

QUADRATURE-MIRROR-FILTER DESIGN FOR THE SUBBAND CODING OF DIGITAL VIDEO SIGNALS

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1 Introduction

This paper explores the design and use of Quadrature-Mirror-Filters (QMF) and Conjugate-Quadrature-Filters (CQF) within a two-dimensional framework for digital video applications, an example of which is the transmission system proposed by Schmidt [1]. The principles of subband coding are well known [2,3] and involve splitting an input signal into M -channel maximally decimated subbands, which in the special case of $M=2$, comprise complementary lowpass and highpass signals, as shown in Figure 1. Due to the non-ideal characteristics of the analysis bank filters and subsequent decimation, the subband signals suffer from three sources of error; namely aliasing, amplitude and phase distortion. The underlying design principle is that by judicious selection of the QMF-CQF structures which comprise the synthesis bank filters at the receiver, all three errors can be cancelled, so leading to the property of Perfect Reconstruction (PR) for the entire system.

The paper will firstly review the basic two-channel subband coding system and its implementation using both QMF and CQF designs. Their features and performance characteristics for the one-dimensional case will be examined and then using the principle of separability, 2-D QMF-CQF implementations will be constructed, as the product of two 1-D transfer functions. Other non-separable solutions are also feasible [2], though these will not be considered in this particular paper. Examples of 2-D QMF-CQF's in band-splitting configurations for video applications have been explored in [1], and will be discussed later.

2 The Subband Coding System (SBC)

A two-channel SBC system can use either QMF's or CQF's as shown in Figure 1. The analysis bank, which splits the input signal into a lowpass and highpass subsignal, comprises two symmetrical linear-phase half-band FIR filters, $H_{AL}(z)$ and $H_{AH}(z)$ respectively. Each subband is therefore bandlimited to approximately $\pi/2$, so that they can be decimated by a factor of two to produce $Y_L(z)$ and $Y_H(z)$ respectively. These signals are then coded, usually in such a manner as to exploit properties of the subband signal. At the receiver, the signals are decoded and up-sampled by a pair of interpolators, before being filtered by the synthesis bank filters $H_{SL}(z)$ and $H_{SH}(z)$, to reconstruct the output $\hat{x}(n)$, that hopefully will be a delayed version of the original input signal $x(n)$. As alluded previously however, the reconstituted signal differs from the input signal due to aliasing, amplitude and phase errors.

3 Quadrature-Mirror-Filters (QMF's)

The symmetrical highpass and lowpass filters have to be designed in such a way that they possess a roll-off which is as sharp as possible. This however, generally implies high filter orders which is not aspired in a real solution involving video bandwidths. Moreover these filters have to be linear-phase FIR filters in order to eliminate phase distortion, with a transition band that is as narrow as possible. The most reasonable filter design consists of slightly overlapping filter bands (Figure 2), where there is no frequency gap in the composite responses, so no information is lost, but some aliasing errors due to the overlap zones will occur. By a careful choice of the synthesis filters $H_{SL}(z)$ and $H_{SH}(z)$, these aliasing errors can be removed, so that the only remaining issue becomes the elimination of amplitude distortion.

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Generally the best approach to obtain the four matching QMF's is to design one of the filters, usually the lowpass-analysis filter $H_{AL}(z)$ and derive the three remaining filters from this prototype filter. This has the advantage that the coefficients of only one filter have to be calculated, from which the other three filters are related [3,5]. To obtain this relationship, we consider the expression for the reconstructed signal $\hat{x}(n)$ at the output of the system:-

$$\hat{X}(z) = 0.5 [H_{AL}(z) \cdot H_{SL}(z) + H_{AH}(z) \cdot H_{SH}(z)] X(z) + 0.5 [H_{AL}(-z) \cdot H_{SL}(z) + H_{AH}(-z) \cdot H_{SH}(z)] X(-z) \quad (1)$$

The first term reflects the actual signal output, while the second term represents the effects of aliasing and imaging in the analysis and synthesis banks respectively. The second expression can be made to disappear totally if the following choice of analysis and synthesis filters is made:-

$$H_{AH}(z) = H_{AL}(-z); \quad H_{SL}(z) = H_{AL}(z); \quad H_{SH}(z) = -H_{AL}(-z) \quad (2)$$

