but rather the \( \frac{1}{2} \). Once we appreciate that students may not realise that multiplying by half is equivalent to dividing by two, this fact ceases to be surprising. Back in 1964, Sawyer [7] commented ‘If we imagine that a pupil understands something, when in fact he does not, we are like a man trying to build on a foundation of air,’ and the phenomenon of ‘the dropped stitch’ was identified by Sheila Tobias in 1978 [9]. If a student fails to understand, for whatever reason, one aspect of simple mathematics (Tobias’s statement of this is
'the day they introduced fractions I had the measles') then more advanced concepts may appear unassailable. The first challenge to those of us who seek to rectify students' mathematical misconceptions is to correctly identify the true cause of these misconceptions.

References


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