Interpretation of geometric shapes – an eye movement study

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INTERPRETATION OF GEOMETRIC SHAPES – AN EYE MOVEMENT STUDY

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Abstract

This paper describes the first in a series of studies which seek to explore the correlation of eye movements with interpretation of geometric shapes. These studies are intended to inform the development of an eye tracking interface for computational tools to support and enhance the fluid interaction required in creative design.

A common criticism of computational design tools is that they do not enable manipulation of designed shapes according to all perceived features. Instead the manipulations afforded are limited by formal structures of shapes. This research examines the potential for eye movement data to be used to recognise and make available for manipulation the perceived features in shapes.

The objective of this first study is to analyse eye movement data with the intention of recognising moments in which an interpretation of shape is made. Results suggest that duration of fixation and distance between successive fixations prove to be consistent indicators of shape interpretation.

Keywords

Eye tracking, shape perception, design

CCS

I.3.6 Interaction techniques
I.5.2 Feature evaluation and selection
I.6.0 Computer-aided design

1 Introduction

Conceptual design involves the creation, exploration and development of design shape alternatives. Designers typically use freehand sketching to support this process, largely because it offers ambiguity and so supports the reinterpretation of shapes that is key to effective shape exploration and development. Commercially available computer aided design (CAD) systems are used during detail design and design definition, but they are not well suited to conceptual design activities because the support they offer for the reinterpretation of shapes is poor. Recent research has explored the feasibility of providing computer support for conceptual design through the integration of shape grammar and computer vision technologies [McKay et al. 2009]. Early results are promising, and a research prototype has been built, but early adoption is limited, in part because the time and intellectual overhead needed to drive such systems interferes with the creative flow. The research reported in this paper is exploring the use of eye tracking technology as a means of providing user interactions that allow designers to focus on their design activity, with the design system as a tool that supports whilst minimising disturbances to their creative flow.

This paper reports results from early experiments investigating what eye movements can reveal about a person’s interpretation of a visual phenomenon. It is concerned with a particular type of visual phenomenon, namely that of shape defined by line against a plain ground. The early stages of this work is concerned with geometric shapes but it is intended that it will progress into the rather less clear world of sketched representations made as part of a creative designing process. This process can be characterised as the creation of design ‘structures’ through the combining of design ‘features’. It is a transformational process revealing ‘moments’ when perceptions are transformed from one state to another. This paper introduces our investigation of structures and features and offers findings from studies using eye tracking that explore this notion of design moments during the interpretation of geometric shapes.

2 Background

2.1 Shape Interpretation in Design

The ability to interpret shape is a fundamental skill in visually creative activities, such as conceptual design. It has been observed that, when sketching, designers often produce series of ideas that are, in places, deliberately ambiguous and open to reinterpretation. These design alternatives are explored visually and can suggest patterns and associations that lead to new avenues of exploration. Schön and Wiggins [1992] describe this as a ‘seeing-moving-seeing’ process where seeing a sketch can result in its reinterpretation according to emergent forms or structures, and this in turn informs the development of future sketches. Exploration in this way typically involves the recognition and transformation of shapes in sketches – such as overall outline shapes or the embedded parts of shapes (so called ‘subshapes’) [Prats et al. 2009]. Interpretation of sketches to recognise alternative shapes or subshapes is a vital element in design exploration and is believed to be a decisive component of innovative design [Suwa 2003].

2.2 Computational Shape Interpretation

Despite the importance of reinterpretation in design exploration it is not readily afforded by the current generation of computational design tools [Henderson 1999]. When a digital product model is created a specific structure is defined using a fixed set of geometric elements, such as edges and surfaces. Reinterpretation of this structure to allow for newly recognised patterns and associations is only straightforward when these emergent forms conform to subsets of the original set of geometric elements. Otherwise, reinterpretation of a model necessitates redefinition according to a new set of elements. A fixed structure such as this can lead to inconsistencies between what can be perceived in a design model and the manipulations allowed. Designers cannot easily manipulate all of the subshapes that they perceive and so cannot take advantage of emergent structures. As a result they are not free to explore the patterns and associations that emerge as a designed shape is being developed. They are restricted to manipulating shapes according to the structure by which the design was initially defined.

