

# A GENERIC FUZZY RULE BASED TECHNIQUE FOR IMAGE SEGMENTATION

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## ABSTRACT

*Many fuzzy clustering based techniques do not incorporate spatial relationships of the pixels, while all fuzzy rule-based image segmentation techniques tend to be very much application dependent. In most techniques, the structure of the membership functions are predefined and their parameters are either automatically or manually determined. This paper addresses the aforementioned problems by introducing a general fuzzy rule based image segmentation technique, which is application independent and can also incorporate the spatial relationships of the pixels. It also proposes the automatic defining of the structure of the membership functions. A qualitative comparison is made between the segmentation results using this method and the popular fuzzy c-means (FCM) applied to two types of images: light intensity (LI) and X-ray of human vocal tract. The results clear show that this method exhibits significant improvements over FCM for both types of images.*

## 1. INTRODUCTION

Classical so-called "crisp", image segmentation techniques while effective when an image contains well-defined structures, such as edges and regular shapes, do not perform nearly so well in the presence of ill-defined data. In such circumstances, the processing of such images that possess ambiguities produces fuzzy regions. Fuzzy image segmentation techniques can cope with the imprecise data well and they can be classified into five classes: fuzzy clustering, fuzzy rule based, fuzzy geometry, fuzzy thresholding, and fuzzy integral based image segmentation techniques [1] but among them the most dominant are fuzzy clustering and fuzzy rule based segmentation techniques. The most popular and extensively used fuzzy clustering techniques are: fuzzy c-means (FCM) [2-3] and possibilistic c-means (PCM) algorithms [4]. These clustering techniques however cannot incorporate human expert knowledge and spatial relation information. Image segmentation without considering the spatial relationships among pixels does not produce good result, as there is a huge amount of overlapping pixel values between different regions. Fuzzy rule based image segmentation techniques can incorporate human expert knowledge, are less computational expensive than fuzzy clustering and able to interpret linguistic as well as numeric variables [5]. But they are very much application dependent and very difficulty to define fuzzy rules that cover all of the pixels. In most techniques, the structures of the membership functions are predefined and their parameters are either manually or

automatically determined [5-9]. In addition to the above-mentioned advantages, a fuzzy rule based image segmentation technique should be both application and image independent, be capable of incorporating spatial information of the regions and be able to define the membership functions and their parameters automatically.

This paper explores a new approach in the development of such a type of fuzzy rule based image segmentation techniques. Section 2 explores the technique used to define the membership function, while the underlying theoretical concepts and fuzzy rule definition and the experimental results are presented in sections 3 and 4 respectively. Finally the discussions and conclusion are provided in section 5.

## 2. DEFINITION OF MEMBERSHIP FUNCTIONS

In this section three types of membership functions are automatically defined to represent respectively the region pixel distributions, the closeness to their centers and their spatial relations. Each membership function possesses a membership value for each region, which indicates the degree of belonging to that particular region. The techniques used to automatically define the structures of the membership functions and hence the membership functions from the region pixel distributions are described in the following subsection

### 2.1. Membership Function for Region Pixel Distributions

In this subsection an attempt is made to automatically define the membership function including its structure from the region pixel distributions. The steps needed to define the membership function are: classification of the sample or the image to be segmented into desired number of regions using manual segmentation or automatically by applying any of the fuzzy clustering algorithms, generation of the gray level pixel intensity histogram for each region and map the frequency for each gray level into [0 1], and approximation of the polynomial for each region. This polynomial represents the membership function for that particular region and the value of the polynomial for each gray level denotes the membership value of that particular gray level value. The cloud image shown in figure 1(a) is divided into two regions namely cloud ( $R_1$ ) and urban scene ( $R_2$ ) using FCM. The membership functions shown in figures 1(b)-1(c) of these two regions are determined from respective region pixel distributions using third order polynomial approximation.

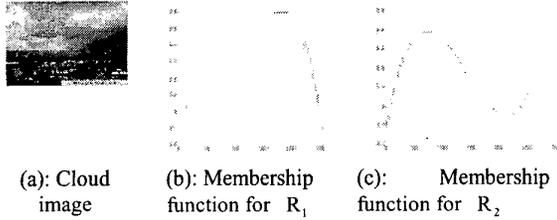


Figure 1: Cloud image and its region membership functions

The degree of belonging of a candidate pixel (the pixel to be classified) to a region is determined from the respective membership function. The structures of the membership functions are automatically generated from the region pixels and hence relieve us from manually defining the structure and parameters of the membership function for each region. The membership function  $\mu_{DR_j}(P_{s,t})$  of the region  $R_j$  for the pixel distribution can be defined as

$$\mu_{DR_j}(P_{s,t}) = f_{R_j}(P_{s,t}) \quad (1)$$

Where  $f_{R_j}(P_{s,t})$  and  $P_{s,t}$  are the polynomial of the region  $R_j$  and the pixel value at the position (s,t) respectively.

