Fuzzy rule for image segmentation incorporating texture features

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FUZZY RULE FOR IMAGE SEGMENTATION INCORPORATING TEXTURE FEATURES

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ABSTRACT

The generic fuzzy rule-based image segmentation algorithm (GFRIS) does not produce good results for images containing non-homogenous regions, as it does not directly consider texture. In this paper a new algorithm called fuzzy rules for image segmentation incorporating texture features (FRIST) is proposed, which includes two additional membership functions to those already defined in GFRIS. FRIST incorporates the fractal dimension and contrast features of a texture by considering image domain specific information. Quantitative evaluation of the performance of FRIST is discussed and contrasted with GFRIS using one of the standard segmentation evaluation methods. Overall, FRIST exhibits considerable improvement in the results obtained compared with the GFRIS approach for many different image types.

1. INTRODUCTION

Image segmentation is the most important and difficult task of digital image processing and analysis systems, due to the potentially inordinate number of objects and the myriad of variations among them. The most intractable task is to define their properties for perceptual grouping, a demand that requires human expert and/or domain specific knowledge to be incorporated to achieve a superior segmentation result. Fuzzy rule-based image segmentation systems can incorporate this expert knowledge, but they are very much application domain and image dependent. The structures of all of the membership functions are manually defined and their parameters are either manually or automatically derived [1]-[5]. Karmakar and Dooley [6][7] proposed a novel generic fuzzy rule based image segmentation (GFRIS) algorithm to address the aforementioned problems. This algorithm however, does not work well for images containing texture, which is for regions that are non-homogeneous and have sharp variations in pixel intensity. Texture is one of the most important attributes of any image that represents the structural arrangements of the surface as well as the relations among them and is widely used in image segmentation [8]. In this paper a new algorithm, fuzzy rules for image segmentation incorporating texture features (FRIST) is proposed by integrating two new membership functions into the set of GFRIS membership functions, based upon the texture features of fractal dimension and contrast. These additional membership functions consider the image domain specific information. The performance analysis of both the GFRIS and FRIST is conducted by applying a superior objective segmentation evaluation technique called the discrepancy based on the number of mis-segmented pixels [9]. The new algorithm is subsequently applied to many different types of images.

The remainder of the paper is organized as follows. Section 2 provides a brief overview of the techniques used to define the membership functions. The definition of the fuzzy rule, and also the determination of the weighting factors and threshold used are presented in Sections 3 and 4 respectively. The evaluation and experimental results are discussed in Section 5, with conclusions provided in Section 6.

2. MEMBERSHIP FUNCTIONS

The GFRIS algorithm uses three types of membership functions to represent the region pixel distributions, the closeness to their centres and the spatial relations among the pixels in a particular region. Each membership function possesses a membership value for every region, which indicates the degree of belonging to that particular region. Full details of these membership functions are given in [6][7]. For the sake of completeness, a brief description of them is now provided.

The approach adopted for the membership function for region pixel distributions is to automatically define the membership function, including its structure from the pixel distributions of a region. This is obtained from the initial segmentation results of the respective region and a polynomial approximation of the pixel distribution of each region. The membership value of a pixel at location $(x,y)$, having a gray level value of $P_{xy}$ in region $R_i$ is defined as:

$$\mu_{ix}(P_{xy}) = f_{ix}(P_{xy})$$

where $f_{ix}(P_{xy})$ is the polynomial for the region $R_i$.

The membership function to measure the closeness of a pixel to a region represents the similarity between the pixel to be classified, called the candidate pixel, and the centre of a region based on the gray level intensity. The membership function reflects the axiom that the closer to a region, the larger the membership value of the candidate pixel and is defined as:

$$\mu_{CR_{ix}}(P_{xy}) = 1 - \frac{|C(R_i) - P_{xy}|}{(2^k - 1)}$$

where $C(R_i)$ is the centre of the region $R_i$, it is assumed that a k-bit gray scale image is used.
The membership function for spatial relations \( \mu_{\text{NS}}(P_{s}, r) \) of the region \( R_{i} \) for the neighborhood radius \( r \) represents the spatial relations between the candidate pixel \( P_{s} \) and its neighbours, and with a total of \( N \) segmented image regions, is defined as:

\[
\mu_{\text{NS}}(P_{s}) = \left( N_{j} \times G_{j} \right) \times \frac{1}{\prod_{j=1}^{N} (N_{j} \times G_{j})}
\]

where \( N_{j} \) and \( G_{j} \) are the number of neighbours and the sum of their inverse distances of the region \( R_{j} \) from the candidate pixel \( P_{s} \), respectively.