Thus, the aliasing introduced by selecting the analysis filters as shown in Figure 2 has been exactly cancelled by the imaging produced by the interpolators and synthesis bank filters. The remaining term in (1) now describes the transfer function of the complete SBC system. The magnitude response $\hat{X}(z)/X(z)$ represents the overall amplitude distortion and directly depends upon the design of the prototype filter. This must have a sharp transition and an amplitude response that ensures that the overall response of the entire SBC system is as flat as possible that is an all-pass response, so that the addition of the lowpass and the highpass response should equal one at all frequencies. This means that the lowpass filter (and the corresponding highpass filter) have to be designed in a way that each branch delivers an attenuation of 6dB (0.5) at the point of intersection of both filter responses, which is at quarter of the sampling frequency. Each of the four filters must therefore have an attenuation of 3dB at the half the Nyquist frequency $\pi/2$, since each branch comprises two cascaded filters.

The most popular method for obtaining the coefficients of an FIR filter with a symmetry around its midpoint is to use the ubiquitous Discrete Fourier Transformation (DFT). The coefficients are evaluated from a windowing function which smoothes the oscillations introduced by the truncation of the frequency spectrum, but makes the resulting transition bandwidth slightly wider. There are many different window functions available. For the purpose of video image transmission, the Blackman and Kaiser-Bessel-windows are the most useful as they ensure a high grade of amplitude smoothness with only a slight widening of the transition bandwidth. The Kaiser-Bessel-window function also possesses the added advantage of having a design parameter which permits a trade off between the sharpness of the cut-off and the smoothness of the frequency response.

4 Conjugate-Quadrature-Filters (CQF's)

A second approach to realising SBC filters is to use Conjugate-Quadrature-Filters (CQF's) [4]. The design and features of these filters differ completely from those of QMF's, starting with the computation of a lowpass product filter which is the basis for both the analysis and synthesis filters $H_{AL}(z)$ and $H_{SL}(z)$ respectively. A good design method for the product filter is to use the Parks-McClellan algorithm (Remes Exchange) which designs symmetrical half-band FIR filters, which have linear phase and equi-ripple in both the passband and stopband (Figure 3). Most of the other design methods have a ripple which tends to increase towards the cut-off frequency. Filters designed with the Parks-McClellan algorithm also have a sharper cut-off than other designs and are symmetrical about half the Nyquist frequency $\pi/2$.

The filter designed using this approach has to be non-negative, since otherwise it cannot be decomposed into the required sub-filters. The non-negative filter response is achieved by adding the peak magnitude value of the equi-ripple to the middle coefficient of the filter's impulse response. The decomposition of the product filter, which incidentally must always have an even order, is achieved by spectral factorisation. The zero plot of a product filter has the following characteristics (Figure 4):

-Double zeros on the left hand side of the unit circle.

-Reciprocal zeros inside and outside the right side of the unit circle, so that every zero inside the unit circle at $re^{j\varphi}$ has a corresponding zero at $r^{-1}e^{-j\varphi}$ outside the unit circle, where r is the radius of the zeros and φ angle of the zeros with respect to the real axis.

-The zeros have to be positioned in complex-conjugate pairs at $re^{j\varphi}$ and $re^{-j\varphi}$

These constraints are fully satisfied by the anti-symmetric and non-negative characteristics of an even order linear-phase FIR product filter of minimum order six, because the decomposed analysis and synthesis lowpass filter must have a minimum order of three. There are two ways to decompose the product filter into two sub-filters (see Figure 5):

-Split the zeros in such a manner that one sub-filter consists only of the zeros inside the unit circle and the other comprises the zeros outside the unit circle. Those on the unit circle are alternately allocated (see Figure 5a).

-Select zeros alternately from outside and inside the unit circle with increasing frequency (see Figure 5b).

The amplitude response of all four sub-filters will be identical, but their phase responses will be different. The sub-filter consisting of all the zeroes inside the unit circle exhibits a minimum phase response, while its counterpart, consisting of zeroes outside the unit circle has a maximum phase response. Both filters therefore no longer possess linear phase, but the combination of the filters in the same transmission system still leads to an overall group delay which is constant. Their magnitude response is the square root of the magnitude response of the product filter, while [4] gives the appropriate lowpass-highpass transformation to obtain the necessary highpass sub-filters.