## 2.2. Membership Function to Measure the Closeness of the Region

Each pixel should be more compact i.e. more close to the belonging region than other regions. The degree of belongingness of a candidate pixel to a region is determined by following the strategy of k-means clustering algorithm. Candidate pixel joins in its nearest region and after joining the center of that region is recomputed. The centroid of a region  $R_j$  can be defined as

$$C(R_j) = \frac{1}{N_j} \sum_{i=1}^{N_j} P_j(i) \quad (2)$$

Where  $N_j$  and  $P_j(i)$  represent the number of pixels and the  $i$ th pixel gray level intensity of the  $j$ th region respectively. The membership function should reflect the relation "the more close to a region the larger membership value the pixel should have". So the membership function  $\mu_{CR_j}(P_{s,t})$ , which determines the degree of belongingness of a candidate pixel  $P_{s,t}$  at a location (s,t) to a region  $R_j$  can be defined as

$$\mu_{CR_j}(P_{s,t}) = 1 - \frac{|C(R_j) - P_{s,t}|}{D} \quad (3)$$

Where the constant  $D$  can be defined as difference of maximum and minimum gray level intensity values of an image i.e. here  $D$  equals to 255. The maximum value of the membership function will be always at the center of the region and the structure of the membership function will be symmetrical around the vertical line passes through the center of the region.

## 2.3. Membership Functions for Spatial Relation

In the previous two sections the membership functions have been developed only based on the feature values i.e. gray level pixel intensities of an image. They don't consider any spatial relationships of the pixels of a region, but there exists strong spatial relations between the pixels of a region. Spatial relations also represents the geometric features of a region and a spatial object contains two descriptors- feature and geometric [11]. There is a large amount of overlapping pixels between the regions. Segmentation does not produce good result without taking into account of these overlapping pixels. The number of overlapping pixels can be trim down by considering the neighborhood relation among a candidate pixel and the classified pixels of the regions i.e. once we get the some region pixels we can easily calculate the neighborhood relation between the candidate pixel and the region pixels. Based on the neighborhood relation the candidate pixel can be assigned to the appropriate cluster or group. The neighborhood relation can mainly be defined using the three techniques- fixed size neighborhoods around candidate pixel, minimum spanning tree and Voronoi tessellation even though there are many ways to define a neighborhood relation [12]. We are interested in fixed size neighborhoods around a candidate pixel, as we need to calculate the number of pixels and their distances from the candidate pixels inside the neighborhood area. The neighborhood configurations of the pixels for  $r=1$ ,  $r=2$  and  $r=4$  are shown in the figures 2(a), 2(b) and 2(c) respectively [13] where  $O$ ,  $\#$  and  $r$  represent the candidate pixel, neighborhood pixels and neighborhood radius respectively.

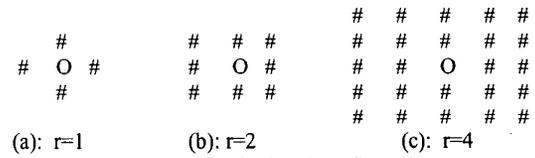


Figure 2: Neighborhood configuration

The number of neighbors would be  $(r+1)^2$  for  $r=1$  otherwise  $(r+1)^2 - 1$ . The main task of the segmentation is to divide the image into desired number of mutually exclusive homogeneous regions. It is thus assumed that the variation of the pixel intensities of a region is in a limited extent but there is a sharp variation of the pixel intensities on the boundaries of the regions that divides the image into some regions. We are interested in determining the spatial relationships among the pixels of a region. So the neighborhood system of a region can be defined as,

**Definition 1 (Neighborhood system)** A neighborhood system with radius  $r$ ,  $\zeta(P_{s,t}, r)$  of a candidate pixel  $P_{s,t}$  is a set of all pixels  $P_{x,y}$  such that  $\zeta(P_{s,t}, r) = \{P_{x,y} \mid (d(P_{x,y}, P_{s,t}) \leq r) \wedge ((P_{x,y} \sim P_{s,t}) \leq T)\}$  where distance  $d(P_{x,y}, P_{s,t}) = |x - s| + |y - t|$ ,  $P_{x,y}$  is a 2D image pixel at Cartesian coordinate  $(x,y)$ , and  $T$  is

the threshold, which denotes the maximum pixel intensity variation of a region.

