1 Introduction

Traditionally, a distinction is made between that what is asserted by uttering a sentence and that what is presupposed. Presuppositions are characterized as those propositions which persist even if the sentence which triggers them is negated. Thus ‘The king of France is bald’ presupposes that there is a king of France, since this follows from both ‘The king of France is bald’ and ‘It is not the case that the king of France is bald’.

Stalnaker (1974) put forward the idea that a presupposition of an asserted sentence is a piece of information which is assumed by the speaker to be part of the common background of the speaker and interpreter. The presuppositions as anaphors theory of Van der Sandt (1992) — currently the best theory of presupposition as far as empirical predictions are concerned (Beaver 1997:983)— can be seen as one advanced realization of Stalnaker’s basic idea. The main insight of Van der Sandt is that there is an interesting correspondence between the behaviour of anaphoric pronouns in discourse and the projection of presuppositions (i.e., whether and how presuppositions survive in complex sentences). Like most research in this area, Van der Sandt’s work concentrates on the interaction between presuppositions and the linguistic context (i.e., the preceding sentences). However, not only linguistic context interacts with presuppositions. Consider:

(1) a. If John buys a car, he checks the motor first.
    b. John walked into the room. The chandelier sparkled brightly.
    c. Mary traded her old car in for a new one. The motor was broken.

All three examples are instances of the notorious bridging phenomenon (Clark 1975). Example (1.a) contains a definite description, the motor, which triggers the presupposition that there is a motor. Intuitively, (1.a) as a whole does not presuppose the existence of a motor; this presupposition is ‘absorbed’ by the antecedent. However, because there is no proper antecedent for this definite description, the theory of Van der Sandt (1992) predicts that the presupposition that there is a motor is accommodated (where accommodation —the term is due to Lewis 1979— amounts to simply adding the presupposition to the context). This fails to do justice to the intuition that the mentioning of a car somehow licenses the use of the motor and that the motor is part of the car which John buys. Thus, for the correct treatment of this example, a rather trivial piece of world knowledge is needed: cars have motors. In the presence of

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such background knowledge an interpreter will be able to construct a bridge between the would-be antecedent (a car) and the presupposition/anaphor (the motor). Example (1.b) can be explained along similar lines; the interpreter has to construct a bridge between a room and the chandelier. Unfortunately, things are a bit more complicated for this example. After all, the interpreter will not be able to use background knowledge such as rooms have chandeliers, since there are many chandelier-less rooms. Example (1.c) illustrates yet another complication: granted that cars have motors, with which of the two cars introduced in the first sentence of example (1.c) should the motor from the second sentence be associated?

For all these examples, the theory from Van der Sandt (1992) predicts that the presuppositions are accommodated, due to the fact that the non-linguistic context is not taken into account. In this article, we want to get a formal grip on the way in which context influences the behaviour of presuppositions. Before we describe how we intend to do this, let us first describe the notion of context we are interested in. There are various uses of the term ‘context’. Bunt (1995) characterizes context as all those factors which are relevant to the understanding of communicative behaviour, and he goes on to distinguish five major dimensions: the linguistic context, the semantic context, the physical context, the social context and the cognitive context. For presuppositions in general, and for bridging in particular, the following seem most relevant: the linguistic context, as this will contain the antecedents from which a bridge has to be constructed, and the cognitive context, which according to Bunt includes the attentional state and the world knowledge of an interlocutor. Throughout this article we will therefore focus on the linguistic and the cognitive context. The resulting, global picture is as follows: an interpreter tries to understand a sentence in some context Γ. This context contains representations of the preceding discourse (the linguistic context) as well as background knowledge (the cognitive context). The interlocutor assumes that parts of her context are, to some extent, public. That is, they form what the interlocutor assumes to be the common ground. In this article, we are particularly interested in how interlocutors use the context to come to an understanding of the current sentence, and how they can adjust their context on the fly, so to speak, when the current sentence calls for such an adjustment. This brings out the extreme flexibility of context in natural language communication: speaker and hearer constantly attempt to align their representations. For more details on the role of context in communication, we refer to Piwek (1998).

The claim that context, and more specifically world knowledge, has an influence on presupposition projection is hardly revolutionary, the question is how to account for this influence. We argue that employing a class of mathematical formalisms known as Constructive Type Theories (CTT, see e.g., Martin-Löf 1984, Barendregt 1992) allows us to answer this question. To do so, we reformulate Van der Sandt’s theory in terms of CTT. CTT differs from other proof systems in that for each proposition which is proven, CTT also delivers a proof object which shows how the proposition was proven. As we shall see, the presence of these proof objects is useful from the presuppositional point of view. Additionally, CTT contexts contain more information than is conveyed by the ongoing discourse, and there is a formal interaction between this ‘background knowledge’ and the representation of the current discourse. This means that the reformulation of Van der Sandt’s theory in terms of CTT is not just a nice technical exercise, but actually creates some interesting new possibilities where
the interaction between presupposition resolution and world knowledge context is concerned. To illustrate this, we show that the resulting system facilitates the treatment of the notorious bridging phenomenon illustrated above. In particular, it will be shown that many of the observations made in Clark’s (1975) seminal ‘bridging’ paper have nice CTT counterparts. We propose to come to a so-called determinate bridge by imposing two conditions: effort and plausibility. Finally, we discuss an explorative study we conducted to find support for our analysis. In this article we will not dig too deep into the formalities of our approach of ‘presupposition projection as proof construction’, for that we refer to Krahmer & Piwek (1997).