### 2.1. Membership functions for fractal dimension

Fractal dimension (FD) is used to estimate the texture in an image. There are many different models for estimating FD. One is called the differential box counting (DBC) method [10], and approximates the fractal dimension based feature (FDF) for developing the membership functions for fractal dimension. The notion of self-similarity is used to estimate fractal dimension. A self-similar set \( A \) is the union of \( N \) mutually exclusive copies of itself that are similar to \( A \) and scaled down by a ratio \( \tau \). The FD of \( A \) can then be defined as:

\[
1 = N_{\tau} \tau^{D} \Rightarrow FD = \frac{\log N}{\log(1/\tau)}
\]

\( N_{\tau} \) is determined using the DBC method in the following way [10]. For an image of size \( M \times M \) to be scaled down to a size of \( 2 \times 2 \times 2 \) where \( 2 \leq x \leq \lceil M/2 \rceil \), the ratio of scale down is \( \tau = x/M \). The image is then extended to 3-D space by introducing a 3\textsuperscript{rd} co-ordinate for the 8-bit gray level intensity of 256 levels. If the image is partitioned into grids of size \( x \times x \times x \), then each grid will have a column of boxes of size \( x \times x \times x \), which implies \( 256/x^3 = x \times x \times x \). If the maximum and minimum gray level values in the \( x \times x \times x \) grid are in the \( x \) and \( k \) boxes, the thickness of the blanket covering the image surface on the grid \( (u,v) \) is:

\[
n_{t}(u,v) = l - k + 1
\]

The contribution from all grids is defined as:

\[
N_{t} = \sum_{u,v} n_{t}(u,v)
\]

FD is estimated from the least square linear fit of \( \log(N_{t}) \) against \( \log(1/\tau) \).

To define the membership function for fractal dimension, the FDF of a candidate pixel \( P_{s} \) is calculated on a window \( W_{s}(s,t) \) of size \( h \times h \) with its centre at \( (s,t) \) rather than the entire image and is defined as:

\[
FDF(P_{s}) = FDF(W_{s}(s,t))
\]

where \( FDF(W_{s}(s,t)) \) denotes the FDF on \( W_{s}(s,t) \) derived using DBC in the following manner. The bound of the box size is chosen as \( 2 \leq x \leq \lceil h/2 \rceil \), the scale down ratio \( \tau = x/h \) and \( x' \) is taken as \( 256 \times [(h+1)/2] \) in order to consider the finer variations of the gray level values, where \( height \) is the height of the image. The value of \( FDF(W_{s}(s,t)) \) will not be the exact fractal dimension of the window \( W_{s}(s,t) \) because the height of the image is used rather than \( h \), the height of the window, in calculating \( x' \). Instead of considering log-log plot, the average value of \( \log(N_{t})/\log(1/\tau) \) is used to obtain the fractal dimension. The membership function \( \mu_{\text{FDF}}(P_{s}) \) of fractal dimension based feature for the region \( R_{i} \) and the pixel \( P_{s} \) can be formulated as:

\[
\mu_{\text{FDF}}(P_{s}) = 1 - \frac{\left| \frac{\text{FDF}_{R_{i}}(P_{s}) - \text{FDF}(P_{s})}{\text{FDF}_{R_{i}}(P_{s})} \right|}{\text{max}(\text{FDF}_{R_{i}}(P_{s}), \text{FDF}(P_{s}))}
\]

where \( \text{FDF}_{R_{i}}(P_{s}) \) and \( \text{FDF}(P_{s}) \) are the fractal dimension-based features for the segmented region \( R_{i} \) and the original image respectively. This membership function considers the image specific information for segmentation. \( \text{FDF}(P_{s}) \) is determined from the ratio of the number of contributory and total grids during \( \text{FDF}_{R_{i}}(P_{s}) \) calculation for each value of \( \tau \).

### 2.2. Membership functions for contrast

Contrast provides the measure of the texture of an image and is measured by considering the dynamic range of gray levels and the polarization of the distribution of black and white on the gray-level histogram. The contrast of a window \( W_{s}(P_{s}) \) in an image is calculated using the technique described in [11]. The membership function for the contrast of the region \( R_{i} \) and the pixel \( P_{s} \) can be defined as:

\[
\mu_{\text{Contrast}}(P_{s}) = 1 - \frac{\left| \frac{\text{Contrast}_{R_{i}}(P_{s}) - \text{Contrast}(P_{s})}{\text{max}(\text{Contrast}_{R_{i}}(P_{s}), \text{Contrast}(P_{s}))} \right|}{\text{max}(\text{Contrast}_{R_{i}}(P_{s}), \text{Contrast}(P_{s}))}
\]

where \( \text{Contrast}_{R_{i}}(P_{s}) \) and \( \text{Contrast}(P_{s}) \) represent the contrast of the portions of the segmented region \( R_{i} \) and the original image covered by the window \( W_{s}(P_{s}) \) respectively.