Now it is needed to define a membership function, which considers the number of neighborhood pixels and the distances between the neighbors and candidate pixel. A membership function  $\mu$  of the spatial relation should possess two characteristics:  $\mu \propto N$  where  $N$  denotes the number of neighbors and  $\mu \propto \frac{1}{d(P_{x,y}, P_{s,t})}$ .

The summation of inverse distances of a region  $R_j$  can be defined as

$$G_{R_j} = \sum_{i=1}^{N_j} \frac{1}{d_i(P_{x,y}, P_{s,t})} \quad (4)$$

Where  $N_j = |\zeta(P_{s,t}, r)|$  = the number of neighborhood pixels of the candidate pixel  $P_{s,t}$  in the region  $R_j$  and  $d_i(P_{x,y}, P_{s,t})$  is the distance between the  $i$ th pixel  $P_{x,y}$  of the region  $R_j$  & the candidate pixel  $P_{s,t}$ .

So considering the number of neighbors ( $N_j$ ) and their sum of inverse distances ( $G_{R_j}$ ) from the candidate pixel ( $P_{s,t}$ ), the membership function  $\mu_{NR_j}(P_{s,t}, r)$  of the region  $R_j$  can be defined as

$$\mu_{NR_j}(P_{s,t}, r) = \frac{N_j \times G_{R_j}}{\sum_{j=1}^{\mathfrak{R}} (N_j \times G_{R_j})} \quad (5)$$

Where  $\mathfrak{R}$  is the desired number of regions of an image.

### 3. FUZZY RULE DEFINITION

The effectiveness of the fuzzy rule plays the vital role for the segmentation result. In this paper, a fuzzy rule is heuristically defined using the three membership functions defined in section 2 and the most wide used fuzzy IF-THEN rule structure.

The overall membership value  $\mu_{AR_j}(P_{s,t})$  of a pixel  $P_{s,t}$  for the region  $R_j$ , which represent the overall degree of belonging to the region  $R_j$ , can be defined by the weighted average of the values of the membership functions  $\mu_{DR_j}(P_{s,t})$ ,  $\mu_{CR_j}(P_{s,t})$  and  $\mu_{NR_j}(P_{s,t})$ .

$$\mu_{AR_j}(P_{s,t}) = \frac{W_1 \mu_{DR_j}(P_{s,t}) + W_2 \mu_{CR_j}(P_{s,t}) + W_3 \mu_{NR_j}(P_{s,t})}{W_1 + W_2 + W_3} \quad (6)$$

Where  $W_1$ ,  $W_2$  and  $W_3$  represent the weight of the membership values for the pixel distribution, closeness to the cluster centers and neighbor relation respectively. The overall membership value  $\mu_{AR_j}(P_{s,t})$  is used in the antecedent

condition of IF THEN RULE and the rule can be defined as,

**Definition 2 (Rule)** IF  $\mu_{AR_j}(P_{s,t})$  supports region  $R_j$  THEN pixel  $P_{s,t}$  belongs to region  $R_j$ .

$\mu_{AR_j}(P_{s,t})$  will give support to the region  $R_j$  if  $\mu_{AR_j}(P_{s,t}) = \max\{\mu_{AR_1}(P_{s,t}), \mu_{AR_2}(P_{s,t}), \dots, \mu_{AR_{\mathfrak{R}}}(P_{s,t})\}$  where  $\mathfrak{R}$  indicates the number of region. As this is the only rule, it is generalized one and can be applied in any type of image.

### 4. EXPERIMENTS

The proposed system and FCM had been implemented using MATLAB 5.3.1 (The Mathworks, Inc.). Two types of images such as light intensity (LI) shown in figure 1 (a) and X-ray image of the human vocal tract shown in figure 4(a) were used in the experiments. For FCM, the initialization of the cluster center was done randomly. The maximum number of iterations, minimum amount of improvement and the value of the fuzzifier ( $m$ ) were taken as 100, 0.00001 and 2 respectively. For our proposed system, GFRIS the membership function defined in section 2.1 was developed using the clusters produced by FCM and their center values were used to initialize the centers of the clusters required to define the membership function described in section 2.2. The values of weights and the threshold were determined empirically and taken as  $W_1 = 1, W_2 = 2, W_3 = 1, T=25$ , and  $W_1 = 1, W_2 = 1.5, W_3 = 1, T=30$  for cloud and X-ray image of the human vocal tract respectively. The segmented results of the original cloud image (figure 1(a)) for two regions namely cloud ( $R_1$ ) and urban scene ( $R_2$ ) produced by FCM and GFRIS are graphically displayed in the figure 3.