A variety of approaches have been proposed that aim to tackle this problem by enabling the manipulation of design models according to perceived structures. For example, Saund and Moran [1994] present a WYPIWYG (What You Perceive is
What You Get drawing system that uses an image interpretation architecture based on token grouping. Tokens are organised into a lattice structure which enables perceptual interpretations of a sketched shape to be specified and manipulated according to simple gestures. Similarly, Gross [2001] presents the ‘Back of an Envelope’ system - a drawing program that automates the recognition of emergent subshapes in a sketch. The system recognises these emergent forms by using standard pattern recognition techniques commonly employed in drawing systems to recognise and identify elements and configurations in freehand input. Jowers et al. [2008] present an approach based on the shape grammar formalism [Stiny 2006], in which shape replacement rules are applied to recognise and manipulate perceived subshapes in a design. These shape rules provide a dual advantage, since as well as enabling the perceived structure of a design to be freely recognised they also formalise the creative process by which a design is generated and thereby enable repetition of the process.

A major limitation to each of these systems lies in the cognitive overhead needed to interface with them. For example, in Saund and Moran’s system, a user is required to learn specific gestures in order to specify a particular perception of a design. In Gross’s system, users select perceived emergent shapes in a design by tracing over them. In Jowers et al.’s system, users specify and manipulate perceived subshapes by defining shape replacement rules. In each of these, additional effort is needed to interact with the system to specify a particular perception of a shape and no matter how small this effort is, it can result in an interruption to the creative flow of the user. A more intuitive, dynamic system, one that better supports a cognitive process of ‘seeing-moving-seeing’, would offer real benefits in avoiding the need for users to explicitly define their interpretation of designed shapes. To this end eye tracking technology presents itself as a potential interface for drawing systems.

Past research has explored the application of eye tracking as an alternative drawing interface, to replace traditional mouse and keyboard input, e.g. Hornof et al. [2004]. Here, it is proposed that eye tracking can serve as an additional interface, augmenting traditional input. It is proposed that eye tracking data can reveal a users interpretation of a shape at a particular moment in time, and that a drawing system can respond to this interpretation by affording appropriate manipulations of the shape. Such an interface would allow designers to focus on their design activity, with the design system as a tool that supports the exploration of designed shapes without disturbing their creative flow.

2.3 Eye tracking and Shape Perception

Eye movements and points of visual fixation can reveal much about how shapes are viewed and interpreted in design. For example, studies suggest that gaze patterns are influenced by information that is apparent in visual stimuli [Henderson and Hollingworth 1998]. Yarbus [1967] illustrated that in addition to this information, eye movements and fixations are also influenced by a viewer’s intent. These studies were based on viewers studying classical paintings, but the results have been successfully replicated for viewers looking at designed shapes [Koivunen et al. 2004].

Research in perceptual localisation suggests that when looking at a simple shape the eye is naturally drawn to the centroid, or centre of gravity [Melcher and Kowler 1999; Vishwanath and Kowler 2003]. A common hypothesis argues that salient points, such as borders or edges, play an important role in this attraction [Itti and Koch 2000]. An alternative hypothesis proposed by Renninger et al. [2007] argues that the attraction occurs due to task-relevant information collected during eye movements.

Studies in visual search suggest that shapes are recognised according to structural decompositions of simpler parts and the spatial relations between the parts [Hoffman and Richards 1984; Biederman 1987]. In creative design, it is believed that designers employ a similar process of decomposition, however the interpreted parts and the relationships between them can change dynamically throughout the design process as alternative interpretations of a shape are recognised [Stiny 2006]. This process of interpretation is similar to the perceptual phenomena exemplified by Kanizsa [1976] figures and the Necker [1832] cube, where a change in interpretation can result in the recognition of alternative structures in a shape.

Studies concerning the viewing of ambiguous shapes, such as the Necker cube, are inconclusive with respect to the relation between eye movements and interpretation. For example, Flam and Bergum [1977] conclude that eye movements and reversals of interpretation are unrelated, while Ellis and Stark [1978] conclude that eye movements precede the reversal of interpretation, and Einhäuser et al. [2004] conclude that eye movements follow reversals.