\section{Presuppositions as Anaphors}

Van der Sandt (1992) proposes to resolve presuppositions, just like anaphoric pronouns are resolved in Discourse Representation Theory (DRT, Kamp & Reyle 1993). In DRT, linguistic contexts are modelled as Discourse Representation Structures (DRSs). A DRS consists of a set of discourse referents and a set of conditions on these referents. The discourse referents can be seen as representatives for the objects which are introduced in the discourse, and the conditions can be seen as assignments of properties to these objects. To resolve presuppositions in DRT, Van der Sandt (1992) develops a meta-level resolution algorithm. The input of this algorithm is an underspecified DRS, which contains one or more unresolved presuppositions. When all these presuppositions have been resolved, a proper DRS remains, which can be interpreted in the standard way.\footnote{In Krahmer (1998), Van der Sandt’s theory is combined with a version of DRT with a partial interpretation. In this way, DRSs which contain unresolved presuppositions can also be interpreted. It is shown that this has several advantages.}

Let us consider the following example, and its Van der Sandtian representation:

(2) If John buys a pantechnicon, he’ll adore the vehicle.

\begin{center}
\begin{tabular}{|c|c|}
\hline
\textbf{x} & \textbf{y} \\
\hline
\text{pantechnicon(x)} & \text{vehicle(y)} \\
\text{buy(john,x)} & \text{adore(john,y)} \\
\hline
\end{tabular}
\end{center}

This DRS consists of a complex condition, containing two sub-DRSs, one for the antecedent and one for the consequent of (2). The antecedent DRS introduces a referent \( x \). This \( x \) stands for a pantechnicon which is bought by John (where ‘John’ is represented by a constant \( \text{john} \) for the sake of simplicity). The definite description \textit{the vehicle} presupposes the existence of a vehicle. This is modelled by adding an embedded, presuppositional DRS to the consequent DRS introducing a referent \( y \) which is a vehicle. The consequent DRS additionally contains the condition that this presupposed vehicle is adored by John. To resolve the presuppositional DRS, we do what...
we would do to resolve a pronoun: look for a suitable, accessible antecedent. In this case, we find one: the discourse referent \( x \) introduced in the antecedent is accessible\(^2\) and suitable since a pantechnicon (i.e., a removal truck) is a vehicle. Exactly how this information can be employed in Van der Sandt's theory is not obvious. For now, we will simply assume that we can bind the presupposition, which results in the following DRS, which can be paraphrased as ‘if John buys a pantechnicon, he’ll adore it’\(^3\).

\[
\begin{array}{c|c|c}
\hline
x & \text{pantechnicon}(x) & \Rightarrow \text{adore}(john, x) \\
\hline
\end{array}
\]

In principle, anaphoric pronouns are always bound. For presuppositions this is different: they can also be accommodated, provided the presupposition contains sufficient descriptive content. Reconsider example (2) again: on Van der Sandt’s approach (globally) accommodating the presupposition associated with the vehicle amounts to removing the presuppositional DRS from the consequent DRS and placing it in the main DRS, which would result in the following DRS.

\[
\begin{array}{c|c|c|c|c}
\hline
\hline
y & \text{vehicle}(y) & x & \text{pantechnicon}(x) & \Rightarrow \text{adore}(john, y) \\
\hline
\hline
\end{array}
\]

This DRS represents the ‘presuppositional’ reading of (2), which may be paraphrased as ‘there is a vehicle and if John buys a pantechnicon, he’ll adore the aforementioned vehicle’. Now we have two ways of dealing with the presupposition in example (2), so the question may arise which of these two is the ‘best’ one. To answer that question, Van der Sandt defines some general rules for preferences, which may be put informally as follows: 1. Binding is preferred to accommodation, 2. Binding is preferred as low as possible, and 3. Accommodation is preferred as high as possible (thus, preferably in the main DRS). The third preference rule seems to suggest that there is more than one way to accommodate a presupposition, and indeed there is. To illustrate this, consider:

(3) It is not true that I adore John’s pantechnicon, since he doesn’t have one!

Here, the definite NP John’s pantechnicon presupposes that John has a pantechnicon. If we globally accommodate this presupposition (that is, the presupposition ‘escapes’ from the scope of the negation and is placed in the main DRS), we would end up with an inconsistent DRS, expressing that John has a pantechnicon, which is contradicted by

\(^2\)In DRT, the generalization is that discourse referents introduced in an antecedent DRS are accessible from the consequent DRS.

\(^3\)This DRS (as the previous one) is presented in the usual ‘pictorial’ fashion. Elsewhere in this paper we also use a linear notation which we trust to be self-explanatory. E.g., in this linear notation the current DRS looks as follows: [\[ x \mid \text{pantechnicon}(x), \text{buy}(john, x) \] \Rightarrow [\text{adore}(john, x)]].
the since-clause. Van der Sandt (1992:367) defines a number of conditions on accommodation, of which consistency is one. Since in the case of (3) global accommodation yields an inconsistent DRS, local accommodation of the presupposition is preferred, where local means within the scope of the negation. The result can be paraphrased as ‘it is not true that John has a pantechnicon and that I adore it, since he doesn’t have one’.

In the next section, we discuss CTT and show how Van der Sandt’s approach can be rephrased in terms of it. In the section thereafter, we will see how the examples in (1), which are problematic for Van der Sandt’s approach as it stands, can be dealt with. We believe that the CTT approach leads to better results than adding a proof system to DRT, as done in e.g., Saurer (1993). The main advantage of CTT is that it is a standard proof system developed in mathematics with well-understood meta-theoretical properties (see Ahn & Kolb 1990 for discussion on the advantages of reformulating DRT in CTT). Moreover, the presence of explicit proof objects in CTT turns out to have some additional advantages for our present purposes. For us, the constructive aspect resides in the explicit construction of proof-objects; we are not necessarily committed to an underlying intuitionistic logic.

3 The deductive perspective

The deductive approach to discourse We introduce CTT by comparing it with DRT; this comparison is based on Ahn & Kolb (1990), who present a formal translation of DRSs into CTT expressions. A context in CTT is modelled as a sequence of introductions. Introductions are of the form $V : T$, where $V$ is a variable and $T$ is the type of the variable. Consider example (4.a) and its DRT representation (4.b) (in the linear notation, cf. footnote 3).

\begin{equation}
\text{(4) a. John drives a vehicle.}
\end{equation}
\begin{equation}
\text{b. } [x \mid \text{vehicle}(x), \text{drives(john,}x)]
\end{equation}

A discourse referent can be modelled in CTT as a variable. A referent is added to the context by means of an introduction which not only adds the variable but also fixes its type. We choose entity as the type of discourse referents. The type entity itself also requires introduction. Since entity is a type, we write: entity:type.