In comparing these two different filter implementations, they are both appropriate for use in a 1-D two-channel SBC system. In the QMF approach, having eliminated both aliasing and phase errors, techniques exist which aim to minimise the residual amplitude distortion, since only trivial solutions are available for the total elimination. Johnson's method [2] formulates an objective measure based upon stopband attenuation and the ability to maintain the power symmetry property of the filters. Computer-aided optimisation algorithms are then employed and this has proven to exhibit very low peak amplitude distortions, typically less than 0.02dB. If all-pass IIR structures are utilised, then [3] has shown that it is possible to eliminate amplitude distortion, though the phase linearity becomes problematic. In the CQF realisation, [4] shows that PR is achievable by eliminating all three errors from the output signal and that in general the CQF implementation possesses superior transition bandwidth and hence a lower order. Their main disadvantage however is, that the linear phase property is only ensured at the output of the whole transmission system, when both subsignals are combined. After transmission, before the signal passes through the synthesis bank, the phase of the subband signal is non-linear. This is especially critical in video applications, where only *one* subband signal is important, such as the band-splitting process in [1]. Here the lowpass path represents a standard resolution TV picture and the highpass path is a digital compressed augmentation signal, which is subsequently used to enable a high definition picture to be obtained at the receiver. The phase distortion introduced would be clearly visible on such a TV picture.

5 Two-dimensional Subband Coding Systems

A one-dimensional filter designed by using one of the above design methods can be extended into a second dimension by using the principle of separability. It is not necessary to extend the dimensions of the filter, but to use two one-dimensional QMF filters, one filtering in the horizontal plane and the other in the vertical direction, both arranged consecutively. The requirement in video applications is that the order of the filter should be as low as feasible, commensurate with minimum coefficient wordlengths, so as to achieve perfect reconstruction (PR). This has meant that direct-form implementations of QMF and CQF structures are not especially robust to wordlength quantisation and PR can be lost. Lattice implementations on the other hand have proved particularly attractive in this application, since provided the polyphase components of the analysis filters exhibit paraunitariness (lossless), PR is structurally induced [2]. Hence, in spite of quantisation the 2-D lattice-QMF SBC system guarantees PR as well as being the lowest complexity realisation possible.

A two-dimensional transmission system is shown in Figure 6, where only 1-D filters are utilised. The first-level analysis bank filters in the horizontal plane and two subbands are generated. The second level of the analysis bank filters both of these subsignals in the vertical direction, so that four subbands in total are derived and transmitted separately. An appropriate arrangement of the synthesis filters recreates the original signal. Figure 7 shows another SBC realisation, which is especially relevant to video applications and in dealing with HDTV compatible transmissions [1]. It includes two additional diagonal filters which are not actually part of the SBC system, but whose role is bandwidth reduction, by the suppression of diagonal frequencies. From the band-splitting process, a two dimensional lowpass signal is derived from a one-dimensional vertical lowpass, proceeded by a one dimensional horizontal lowpass filter. The two wedge-shaped elements remaining, which are illustrated in Figure 8, will after decimation fold back into exactly the same bandwidth as the low-pass signal, so that it enables HDTV transmissions to be compatible in terms of bandwidth with conventional TV transmissions [6,7]. An interesting extension to the systems shown in Figures 6 and 7 is that further levels of QMF decomposition can be implemented, so enabling finer resolution to be achieved in both the horizontal and vertical directions. This produces a multi-resolution binary tree, which is a direct realisation of the Discrete Wavelet Transform (DWT), whose performance as a viable data compression algorithm for image and video applications is gaining considerable attention.

6 Conclusion

This paper has explored the various filter design options available for implementing QMF and CQF structures in a 2-D subband coding system. They have both been proven to provide very good or even perfect reconstruction properties, though in the case of QMF, there is a small amount of amplitude distortion, while in the CQF the phase linearity is lost during subband coding. The SBC system and the filters can be easily extended to process two-dimensional signals, without any loss of their reconstruction properties.

Acknowledgements

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References

- [1] G. Schmidt, L. S. Dooley, W. P. Buchwald, "A PALplus Compatible HDTV Encoder System," IEEE Transactions on Consumer Electronics, Vol. 40, No. 3, August 1994, pp207-215.
- [2] P. P. Vaidyanathan, "Multirate Systems and Filter Banks", Prentice Hall, 1993.
- [3] P. P. Vaidyanathan, "Quadrature Mirror Filter Banks, M-Band Extensions and Perfect-Reconstruction Techniques," IEEE ASSP Magazine, July 1987
- [4] M. J. Smith, T. P. Barnwell, "Exact Reconstruction Techniques for Tree-Structured Subband Coders," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-34, No. 3, June 1986.
- [5] P. P. Vaidyanathan, "Theory and Design of M -Channel Maximally Decimated Quadrature Mirror Filters with Arbitrary M , Having the Perfect-Reconstruction Property," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-35, No. 4, April 1987.
- [6] Michael Meißner, "Quadrature-Mirror-Filter Design for Subband Coding of Video Signals," Diploma Thesis, University of Glamorgan, Fachhochschule Braunschweig/Wolfenbüttel, April 1995.
- [7] G. Schmidt, L. S. Dooley, W. P. Buchwald, "Design of 2-D digital video filters for a PALplus-Compatible HDTV Transmission Encoder," 14th IEE Saraga Colloquium on Digital and Analogue filters and filtering systems, London, November 1994.