The research described in this paper is based on an assumption that there is a relation between eye movements and shape interpretation, and from this relation that interpretation can be deduced from eye movements. In particular, the study presented here explores eye movements with the intention of identifying moments at which an interpretation of an ambiguous shape takes place. This identification is a first step to developing an interface for drawing systems that enables fluid interaction with perceived shapes and structures.

3 Methodology

The study described here is concerned with analysing eye movement data with the objective of recognising moments at which interpretations of ambiguous shapes are made. To this end, participants in the study were asked to observe a series of shapes and perform set tasks, which naturally lead them to interpret the shapes in different ways. In total, 11 participants were included in the study - these were a mixture of students and university staff, both male and female, with varied research interests, and ages ranging from early 30s to late 40s. Due to inaccuracies in calibration, data from 4 of these had to be discounted and consequently, the results presented here are drawn from the data of 7 participants.

3.1 Setup

During the study eye movement data was collected with a Tobii X120 eye tracker (accuracy 0.5°, drift < 0.3°, binocular tracking, data rate 120 Hz). This equipment is nonintrusive and includes a head-motion compensation mechanism that allows for a freedom of movement of 0 x 22 x 30cm. Despite this, participants were asked to keep as steady as possible in order to ensure eye movements were consistently captured. A dual VDU arrangement allowed the facilitator to monitor the participants in order to ensure that eye movement data was captured at all times.
Participants were encouraged to think-aloud whilst conducting the tasks and a webcam with microphone was used to record any verbalisations as well as any interaction between the facilitator and the participants.

3.2 Shape interpretation tasks

Participants were asked to complete three tasks which were designed to encourage reinterpretation of ambiguous shapes. These were based on studies by Yarbus [1967], in which participants were given different objectives that then influenced their eye movements when viewing pieces of classical art. Here, however, participants were not presented with meaningful art forms. They were instead presented with abstract images composed of geometric primitives, such as squares and triangles, which were highly suggestive and open to reinterpretation [Stiny 2006].

Generally speaking, the three tasks can be classified according to three visual strategies: i) shape detection; ii) shape recognition; and iii) shape reinterpretation. Here, shape detection is concerned with searching for a particular shape; shape recognition is concerned with applying a meaning or interpretation to a shape; shape reinterpretation is concerned with recognising an alternative meaning or interpretation of a shape. These three strategies are not distinct and cannot be considered independently, however each of the three tasks was intended to emphasise one strategy over the others.

Task 1 – Shape detection

Task 1 was a search task in which participants were asked to detect specific shapes in a series of three images, under no time constraint. The three images were composed of geometric primitives, and before each one was displayed the participants were shown instructions specifying the shape to search for. They were also instructed to vocalise when they had found the shape by saying "Now", and then focus on it for a few seconds.

The image in Figure 1a) was the first image shown to the participants. In this image, participants were asked to find the square. The second image shown to the participants was similar to this, but contained a single triangle which they were asked to find. The image in Figure 1b) was shown to the participants three times, and each time they were shown the image they were asked to search for a different shape. These were an arrow, a pacman shape, and a T. In the image in Figure 1b), the three shapes emerge through figure-ground reversal or as a result of overlapping and intersecting geometric primitives. As a result, in order to find these shapes it was necessary for participants to reinterpret the shapes in the image.

Task 2 – Shape recognition:

Task 2 was a natural viewing task in which participants were asked to look at a series of three images and describe the shapes that they contain. The three images were all composed of overlapping polygons. For example, one of these images is illustrated in Figure 2, and is composed of three rectangles. Participants had 10 seconds to view each image and name the shapes that they recognised while performing the task.

Task 3 – Shape reinterpretation:

Task 3 was also a search task and, similar to Task 1, participants were asked to detect a specific shape in a series of three images, under no time constraint. As in Task 1, participants were instructed to vocalise when they had found the shape by saying “Now”, and then focus on it for a few seconds.