The type entity should only be used in the introduction $x:entity$ if entity:type is already part of the context. This way, one introduction depends on another introduction, hence a context is an ordered sequence of introductions. The type type also requires introduction. The introduction is, however, not carried out in the context; it is taken care of by an axiom which says that type:☐ (where ☐ is to be understood as the ‘mother of all types’) can be derived in the empty context ($\varepsilon \vdash \text{type : ☐}$).

DRT’s conditions correspond to introductions $V : T$, where $T$ is of the type prop (short for proposition, which comes with the following axiom: $\varepsilon \vdash \text{prop : ☐}$). For instance, the introduction $y : (\text{vehicle} \cdot x)$ corresponds to the condition vehicle$(x)$. The type vehicle · $x$ (of type prop) is obtained by applying the type vehicle to the object $x$. Therefore, it depends on the introductions of $x$ and vehicle. Since vehicle · $x$ should be of the type prop, vehicle must be a (function) type from the set of entities into propositions, i.e., vehicle : entity → prop.
The introduction \( y : (\text{vehicle} \cdot x) \) involves the variable \( y \) (of the type \( \text{vehicle} \cdot x \)). The variable \( y \) is said to be an inhabitant of \( \text{vehicle} \cdot x \). Curry and Feys (1958) came up with the idea that propositions can be seen as classifying proofs (this is known as the ‘propositions as types — proofs as objects’ interpretation). This means that the aforementioned introduction states that there is a proof \( y \) for the proposition \( \text{vehicle} \cdot x \). The second DRS condition \( (\text{drive}(john, x)) \) can be dealt with along the same lines. Assume that \( \text{drive} \) is a predicate which requires two arguments of the type \( \text{entity} \), this yields \( z : \text{drive} \cdot x \cdot john \). (The · (representing function application) is left-associative, thus \( f \cdot x \cdot y \) should be read as \((f \cdot x) \cdot y\)). In sum, the CTT counterpart to the DRS \((4.b)\) consists of the following three introductions: \( x : \text{entity}, y : \text{vehicle} \cdot x, z : \text{drive} \cdot x \cdot john \).

**Dependent Function Types** In DRT, the proposition *Everything moves* is translated into the implicative condition \( [x \mid \text{thing}(x)] \implies [ \mid \text{move}(x)] \). In CTT, this proposition corresponds to the type \((\Pi x : \text{entity} \cdot \text{move} \cdot x)\), which is a dependent function type. It describes functions from the type \( \text{entity} \) into the type \( \text{move} \cdot x \). The range of such a function \( \text{move} \cdot x \) depends on the object \( x \) to which it is applied. Suppose that we have an inhabitant \( f \) of this function type, i.e., \( f : (\Pi x : \text{entity} \cdot \text{move} \cdot x) \). Then we have a function which, when it is applied to an arbitrary object \( y \), yields an inhabitant of the proposition \( \text{move} \cdot y \). Thus, \( f \) is a constructive proof for the proposition that *Everything moves*.

Of course, function types can be nested. Consider the predicate \( \text{drive} \). Above we suggested to introduce it as a function from entities (‘the driver’) to entities (‘the thing being driven’) to propositions. One could, however, argue that the second argument of \( \text{drive} \) (‘the thing being driven’) can only be a vehicle. In that case, \( \text{drive} \) would have to be introduced as function from entities to entities to another function (i.e., the function from a proof that the second entity is a vehicle to a proposition), that is \( \text{drive} : (\Pi y : \text{entity}.(\Pi e : \text{entity}.(\Pi p : \text{vehicle} \cdot x \cdot \text{prop}))) \). We will abbreviate this as \( \text{drive} : ([y : \text{entity}, x : \text{entity}, p : \text{vehicle} \cdot x] \Rightarrow \text{prop}) \).

**Deduction** The core of CTT consists of a set of derivation rules with which one can determine the type of an object in a given context. These rules are also suited for searching for an object belonging to a particular type. There is, for instance, a rule which is similar to modus ponens in classical logic (in the rule below, \( T[x := a] \) stands for a \( T \) such that all free occurrences of \( x \) in \( T \) have been substituted by \( a \). Furthermore, \( \Gamma \vdash E : T \) means that in context \( \Gamma \), the statement \( E : T \) is provable):

\[
\frac{\Gamma \vdash F : (\Pi x : A.B) \quad \Gamma \vdash a : A}{\Gamma \vdash F \cdot a : B[x := a]}
\]

For instance, if a context \( \Gamma \) contains the introduction \( b : \text{entity} \) as well as the introduction \( g : (\Pi y : \text{entity} \cdot \text{move} \cdot y) \) (‘everything moves’), then we can use this rule to find an inhabitant of the type \( \text{move} \cdot b \). In other words, our goal is to find a substitution \( S \) such that \( \Gamma \vdash P : \text{move} \cdot b[S] \). The substitution \( S \) should assign a value to \( P \). \( P \) is a so-called gap.

\(^4\)In Piwek (1997, 1998) it is shown how these same gaps can be used in the analysis of questions. Piwek argues that questions introduce gaps, which can be filled by extending the context of interpretation.

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The deduction rule tells us that \((g \cdot b)\) can be substituted for \(P\), if \(\Gamma \vdash g : (\Pi y : entity, move \cdot y)\) and if \(\Gamma \vdash b : entity\). Both so-called judgements are valid, because we assumed that \(g : (\Pi y : entity, move \cdot y)\) and \(b : entity\) are members of \(\Gamma\). Thus, we can conclude that \(\Gamma \vdash (g \cdot b) : move \cdot b\).

**Presuppositions as Gaps** Van der Sandt’s presuppositional DRSs can be seen as a kind of ‘proto DRSs’ for which the presuppositional representations have not yet been resolved. Only after resolution and/or accommodation of the presuppositions is a proper DRS produced. Analogously, in CTT terms, a construction algorithm could translate a sentence into a proto type before a proper type (of the type prop) is returned. This proper type (i.e., proposition) can then be added to the main context by introducing a fresh proof for it. Let us reconsider example (2), repeated below as (5), together with the appropriate proto type for this sentence in (6).

(5) If John buys a pantechnicon, he’ll adore the vehicle.