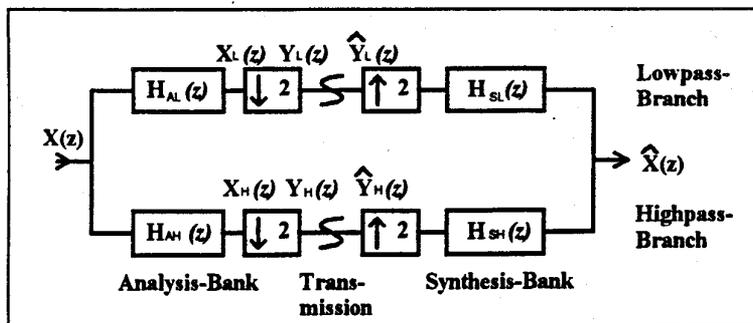


Fig. 1: Two-channel subband coding bank

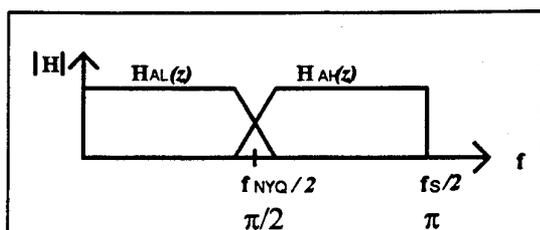


Fig. 2: Overlapping analysis filter

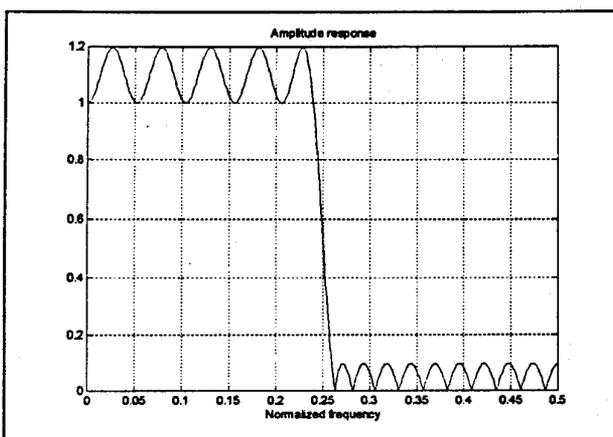


Fig. 3: Amplitude response of the anti-symmetrical, equi-ripple FIR half-band product filter

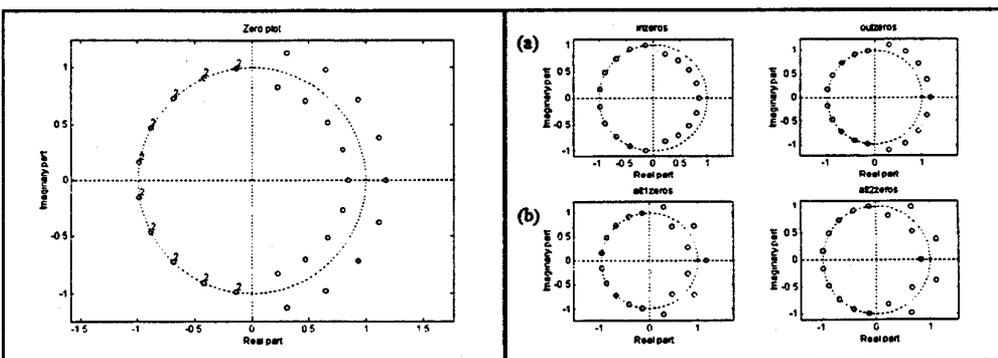


Fig. 4 (left): Zero plot of the resulting product filter zeros in the z-plane

Fig. 5 (right): Decomposition of the product filter zeros for the analysis and synthesis lowpass.

(a): zeros selected separately from inside and outside the unit circle,

(b): zeros selected alternately.