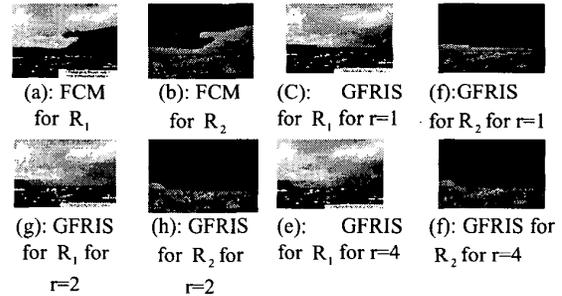


Figure 3: The segmented results of the cloud image into two regions produced by FCM and GFRIS

From the results it is visually shown that GFRIS separated almost the whole cloud from the image and produced significantly better results than FCM because FCM did not consider the spatial relationships among the pixels of a region. GFRIS also showed better results for larger values of neighborhood radius  $r$ , because the pixels of  $R_1$  (cloud) are homogeneous and very much spatially correlated.

Another experiment was performed using an X-ray image of the human vocal tract shown in figure 4(a) and its segmentation results into two regions namely human vocal tract ( $R_1$ ) and background ( $R_2$ ) produced by FCM and GFRIS are presented in the figures 4(b) – 4(i). It is visually evident that the proposed technique GFRIS considerably outperformed FCM. There are no isolated pixels at all in the regions produced by GFRIS

whereas the regions (figures 4(b) -4(c)) produced by FCM contain significant amount of isolated pixels.

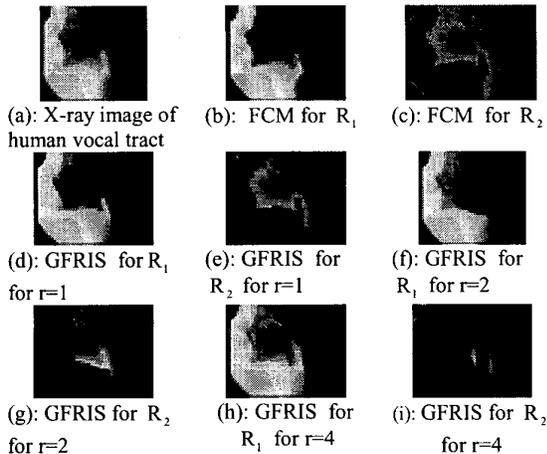


Figure 4: X-ray image of the human vocal tract and its results for two regions produced by FCM and GFRIS

This also ensures that the spatially related pixels had been classified successfully by GFRIS. The image shown in figure 3(a) contains two regions such as human vocal tract (lips, tongues, teeth) and background. The soft part of the human vocal tract is not clearly visible and has low local contrast pixels [14]. Almost the whole of the vocal tract had been successfully separated by GFRIS using  $r=4$ , which ensures the larger values of  $r$ , the better representation of the spatial relation. It also considered the underlying meaning of data better while FCM did not consider at all.

## 5. DISCUSSIONS AND CONCLUSION

In this paper a general fuzzy rule based image segmentation technique has been proposed. The proposed technique includes the spatial relationships among the pixels. It is also image and application independent like the standard fuzzy clustering algorithm FCM. The results have shown that it has represented the spatial relationships as well as the underlying meaning of the data better than FCM. Only one fuzzy rule is capable to classify all the pixels. The structures of the membership functions have been automatically derived and there is no need of defining the parameters. It is visually apparent that this system has shown promising result over FCM.

The values of the weighting factors  $W_1$  &  $W_2$ , and  $W_3$  imposes the importance of the feature based and spatial information. There is a trade-off between feature based and spatial information. It depends on the application, which one is assigned to how much importance. It is apparent that feature-based information should be given more emphasis than spatial information as it represents more human visual perspective than spatial information. Another parameter is the value of the threshold  $T$ , which represents the maximum amount of pixel intensity variation between neighbor and candidate pixels. It is intuitively determined that the suitable range of the value of  $W_2$  and  $T$  are 1.5 to 2, and 25-30 respectively and others are 1.

It also needs more research for determining the suitable values of weighting factors and the threshold. As the proposed technique is fuzzy rule based technique, it is capable to incorporate any types of attributes of any special application. It is also possible to add membership function from the high level semantics of an object for object based image segmentation. Like FCM the proposed technique needs to provide the desired number of regions. It also needs more investigation to automatically determine the optimum number of regions.

## 6. REFERENCES

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