(6) \([x : entity, y : pantechnicon \cdot x, z : buy \cdot x \cdot john] \Rightarrow (adore \cdot Y \cdot john)\]

In words: if \(x\) is an entity and \(y\) a proof that \(x\) is a pantechnicon and \(z\) is a proof that it is bought by John, then there exists a proof that John adores \(Y\), where \(Y\) is a gap to be filled by an entity for which we can prove that it is a vehicle. The (subscripted) presuppositional annotation consists of a sequence of introductions with gaps.

**Filling the Gaps: Binding v. Accommodation** Suppose we want to evaluate the CTT representation (6) given some context \(\Gamma\). Before we can do that we have to resolve the presupposition by filling the gap. For this purpose, we have developed an algorithm which operates on proto-types and CTT contexts, based on Van der Sandt’s presupposition resolution algorithm (see Krahmer & Piwek (1997) for technical details). The first thing we do after starting the resolution process, is try to ‘bind’ the presuppositional gap. The question whether we can bind the presupposition triggered by the vehicle in example (5) can be phrased in CTT as follows: is there a substitution \(S\) such that the following can be proven?

(7) \(\Gamma, x : entity, y : pantechnicon \cdot x, z : buy \cdot x \cdot john \vdash (Y : entity, P : vehicle \cdot Y)[S]\)

In words: is it possible to prove the existence of a vehicle from the global context \(\Gamma\) extended with the local context (the antecedent)? The answer is: that depends on \(\Gamma\). Suppose for the sake of argument that \(\Gamma\) itself does not contain any vehicles, but that it does contain the information that a pantechnicon is a vehicle. Technically, this means that the following function is a member of \(\Gamma\):

(8) \(f : ([a : entity, b : pantechnicon \cdot a] \Rightarrow (vehicle \cdot a))\)

with the answer provided by the dialogue participant. A question is answered, when the associated gaps can be filled.
Given this function, we find a substitution $S$ for (7), mapping $Y$ to $x$ and $P$ to $(f \cdot x \cdot y)$ (which is the result of applying the aforementioned function $f$ to $x$ and $y$). So we fill the gaps using the substitution $S$, remove the annotations (which have done their job) and continue with the result:

(9) $[x : \text{entity}, y : \text{pantechnicon} \cdot x, z : \text{buy} \cdot x \cdot \text{john}] \Rightarrow (\text{adore} \cdot x \cdot \text{john})$

Thus, intuitively, if an interpreter knows that a pantechnicon is a vehicle, she will be able to bind the presupposition triggered by the definite the vehicle in (5).

Now suppose the interpreter does not know that a pantechnicon is a vehicle. That is, $\Gamma'$ does not contain a function mapping pantechnicons to vehicles. Then, still under the assumption that $\Gamma$ itself does not introduce any vehicles, the interpreter will not be able to prove the existence of a vehicle. Intuitively this means that the interpreter is faced with an expression containing an unsatisfied presupposition. In that case, she might come to the conclusion that her context is not rich enough and that something (namely a vehicle) is missing from it. She can then try to accommodate the existence of a vehicle by replacing the gaps $Y$ and $P$ with fresh variables, say $y'$ and $p'$, and extending the context $\Gamma$ with $y' : \text{entity}, p' : \text{vehicle} \cdot y'$. Of course, it has to be checked whether this move is adequate, whether the accommodation is consistent, etc.

4 **Bridging**

Let us take stock. We claim that context, and more in particular, world knowledge plays a role in presupposition projection. However, there are very few, if any, theories of presupposition which account for the interaction between presuppositions and context/world knowledge. We have argued that the deductive perspective of CTT offers an attractive framework to model this interaction. Our starting point is a reformulation of Van der Sandt’s presupposition resolution algorithm tailored to CTT. In this section we want to illustrate the formal interaction between world knowledge and presupposition resolution, by focusing on the bridging phenomenon:

So, what is bridging precisely? Clark (1975) describes it in terms of an interpreter who is looking for an antecedent, but cannot find one ‘directly in memory’. “When this happens, he is forced to construct an antecedent, by a series of inferences, from something he already knows. (…) The listener must therefore bridge the gap from what he knows to the intended antecedent.” (Clark 1975:413). We want to make these general ideas more precise. In particular, we want to spell out the notion of inference that is involved. Clark himself contends that the bridging-inferences are similar in nature to what Grice (1975) has called ‘implicatures’. From the current perspective, there are two kinds of inferences relevant for bridging. The most straightforward one would simply be inference in CTT. We take it that a CTT context $\Gamma$ represents the information an agent has ‘directly in memory’. Inferred information corresponds with objects that can be constructed from objects in $\Gamma$ using the deduction rules of CTT. However, there is also a second kind of inference present in the approach to presuppositions sketched

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5 Elsewhere (in Krahmer & Piwek 1997) we have shown that the CTT approach also yields interesting results for the interaction between presupposition projection in conditionals and world knowledge.
above: accommodation (which bears a close resemblance to abduction in the framework of Hobbs et al. 1993, Krause 1995). We claim that both kinds of inference play a role in bridging. Let us discuss each in somewhat more detail.

‘Inference’ as Deduction in CTT From this perspective, bridging amounts to using world knowledge to fill gaps. Consider first example (10.a) with its CTT representation given in (10.b).

(10) a. If John buys a car, he checks the motor first.
   b. \[ x : \text{entity}, y : \text{car} \cdot x, z : \text{buy} \cdot x \cdot \text{john} \Rightarrow \]
   \[ (\text{check} \cdot Y \cdot \text{john})[Y : \text{entity}, P : \text{motor} \cdot Y] \]

Before we can add this expression to some context \( \Gamma \), we have to resolve the presuppositional expression. To do so, we first search for a substitution \( S \) such that the following can be proven:

(11) \( \Gamma, x : \text{entity}, y : \text{car} \cdot x, z : \text{buy} \cdot x \cdot \text{john} \vdash (Y : \text{entity}, P : \text{motor} \cdot Y)[S] \)