The images the participants were shown were the same three images used in Task 2, an example of which is illustrated in Figure 2. In all three images, the participants were asked to find the same six-sided polygon, illustrated in Figure 3. This shape was shown to participants at the beginning of the task, and in order to detect it participants had to reinterpret the shapes they had previously seen in Task 2.

3.3 Analysis

Eye movement data was analysed according to the duration of fixation points and the distance between successive fixation points. Longer fixation durations indicate that the stimulus is engaging in some way, while shorter durations are indicative of search [Just and Carpenter 1976]. Short distances between successive fixation points indicate a period of localised learning, while long distances suggest that the eye is attracted to distinct areas of the stimulus [Goldberg et al. 2002]. Based on this, it was expected that if an interpretation of a shape was sufficiently engaging then it would result in an increase in duration of fixations together with a decrease in distance between successive fixations.

An extract of the data is presented in Table 1. Fixation points were defined with a minimum fixation duration set at 150 ms and a fixation radius of 50 pixels. In Table 1, the Time values indicate, in minutes and seconds, the timestamp at the start of a fixation, the X and Y values are the screen coordinates, in pixels,
of the mean central point of a fixation, the Duration values are the duration, in milliseconds, of a fixation, and the Distance values are the distance, in pixels, calculated between the centre points of successive fixations. This data was plotted in graphs that visualise the duration of fixations (in milliseconds) and distances between successive fixations (in pixels) as they change over time. Examples of these graphs are presented and discussed in the next section.

Table 1: Extract of participants’ data.

<table>
<thead>
<tr>
<th>Time (mm:ss)</th>
<th>X (px)</th>
<th>Y (px)</th>
<th>Duration (ms)</th>
<th>Distance (px)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:31.454</td>
<td>524.19</td>
<td>486.00</td>
<td>966</td>
<td></td>
</tr>
<tr>
<td>10:31.904</td>
<td>438.91</td>
<td>495.53</td>
<td>533</td>
<td>85.812</td>
</tr>
<tr>
<td>10:32.453</td>
<td>441.88</td>
<td>623.29</td>
<td>566</td>
<td>127.80</td>
</tr>
<tr>
<td>10:33.170</td>
<td>447.96</td>
<td>481.35</td>
<td>866</td>
<td>142.08</td>
</tr>
<tr>
<td>10:33.836</td>
<td>443.04</td>
<td>479.41</td>
<td>366</td>
<td>5.28</td>
</tr>
<tr>
<td>10:34.319</td>
<td>449.30</td>
<td>596.81</td>
<td>600</td>
<td>117.56</td>
</tr>
<tr>
<td>10:34.877</td>
<td>446.40</td>
<td>639.00</td>
<td>416</td>
<td>42.29</td>
</tr>
<tr>
<td>10:35.368</td>
<td>454.34</td>
<td>474.09</td>
<td>533</td>
<td>165.10</td>
</tr>
<tr>
<td>10:36.159</td>
<td>465.07</td>
<td>460.70</td>
<td>949</td>
<td>17.16</td>
</tr>
<tr>
<td>10:36.875</td>
<td>463.09</td>
<td>454.96</td>
<td>600</td>
<td>6.08</td>
</tr>
</tbody>
</table>

Table 2: Extract of participants’ transcript.

<table>
<thead>
<tr>
<th>Time (mm:ss)</th>
<th>Speaker</th>
<th>Transcript and actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.56</td>
<td>Start task</td>
<td></td>
</tr>
<tr>
<td>11.00</td>
<td>Participant</td>
<td>humming to self</td>
</tr>
<tr>
<td>11.02</td>
<td>Participant</td>
<td>“Now”</td>
</tr>
<tr>
<td>11.03</td>
<td>Participant</td>
<td>“It’s in front of me”</td>
</tr>
<tr>
<td>11.06</td>
<td>Facilitator</td>
<td>“Very good”</td>
</tr>
<tr>
<td>11.08</td>
<td>End task</td>
<td></td>
</tr>
</tbody>
</table>

In addition to the numerical data, the verbalisations made by participants during the tasks were transcribed and analysed. Participants were encouraged to express their interpretations of images as they viewed them and, especially, in Task 1 and Task 3, they were instructed to indicate the moments at which requested shapes were found, as illustrated by the extract in Table 2. The transcripts provide further qualitative data to support conclusions drawn from the quantitative data discussed in the Section 4.