### 3. DEFINING FUZZY RULE

The overall membership value \( \mu_{\text{NS}}(P_{s}, r) \) of a pixel \( P_{s} \) for region \( R_{i} \), represents the overall degree of belonging to that region, and is defined by the weighted average of the five individual membership function values \( \mu_{\text{NS}}(P_{s}, r), \mu_{\text{FDF}}(P_{s}), \mu_{\text{Contrast}}(P_{s}), \mu_{\text{NS}}(P_{s}, r'), \) and \( \mu_{\text{Contrast}}(P_{s}, r') \).

\[
\mu_{\text{NS}}(P_{s}, r') = \frac{w_{1}\mu_{\text{NS}}(P_{s}) + w_{2}\mu_{\text{FDF}}(P_{s}) + w_{3}\mu_{\text{Contrast}}(P_{s})}{w_{1} + w_{2} + w_{3}}
\]

Where \( w_{1}, w_{2}, \) and \( w_{3} \) are the weights assigned to the respective membership functions.
where $w_1, w_2, w_3, w_4, w_5$, and $w_6$ are the weightings of the membership values for pixel distribution, closeness to the cluster centres, neighbourhood relations, fractal dimension and contrast respectively.

**Definition 1 (Rule)** IF $\mu_{ab_i}(P_{i,r}) = \max_{i=1}^{n} \mu_{ab_i}(P_{i,r})$ THEN pixel $P_{i,r}$ belongs to region $R_j$.

### 4. Determining the Parameters

The weighting factors $w_1, w_2$, and $w_3$, and threshold $T$ for the neighbourhood system are automatically determined using the algorithm described in [7]. The other two weighting factors $w_4$ and $w_5$ are approximated based on the FD of the entire image and the standard deviations ($\text{std}$) of pixel intensities of the initially segmented regions, as follows:

$$w_4 = w_5 = \frac{\text{FD} - 2}{\text{var(\text{std})}}$$

(11)

Since $2 \leq \text{FD} \leq 3$ the topological dimension of the image (2) is deducted from the FD, thereby keeping the original contribution of the fractal within $[0,1]$. This ensures that the contributions of all the weights are constrained within their limits. From the observations, it was found that the regions of high texture suppressed the regions containing less texture because they produced higher FD values. Since the standard deviation approximates the texture, the weights $w_4$ and $w_5$ are normalised using the variance of the standard deviations $\text{var(\text{std})}$ of the initially segmented regions, to minimize this effect. This has been experimentally tested upon various image types.

### 5. Experimental Results

Both the new FRIS and GFRIS algorithms were implemented using MATLAB 6.0 (The Mathworks, Inc.). A number of different image types were used in the experiments, but only two are included in this paper, namely the cloud shown in Fig. 1(a), which comprises one homogeneous and one non-homogeneous region, and the Brodatz texture image shown in Fig. 1(c), which contains two separate textural regions.

![Fig. 1: (a) Cloud image, (b) Ref: images for cloud, (c) Brodatz textures, (d) Ref: images (d60 and d98) for Brodatz textures.](image)

As alluded previously, quantitative evaluation of the segmentation process was achieved using *discrepancy based on the number mis-segmented pixels* [9]. Type I, $error_I$, represents the percentage error of all $n^I$ region pixels that are not classified in the $n^I$ region, whereas Type II, $error_{II}$, is the percentage error of all other region pixels wrongly classified in the $n^I$ region.

For both GFRIS and FRIS, the membership function for region pixel $\mu_{ab_i}(P_{i,r})$ was developed using the clusters produced by the initial segmentation results using the fuzzy c-means (FCM) algorithm [12]. The centre values were used to initialize the centres of the clusters required to define the membership function for the closeness of a region $\mu_{ab_i}(P_{i,r})$.

The neighborhood radius $(r)$ was taken as $1, 2$ and $4$, but only the results for the $r = 1$ and $2$ cases are included in this paper, with the size of the window $W_{ab}(s,r)$ being $4 \times 4$. The results of segmenting the cloud image (Fig. 1(a)) into two regions namely, cloud ($R_1$) and urban scene ($R_2$) using GFRIS and FRIS are shown in Fig. 2. The numerical segmentation results of the cloud image segmentation with respect to manually segmented reference images (Fig. 1(b)) are shown in Table 1.