Let us assume that \( \Gamma \) (a model of the agent’s ‘direct memory’) does not contain a sufficiently salient motor. Then the interpreter will try to ‘bridge the gap from what he knows to the intended antecedent’. When does he succeed in this, i.e., when can the motor be understood as a bridging anaphor licensed by the introduction of a car? The answer is simple: if the interpreter knows that a car has a motor. Modelling this knowledge could go as follows. \( \Gamma \) contains two functions: one function which maps each car to an entity, \( f : ([a : \text{entity}, b : \text{car} \cdot a] \Rightarrow \text{entity}) \), and one function which states that this entity is the car’s motor \( g : ([a : \text{entity}, b : \text{car} \cdot a] \Rightarrow (\text{motor} \cdot (f \cdot a \cdot b))) \).
Using these functions, we find a substitution \( S \) in (11), mapping \( Y \) to \( f \cdot x \cdot y \) and \( P \) to \( g \cdot x \cdot y \). We can look at the resulting proof objects as the ‘bridge’ that has been constructed by the interpreter; it makes the link with the introduction of a car explicit (by using \( x \) and \( y \)) and indicates which inference steps the user had to make to establish the connection with the motor (by using the functions \( f \) and \( g \)). Thus, we can fill the gaps, assuming that the proofs satisfy certain conditions. Of course, they have to satisfy the usual Van der Sandt conditions (such as consistency). Additionally, the bridge itself has to be ‘plausible’. Below we will return to the issue of constraints on building bridges.

‘Inference’ as Accommodation Let us now consider a somewhat more complex example (after Clark 1975:416).

(12) John walked into the room. The chandelier sparkled brightly.

Let us assume that the first sentence of (12) has already been processed, which means that the context \( \Gamma \) contains the following introductions: \( x : \text{entity}, y : \text{room} \cdot x, z : \text{walk} \cdot x \cdot \text{john} \). At this stage, we want to deal with the CTT representation of the second sentence, given below.

(13) \( q : \text{sparkle} \cdot Y[Y : \text{entity}, P : \text{chandelier} \cdot Y] \)
We want to resolve the presupposition triggered by *the chandelier* in the context \( \Gamma \) (assuming that \( \Gamma \) does not introduce any (salient) chandeliers). When would an interpreter be able to link *the chandelier* to the room John entered? Of course, it would be easy if she had some piece of knowledge to the effect that every room has a chandelier (if her \( \Gamma \) contained functions which for each room produce a chandelier). However, such knowledge is hardly realistic; many rooms do not have a chandelier.

In a more lifelike scenario, the following might happen. The interpreter tries to prove the existence of a chandelier, but fails to do so. However, the interpreter knows that a chandelier is a kind of lamp and the existence of a lamp *can* be proven using the room just mentioned and the background knowledge that rooms have lamps. Formally, and analogous to the ‘motor’ example, one function which produces an entity for each room; \( f : (\{a : \text{entity}, b : \text{room} \cdot a\} \Rightarrow \text{entity}) \), and one which states that this entity is a lamp; \( g : (\{a : \text{entity}, b : \text{room} \cdot a\} \Rightarrow (\text{lamp} \cdot (f \cdot a \cdot b))) \). Since the speaker has uttered (12) the interpreter will *assume* that (one of) the lamp(s) in the room is a chandelier (compare Clark 1975:416). In terms of the CTT approach, this could go as follows: first, the interpreter *infers* (deduces) that the room which John entered contains an entity which is a lamp (applying the aforementioned piece of knowledge; the functions \( f \) and \( g \)), and then binds part of the presupposition by filling the \( Y \) gap with \( f \cdot x \cdot y \) (the inferred lamp). The remaining part of the presupposition (that the lamp is in fact a chandelier) is now *accommodated* in the Van der Sandtian way by filling the gap with a fresh variable.

**Bridging as a Determinate Process**  
Clark (1975) claims that bridging is a *determinate* process. In theory, however, background knowledge will license a number of bridges. In CTT terms, there will often not be *one* way to fill a presuppositional gap, but there will be many. Clark discusses the following example:

(14) Alex went to a party last night. He is going to get drunk again tonight.

Here *again* triggers the presupposition that Alex was drunk before. According to Clark, we assume that “*every time Alex goes to a party, he gets drunk*”. In our opinion, this assumption is too strong, we feel that one would merely assume that Alex was drunk at the party he visited last night (compare the ‘chandelier’ case). But that is not the point here. Clark (1975:419-420) goes on to notice that there are theoretically conceivable alternatives for his assumption which interpreters, however, would never construct: “(…) we could have assumed instead that every time he [Alex, P&K] goes to party he meets women, and all women speak in high voices, and high voices always remind him of his mother, and thinking about his mother always makes him angry, and whenever he gets angry, he gets drunk” We would like to stress that the problem of determinacy is not restricted to bridging. Consider the following example from Lewis (1979:348):

(15) The pig is grunting, but the pig with the floppy ears is not grunting.

Apparently, this sentence can only be uttered when there are (at least) two pigs in ‘direct memory’. Nevertheless, each of the definite descriptions can be understood as referring to a determinate pig. Lewis argues that *salience* is the relevant notion here:
he argues that the pig is the most salient pig, while the pig with the floppy ears is the most salient pig with floppy ears. In other words: the interpreter has to find the most salient antecedent for the respective descriptions in order to guarantee determinedness. However, in the case of bridging, salience is a necessary, but certainly not a sufficient condition to guarantee determinedness. We propose to use two (groups of) conditions to come to a determinate bridge, related to the effort an interpreter needs to construct a bridge and the plausibility of the constructed bridges.

The Effort Condition  To begin with the former: as noted above, Clark (1975:420) claims that interpreters do not draw inferences ad infinitum, and to model this he proposes a general stopping rule, which says essentially that the interpreter builds the shortest possible bridge that is consistent with the context. We take this constraint to subsume two conditions. The first of these conditions boils down to the following rule: use your ‘informational resources’ as frugal as possible. In CTT terms: if a gap can be filled with more than one proof object, fill it with the one with the lowest complexity. As a (rather informal) illustration, consider:

(16) A married couple is strolling through the park. The man looks in love.