4 Results

The objective of this research is to explore the possibility of using eye tracking as an interface for a computational design system. It is intended that eye movements will be used to inform such a system with respect to users’ interpretations of designed shapes. To this end, the study reported here is concerned with recognising patterns in eye movements that correspond with moments at which an interpretation of a shape is made, termed the “Now” moments. The following discussion explores the significance of participants’ data with respect to the three visual strategies of shape detection, shape recognition, and shape reinterpretation.

4.1 Task 1 – Shape detection

Figure 5 illustrates three graphs plotting participants’ data from Task 1. Similar graphs were plotted for each participant and each task. In these graphs duration of fixations and distance between successive fixations are plotted against time. Duration is plotted as a dashed blue line and corresponds to the axis on the left-hand side of the graphs, while distance is plotted as a continuous red line and corresponds to the axis on the right-hand side. Also plotted on the graphs are the “Now” moments – moments at which a target shape is detected. These moments were specified by considering transcripts of participants’ vocalisations, as illustrated in Table 2. In the first of these graphs, the target shape was a square which was to be found in the image in Figure 1a), in the second graph the target shape was a triangle, and in the third the target shape was a pacman shape which was to be found in the image in Figure 1b).

Figure 4: Example of scan path data

Figure 5: Plots of participants’ data from Task 1.

In each of these three graphs, it is clear that “Now” moments follow immediately after an increase in the duration of fixation and a decrease in the distance between successive fixations. A similar pattern was visible in more than 70% of the graphs plotted based on the data from this study, which confirms our
expectation based on the literature [Just and Carpenter 1976; Goldberg et al. 2002].

An anomalous graph, that does not follow this trend, is illustrated in Figure 6. Here, the target shape was a square which was to be found in the image in Figure 1a). Unlike the graphs in Figure 5, the “Now” moment in this graph is not immediately preceded by a decrease in the distance between successive fixations. However, analysis of the participant’s scan path data suggests that this anomaly is caused by a significant delay between the detection of the target shape and the verbal confirmation of this detection. The fixation point labelled 5 in the graph was located within the target shape – the square – and immediately following this fixation there was an increase in duration of fixation and decrease in distance between fixations. This indicates that the target shape was detected at this moment, and the participant only verbalised the detection after further visual search of the image. Similarly to point 5, fixation points labelled 12 and 18 were also located within the target shape.

It is likely that delay between detection of the target shape and the verbalisation of this detection is common across all participants. As a result, the verbal data cannot be considered a primary source of data for this analysis, and instead must be used only to support conclusions drawn from the quantitative data.

4.2 Task 2 – Shape recognition

Figure 7 illustrates three graphs plotting participants’ data from Task 2. In this task natural viewing of the images was encouraged and participants were not asked to detect or focus on any particular shape. Instead they were free to recognise any shapes in the images, and were asked to verbalise the shapes that they recognised. The three graphs illustrated are based on data collected as participants viewed the image in Figure 2.

The graphs from Task 2 were analysed independently of the verbal data in order to identify fixation patterns as identified in analysis of Task 1. If such patterns correlate to moments of interpretation, so called “Now” moments, then they should be apparent in the graphs at moments that shapes are recognised. In the first and second graphs in Figure 7 three “Now” moments were identified, while in the last graph in Figure 7 four “Now” moments were identified. Each of these moments follows an increase in the duration of fixation and a decrease in the distance between successive fixations.

With these moments identified, the verbal data was consulted in order to verify that they are indeed moments at which shapes were recognised in the image. Each of the three participants recognised the image in Figure 2 as three overlapping rectangles. In particular, the verbal data corresponding to the first graph in Figure 7 revealed that this interpretation was made at time 09:56, the verbal data for the second graph in Figure 7 revealed that this interpretation was made at time 9:05, and the verbal data revealed that the interpretation in the last graph in Figure 7 was made at time 9:26, and also at time 9:32 referring to another interpretation. Small discrepancies in these two sources of data can be attributed to delays in verbal responses, as discussed in the previous section. Most of these moments of shape recognition correlate with identified “Now” moments in the graphs.