![Fig. 2: The segmented results of the cloud image into two regions by GFRIS (a) to (b), and FRIS (c) to (d).](image)

**Table 1: Error percentage for region $R_1$ of cloud (cloud) and Brodatz (d60) image segmentations**

<table>
<thead>
<tr>
<th>Image</th>
<th>Algorithm</th>
<th>Error I</th>
<th>Error II</th>
<th>Algorithm</th>
<th>Error I</th>
<th>Error II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloud</td>
<td>GFRIS $r=1$</td>
<td>7.33</td>
<td>17.05</td>
<td>FRIS $r=1$</td>
<td>10.56</td>
<td>4.487</td>
</tr>
<tr>
<td>Cloud</td>
<td>GFRIS $r=2$</td>
<td>1.73</td>
<td>21.25</td>
<td>FRIS $r=2$</td>
<td>9.106</td>
<td>4.230</td>
</tr>
<tr>
<td>Cloud</td>
<td>GFRIS $r=4$</td>
<td>1.80</td>
<td>23.62</td>
<td>FRIS $r=4$</td>
<td>7.287</td>
<td>4.038</td>
</tr>
<tr>
<td>Cloud</td>
<td>GFRIS $r=1$</td>
<td>33.99</td>
<td>17.11</td>
<td>FRIS $r=1$</td>
<td>28.247</td>
<td>15.718</td>
</tr>
<tr>
<td>Brodatz</td>
<td>GFRIS $r=2$</td>
<td>33.16</td>
<td>19.47</td>
<td>FRIS $r=2$</td>
<td>25.488</td>
<td>18.687</td>
</tr>
<tr>
<td>Brodatz</td>
<td>GFRIS $r=4$</td>
<td>26.65</td>
<td>21.96</td>
<td>FRIS $r=4$</td>
<td>19.97</td>
<td>16.269</td>
</tr>
</tbody>
</table>

In Table 1, only the error rates for region $R_1$ are shown since the error rates of the other region $R_2$ are simply the reverse order of $R_1$. The segmentation results for the cloud image using GFRIS showed that region $R_1$ (Fig. 2(a) and (b)) contained a large number of misclassified pixels from region $R_2$, which has sharp variations in pixel intensity. Type II error rates for region $R_1$ using GFRIS (Table 1) were higher than type I error rates. Almost all of the misclassified pixels, including the
text caption were correctly classified using FRIST (Fig. 2(c) and (d)). The average error rates for both techniques are graphically

![Graph showing average error rates for GFRIS and FRIST for cloud (GFRISC and FRISTC) and Brodatz (GFRISB and FRISTB) images texture segmentation.](image1)

Fig. 3: Average error rates of GFRIS and FRIST for cloud (GFRISC and FRISTC) and Brodatz (GFRISB and FRISTB) images texture segmentation.

shown in Fig 3. From Fig. 2 and 3, it is clear that FRIST achieved considerable improvements over the GFRIS.

![Images of segmented Brodatz texture using GFRIS and FRIST with different radii](image2)

Fig. 4: Segmented results of Brodatz texture into two regions using GFRIS (a) and (c), and FRIST (b) and (d).

A second series of experiments was performed using the Brodatz texture image (Fig. 1(c)). The segmentation results for the two separate regions namely, 600 $L_r$ and 498 $L_r$ produced by the GFRIS and FRIST are presented in Fig. 4. The error and average error rates of $d_0$ segmentation with respect to the manually segmented reference images (Fig. 1(d)) are shown in Table 1 and Fig. 3 respectively. The segmented results obtained using FRIST for all values of $r$ are again considerably better than GFRIS. Note, that it was shown in [7], that GFRIS consistently provided superior results to both FCM [12] and probabilistic c-means (PCM) [13] algorithms for many different image types. Since the proposed technique is fuzzy rule based, it is capable of incorporating any type of attribute of any special application domain. It is possible to add membership functions for high-level semantics of an object for object based image segmentation. More research however is required in order to automatically determine the explicit number of regions in an image.

7. REFERENCES


6. CONCLUSIONS

This paper has outlined the development of a new general fuzzy rule-based image segmentation technique incorporating texture based upon fractal dimension and contrast. A new algorithm titled fuzzy rules for image segmentation incorporating texture features (FRIST), has been proposed and both a quantitative and qualitative analysis have been undertaken to compare it with the generic approach (GFRIS). The experimental results have shown that FRIST outperformed GFRIS for many different image