Let us assume that the interpreter has evaluated the first sentence and added its representation to $\Gamma$. Thus, $\Gamma$ contains an introduction for a married couple, say $x$. Now say that the interpreter has (at least) the following background knowledge: ‘every married couple contains a man’ (call this function $f_1$), ‘every married couple contains a woman’ ($f_2$) and ‘everyone has a father’ ($g$), assuming that fathers are male. If the interpreter now looks for an antecedent for the presupposition triggered by the man, she will find $f_1 \cdot x$ (the male half of the married couple), but also $g \cdot (f_1 \cdot x)$ (the father of the male half of the married couple), not to mention $g \cdot (f_2 \cdot x)$ (the father of the female half of the married couple). However, the proof-object $f_1 \cdot x$ is clearly less complex (makes thriftier use of the ‘informational resources’) than the last two proof-objects, and therefore the interpreter uses this object to fill the presuppositional gap, and not the other two.

The second condition subsumed by Clark’s stopping rule can be put as follows: make as few assumptions as possible (accommodate as little as possible). Example (14) provides an illustration of this second condition, which on the current approach follows from the general view of accommodation as a repair strategy (modelled by a preference for resolution/binding over accommodation). Factors like recency are also related to minimizing effort; it takes more effort to build a bridge to an antecedent occurring ten sentences ago then to one occurring in the previous sentence.

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$^6$The complexity of a proof object is defined as the number of unbound variables in the proof object. A variable which occurs as part of a proof object and which is not bound by a $\lambda$ operator corresponds to an object in $\Gamma$. $\lambda$ abstraction basically introduces a hypothetical object into a proof, e.g., we can proof $p \rightarrow q$ from $q$ by starting with a proof $a$ for $q$ and then introducing a hypothetical proof $x$ for $p$: $(\lambda x : p.a) : (p \rightarrow q)$. Thus we obtain a function, which when it is applied to some proof $y$ of $p$ yields a proof for $q$, namely $(\lambda x : p.a) \cdot y = a$. Our complexity measure takes into account the amount of information needed from $\Gamma$ to construct the proof-object and also how many times an object has been used. Assume for instance that $\Gamma = p : prop,q : prop,a : p,f : p \rightarrow q$ (in words, $p$ and $q$ are propositions, and $a$ is a proof of $p$ and $f$ is a proof of $p \rightarrow q$, respectively). We denote the complexity of object $a$ by $C(a)$. For the proof $a$ of $p$ we have: $C(a) = 1$. For $q$ we have $C(f \cdot a) = 2$. Note that for tautologies there are proofs whose complexity is equal to zero (we need no premises to proof them).
The Plausibility Condition
Suppose that we can fill the gap associated with a bridging anaphor with two objects which are indistinguishable under the effort-conditions: the utterance is ambiguous between two different assertions. However, if one of the assertions is less ‘plausible’ than the other, this helps us to select a determinate reading. Consider the following mini-dialogue:

(17) John: Why did Tom drive Mike’s car and not his own?  
    Bill: Because the motor had broken down.

John’s question presupposes that Tom drove Mike’s car and not his own. The description the motor can either be licensed by Mike’s car or by Tom’s car. If John uses the background knowledge that one cannot drive a car with a broken motor, he will be able to derive an inconsistency from Bill’s answer (combined with the presupposition of his question) when he takes Mike’s car as an ‘antecedent’ for the motor, but not when the motor is part of Tom’s car. Based on consistency requirements, only the interpretation of Bill’s utterance which answers John’s question is selected. We feel that relevance (e.g., answerhood) often is a side-effect of consistency, which we take to be a minimal condition of plausibility. Another illustration of the plausibility condition is the following:

(18) Mary traded her old car in for a new one. The motor was broken.

In this example, the motor can be licensed by both Mary’s old car and by her new car. Nevertheless, one has a very strong tendency to interpret the motor as referring to Mary’s old car. To check these intuitions, we conducted a small enquiry via email among Dutch subjects with (a Dutch version) of (18) as well as some other examples. After reading the example, the subjects were presented with the following query: Of which car was the motor broken? ☐ The old one, or ☐ The new one. Subjects were asked to provide the first answer that came to their mind. The results were unequivocal: 2 subjects choose the new car, while 48 interpreted the motor as referring to Mary’s old car. How can we account for this? One possibility is this: the 48 subjects have the ‘knowledge’ that a new car has a working motor. If an interpreter has such background knowledge in her, than she will be able to derive an inconsistency when the description the motor is linked to Mary’s new car. Hence this resolution is not plausible, and thus rejected in favour of the other resolution.

In the two examples discussed so far, two possible readings remained after applying the effort condition, one of which could later be ruled out due to plausibility (consistency) with the side-effect that the selected reading is ‘relevant’ (i.e., provides an explanation for the event described in the first sentence). However, it can also happen

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7 The questionnaire (consisting of four multiple-choice questions) was returned by 50 native speakers of Dutch working at IPO. It included participants from a number of different backgrounds: (computational) linguists, perception scientists (vision, auditory), physicists, computer scientists, psychologists, ergonomists, secretaries and management.

8 Notice that it can happen that both the effort and the plausibility conditions put together fail in selecting one most preferred proof-object. In that case an unresolvable ambiguity results; no determinate bridge can be constructed. The following provides an illustration of this: ?? If John buys a car and a motorbike, he’ll check the engine first. There are two potential antecedents for the presupposition triggered by the engine which are indistinguishable under both the effort and the plausibility condition. As a result, the sentence is odd (marked by the double question mark).
that exactly one reading remains under the effort conditions, although this reading is rather implausible, e.g., it is inconsistent with the world-knowledge of the interpreter. Consider example (19), from Asher & Lascarides (1996:16).

(19) a. I met two interesting people last night who voted for Clinton.
   b. The woman abstained from voting in the election.

The only available antecedent for the woman in (19.b) is one of the two people mentioned in (19.a). As Asher & Lascarides point out, the only available binding reading “(...) results in an inconsistency that makes the discourse sound strange”. Nevertheless, this (inconsistent) reading is the preferred (only) one. They use this example to indicate a difference with the abductive framework proposed in Hobbs et al. (1993). In that framework, presuppositional and asserted material are treated on a par. As a consequence Hobbs et al. predict a reading in which an antecedent for the woman is accommodated. Here our approach makes the same prediction as Asher & Lascarides’ approach. The plausibility condition selects the most plausible reading from the readings which passed the effort condition (those readings requiring least effort). Obviously, if only one reading survived the effort condition, it is by definition the most plausible reading.