The additional “Now” moments identified in the graphs also correlate to moments of shape recognition, as evidenced by the verbal data. For example, the verbal data corresponding to the last graph in Figure 7 is presented in Table 3. Each of the “Now” moments in the graph corresponds to a vocalisation in this transcript.

4.3 Task 3 – Shape reinterpretation

Analyses of the data from Task 1 and 2 has suggested that fixation patterns can be used to identify moments at which shapes are detected and recognised. The visual strategies of shape detection and shape recognition both include elements of shape interpretation, and Task 3 was concerned with exploring fixation patterns that result when such an interpretation is changed through reinterpretation. In particular the participants
were asked to reinterpret the shapes from Task 2 in order to
detect the shape in Figure 3.

Figure 8 illustrates a graph plotting a participant’s data from
Task 3, which was collected as the participant viewed the image
in Figure 2. In the graph, a “Now” moment was identified based
on the fixation patterns – it follows an increase in the duration of
fixation and a decrease in the distance between successive
fixations. The verbal data for this participant is presented in
Table 2 and the “Now” moment identified in the graph
correlates with this. The participant who produced this data also
produced the data plotted in the first of the graphs in Figure 7.
However, despite these two graphs being based on two sets of
data in which two different interpretations of the same shape are
made they provide no insight into the particular interpretations
made. The graphs can only identify the moments at which an
interpretation was made.

![Figure 8: A plot of a participant’s data from Task 3.](image)

In order to analyse participants’ interpretations of a shape it is
necessary to consider alternative eye movement data. For example, some conclusions can be drawn by considering the
scan path data. In Figure 9 the scan path data for three of the
participants as they viewed the shape in Figure 2 is illustrated.
Here, two different interpretations of the shape are presented
which result from the participants responding to Task 2 and
Task 3.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>three rectangles, six-sided polygon</td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 9: Records of eye movements by three different subjects. The images on the left column were interpreted differently from those on the right column.](image)

The column on the left-hand side of Figure 9 illustrates the scan
paths of the participants as they respond to Task 2. Here, the
participants were asked to detect the shape illustrated in Figure 3,
and there is a clear difference in the scan path that results from
this reinterpretation of the shape. Here, durations of fixations are
typically longer and the distance between successive fixations is
shorter. The scan paths are also concentrated inside the target
shape. Note that participant C erroneously detected the
alternative target shape in the bottom left corner of the image.

It is clear from these scan paths that the interpretation of shape
in the right-hand column is different from the interpretation on
the left-hand side. Also it is clear that in the right-hand side
column participant C made a different interpretation from
participant A and B. This suggests that it may be possible to
deduce interpretations of shape by considering scan path
patterns.

5 Discussion and future work

This paper has explored how the human eye examines
geometrical shapes according to three visual strategies – shape
detection, shape recognition, and shape reinterpretation. Our
initial results suggest that, to some extent, eye movements
reflect these three visual strategies. These results support the
hypothesis that eye movements could be used to select
interpreted shapes in a design, in a natural and non intrusive way.
Records of eye movements show that duration of fixations and
distances between successive points of fixation can be used as
indicators of shape interpretation. The work presented here
focuses on the moment that a shape is interpreted. Our
continuing research will examine i) additional indicators in order
to get a more robust model on ‘when’ a shape is interpreted, and
ii) ways to determine ‘which’ shape is interpreted.

The ultimate goal of this research is to develop a computational
drawing system that uses eye movement data to recognise any
shape interpreted in a design. Figure 10 illustrates a simple
element of a potential application of such a system. Users
would not be restricted to manipulating shapes according to the
structure by which the design was initially defined – in this
element, two squares – but they would be free to manipulate all
of the shapes that they interpret – for example, the emergent L
shape. Reinterpretation of shapes is a vital element in design
exploration. Computational drawing systems that incorporate
eye tracking technology have the potential to establish a more
cognitive-friendly interaction between designer and computer.
Such interaction would allow designers to focus on their design
activity, with the design system as a tool that supports without
disturbing their creative flow.
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