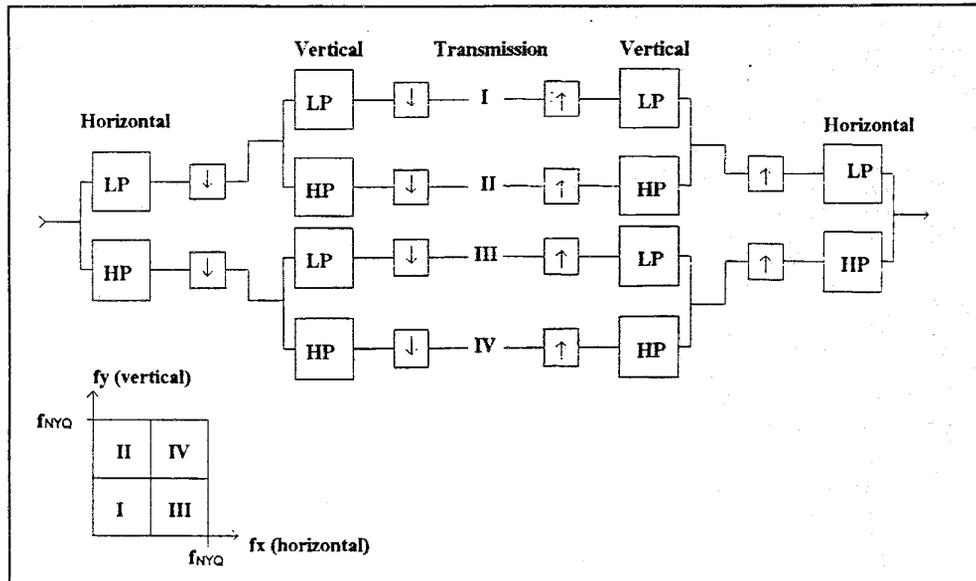


Fig. 6: Two-dimensional subband coding system.

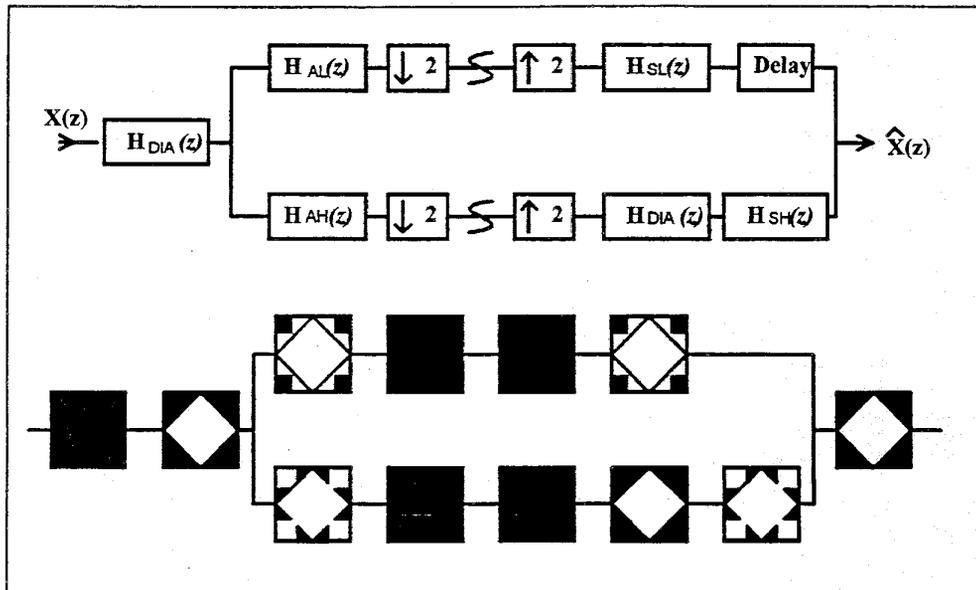


Fig. 7: Two-dimensional subband coding system with relevant signal spectra and diagonal filters.

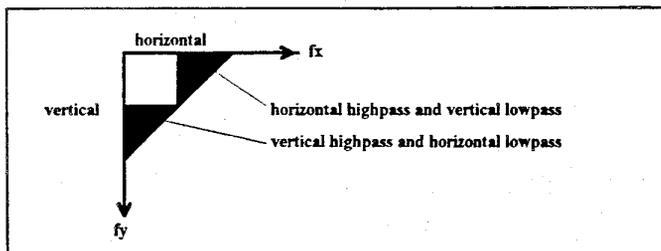


Fig. 8: Wedge-shaped high frequency components.