The picture that emerges is one where interpretation involves two stages: first a stage where some readings (if any) are selected on the basis of effort and then a second stage in which the interpreter selects the most plausible reading. Accommodation is only an option if the effort condition yields no binding reading whatsoever. Note that this approach gives a particular meaning to the idea that an interpreter tries to make sense of what has been said. The process of making sense is constrained by simple effort conditions. In other words, although for a particular utterance there may be theoretically possible readings that ‘make sense’ (e.g., are ‘explanatory’ with respect to the preceding discourse), these readings may simply not be available, because there are other readings which, although they make less sense, present themselves more readily to the interpreter. Let us illustrate this with another example. Consider:

(20) John moved from Brixton to St. John’s Wood. The rent was less expensive.

Matsui (1995) found that people interpret the rent as anaphoric to the rent in St. John’s Wood. What is more, Matsui’s experiment showed that this preference even overrides the default knowledge present in the subjects of the experiment that the rents are generally more expensive in St. Johns Wood than in Brixton. Asher & Lascarides (1996:10) argue that: “(...) intuitively, one prefers explanations of intentional changes (in this case, moving house) to simple background information that sets the scene for the change”. If the rent is anaphoric to the rent in St. John’s Wood, we get an explanation for John’s removal. But are they right? Consider:

(21) John moved from Brixton to St. John’s Wood. The rent was more expensive.

If we apply the Asher & Lascarides analysis to this example (‘explanation preferred’), then the prediction is that an interpreter should prefer a reading where the rent refers to the rents in Brixton, since the rents in Brixton being more expensive than those in St.
John’s Wood would provide a good reason for John’s removal. In the aforementioned experiment we tested Asher & Lascarides’ prediction with example (22)\(^9\).

(22) John moved from Horst to Maasbree. The rent was more expensive.

After reading this example, subjects were asked the following question: Where is the rent more expensive? Horst, or Maasbree. Again subjects were asked to provide the first answer that came to their mind. The results for this particular question were as follows: Horst: 8, Maasbree: 42. These results (\(\chi^2 = 23.12, p < 0.001\)) are the opposite of Asher & Lascarides’ prediction: the rent is interpreted as the rent of John’s new domicile in Maasbree/St. John’s wood.

Let us now sketch an alternative explanation in terms of our framework. The basic idea is that the effort condition suggests the most salient antecedent as the most likely candidate. In this case, salience is influenced by the temporal interpretation of the two sentences. In particular, the notion of a reference event, as proposed in Hinrichs (1986) may play a central role. In line with the rules provided by Hinrichs, this reference event will be located immediately after the moving event. In other words, for the reference event it holds that John is living in Maasbree and thus he is paying the rent in Maasbree. According to Hinrichs, if a sentence expresses a state, then the reference event of the previous sentence should be temporally included in this state. Thus, the state expressed by ‘The rent was more expensive’ should include the referent event where John lives in Maasbree. This is what makes the reading where the rent refers to the rent in Maasbree the most preferred reading.\(^{10}\) As regards to plausibility, neither the reading where the rent refers to the rent in Horst nor the one where it refers to the rent in Maasbree is implausible (leads to an inconsistency), thus the reading preferred by the effort condition is retained. Note that the same story can be told for Asher & Lascarides’ example. Sometimes, plausibility can override the ordering suggested by effort:

(23) John moved from Horst to Maasbree. The rent was too expensive.

In this case, the reading where John moves to a place where the rent is too expensive for him has little plausibility; background knowledge like ‘one cannot pay things which are too expensive’ will rule out this reading.

5 Related work

In this section we compare our proposals with some related work from the literature. Various authors have studied presuppositions from a proof-theoretic perspective. In Ranta (1994), CTT is extended with rules for definite descriptions. Ahn (1994) and Beun & Kievit (1995) use CTT for dealing with the resolution of definite expressions. The latter focus on selecting the right referent (which can come not only from the linguistic context, but also from the physical context) using concepts such as prominence and agreement. Krause (1995) presents a type-theoretical approach to presupposition.

\(^9\)Horst and Maasbree are two small Dutch towns which are not very well known.

\(^{10}\)A somewhat comparable approach is advocated in Poesio (1994). Poesio shows how shifts in the focus of attention influence the interpretation of definite descriptions.
His theory not only allows binding of presuppositions, but also has the possibility to globally accommodate them using an abductive inferencing mechanism. One important difference with our approach is that we simply take the entire theory of Van der Sandt and rephrase it in terms of CTT. Apart from that, we also want to show that this reformulation paves the way for a formalized influence of background knowledge on presupposition projection (in particular: bridging). In general, we believe that the CTT approach leads to better results than adding a proof system to DRT, as done in e.g., Saurer (1993). The main advantage of CTT is that it is a standard proof system developed in mathematics with well-understood meta-theoretical properties. And, as we have shown, the presence of explicit proof objects in CTT has some additional advantages for the treatment of bridging.

We are aware of three formal approaches to bridging: the abductive approach (Hobbs 1987 and Hobbs et al. 1993), the lexical approach (Bos et al. 1995) and the rhetorical approach (Asher & Lascarides 1997).

**Abduction: Chandeliers Revisited** We have analysed example (12) in terms of CTT deduction and accommodation, where the latter is similar to the notion of implicature argued for by Clark. An analysis of (12) in terms of implicature has also been presented in Hobbs (1987). Though our approach is similar to his in spirit, it differs in the details. Hobbs suggest the following basic scheme for implicature: IF P is mutually known, (P&R) → Q is mutually known, and the discourse requires Q, THEN assume R as mutually known and CONCLUDE Q. In case of the chandelier example, Q is instantiated with ‘there is a chandelier’, P with ‘there is a lamp’ and R with ‘in the form of a branching fixture’. Uttering the first sentence makes it mutually known that there is a lamp. Hobbs now explains the use of the definite the chandelier in the second sentence as follows: the interpreter ‘accommodates’ that this lamp has the form of a branching fixture, and thus can derive the presence of a chandelier.

Let us compare this to the present approach. We also assume that the first sentence introduces a lamp. And similarly, the definite in the second sentence requires the presence of a chandelier. Furthermore, we assume that it is part of the world knowledge of the interlocutors that chandeliers are lamps. The idea is now that the interpreter is licensed to assume that the lamp in question is a chandelier in virtue of (i) the fact that the linguistic context contains a salient lamp, (ii) the fact that chandeliers are lamps and (iii) the fact that a chandelier is presupposed. One important difference is that we do not require a decomposition of chandelier into a lamp in the form of a branching fixture. A further difference with the approach of Hobbs et al. is that on our approach it is the independently motivated presupposition resolution algorithm which drives the bridging process.

**Is Bridging a Lexical Phenomenon?** Bos et al. (1995) treat bridging as a lexical phenomenon. They combine a version of Van der Sandt’s presupposition resolution algorithm with a generative lexicon (Pustejovsky 1995). In this way each potential antecedent for a presupposition is associated with a qualia-structure indicating which ‘concepts’ can be associated with the antecedent. As they put it, a qualia-structure can be seen as a set of lexical entailments. Our main objection to this approach is that not all implied antecedents are lexical entailments. The examples in (24) illustrate the
importance of non-lexical background knowledge.

(24) a. Yesterday Chomsky analysed a sentence on the blackboard, but I couldn’t see the tree.
   b. Yesterday somebody parked a car in front of my door, and the dog howled awfully.

For most people trees have as much to do with sentences as dogs have with cars. Yet, both these examples can have a bridging reading, given a suitable context. The a. example requires some basic knowledge concerning formal grammars which most readers of this paper presumably will have. For them, (24.a) is a perfectly normal thing to say under its bridging reading (because all of them have a mental function mapping sentences to trees). Likewise, (24.b) can be understood in a bridging manner given the ‘right’ background knowledge. Suppose, it is well-known between speaker and interpreter that the former lives opposite a home for stray animals somewhere in the countryside, and all cars which stop in front of this home for lost animals either drop a dog or pick one up. In this context, the interpreter will have no trouble constructing the required bridge (since she has a mental function which produces a dog for each car stopping in front of the speaker’s door).

Bridging as a Byproduct of Computing Rhetorical Structure  In the previous section we already discussed the work of Asher & Lascarides (1997), who claim that bridging is determined by rhetorical structure. We have illustrated that the predictions of Asher & Lascarides in connection with examples (20) – (22) are not in accord with the results we obtained in a small observational study. This is not to say that we disagree with the basic tenet from Asher & Lascarides that discourse structure, and in particular, rhetorical relations between sentences play a factor in bridging. For example, we believe that their observation concerning examples like *John was going to commit suicide. He got a rope.* (Charniak 1983) is correct; the fact that the second sentence is somehow constructed as ‘background’ to the first may be useful to infer that the rope is going to be used for the suicide (essentially this is due to Grice (1975)’s maxim of relevance). We do not agree, however, with the claim that rhetorical structure is the driving force behind bridging (that it is a “byproduct of computing rhetorical structure”, Asher & Lascarides 1996:1). In our opinion, rhetorical structure is certainly an influence on bridging, but we believe that world knowledge, as well as factors like effort and plausibility play a more substantial role, as we have tried to argue in connection with examples (20) – (22).

6 Conclusion

We have discussed a deductive variant of Van der Sandt’s presuppositions as anaphora theory. In this new perspective presuppositions are treated as gaps, which have to be filled using contextual information. This information can come from the linguistic context, that is the preceding sentences (as in Van der Sandt 1992). But presuppositional gaps can also be filled using the non-linguistic context, i.e., world knowledge. As an illustration of the formal interaction between world knowledge and presupposition, we
have applied our deductive approach to Clark’s bridging cases. We distinguished two cases: if the ‘bridge’ between presupposition and would-be antecedent is fully derivable using context (including world knowledge), the presupposition associated with the anaphor can be bound. This means that binding plays a more substantial role than in Van der Sandt’s original theory, as presuppositions can be bound to both inferred and non-inferred antecedents. If the ‘bridge’ between anaphor and antecedent is not fully derivable, the ‘missing link’ will be accommodated. So, accommodation is still a repair strategy, as in Van der Sandt’s original approach, but now there is generally less to repair. In most cases, accommodation will amount to ‘assuming’ that an object of which the existence has been proven satisfies a more specific description (in the case of (12), that the lamp whose existence has been proven on the basis of a room is in fact a chandelier).

Our approach to bridging resembles the abductive approach advocated by Hobbs et al. (1993), but there are also a number of important differences. We take it that the presuppositionhood is the driving force behind bridging. As a consequence we make a strict separation between presupposed and asserted material, thereby avoiding the problems Hobbs and co-workers have with examples like (19). Additionally, we are not committed to lexical decomposition, such as chandeliers are lamps with a branching fixture. The knowledge that chandeliers are lamps is sufficient. We have tried to argue that bridging is not a purely lexical process (as opposed to Bos et al. 1995), and that bridging is not a byproduct of rhetorical structure determination (as opposed to Asher & Lascarides 1997). This is not to say that lexical matters or rhetorical relations have no relevance for bridging, we feel that they are two of the many factors which play a role in bridging.

It is well known that bridging is not an unrestricted process. Therefore we have tried to formulate two general constraints/filters on the bridging process partly inspired by the informal observations from Clark (1975): effort (the ‘mental capacity’ the interpreter needs to construct a bridge) and plausibility (the relative admissibility of the constructed bridges). We have modelled the effort constraint in CTT terms as follows: if a gap can be filled by more than one proof object, order them with respect to proof complexity (the one with the lowest complexity first). Moreover, use the ‘informational resources’ as frugal as possible (accommodate as little as possible). This latter condition is ‘hard-coded’, as it were, in the Van der Sandtian resolution algorithm. We take it that factors like recency and salience are also relevant here. The plausibility condition is modelled as a simple consistency condition, with relevance as a side-effect. We have seen that these two simple conditions help in arriving at a determinate bridge. The precise formulation of these two conditions and the interplay between them will be the subject of further research. Additionally, we are interested in further empirical validation of our predictions along the lines of the observational study described in this paper.

References


