Fuzzy image segmentation combining ring and elliptic shaped clustering algorithms

Conference or Workshop Item

Fuzzy Image Segmentation combing ring and elliptic shaped clustering algorithms

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Abstract

Results from any existing clustering algorithm that are used for segmentation are highly sensitive to features that limit their generalization. Shape is one important attribute of an object. The detection and separation of an object using fuzzy ring-shaped clustering (FKR) and elliptic ring-shaped clustering (FKE) already exists in the literature. Not all real objects however, are ring or elliptical in shape, so to address these issues, this paper introduces a new shape-based algorithm, called fuzzy image segmentation combing ring and elliptic shaped clustering algorithms (FCRE) by merging the initial segmented results produced by FKR and FKE. The distribution of unclassified pixels is performed by connectedness and fuzzy c-means (FCM) using a combination of pixel intensity and normalized pixel location. Both qualitative and quantitative analysis of the results for different varieties of images proves the superiority of the proposed FCRE algorithm compared with both FKR and FKE.

1. Introduction

Image segmentation is an important research area because it plays a fundamental role in image analysis, understanding and coding [1]. Segmenting an image is the most challenging and difficult task because there exist different objects and a huge variations between them which makes it very difficult to segment all objects using a general framework [2].

The effectiveness of a clustering algorithm [3]-[5], is solely dependent on the type of the features used and the information about the domain of the objects in that image. This raises an interesting question about which type of feature produces better results for which type of image. This indicates the requirement of incorporating shape information in the segmentation process. To address this issue detection and separation of ring-shaped clusters using fuzzy clustering (FKR) [6] and fuzzy clustering of elliptic ring-shaped clusters (FKE) [7] algorithms were introduced. The former can only segment objects which are ring, compact spherical or a combination of ring-shaped objects and it does not produce better segmented results for objects having other shapes. To improve the quality of the shape-based segmentation process, Gath and Hoory (1995) [7] proposed an alternative shape-based algorithm, called fuzzy clustering of elliptic ring-shaped clusters (FKE) considering the elliptic shape information. Since ellipse is a generalized form of circle (ring), this increases the application area of shape-based segmentation as it is able to detect and separate both elliptical or ring shaped or a combination of both objects. The main problem is that most natural objects are neither ring nor elliptical in shape, and so for this reason FKR algorithm will produce improved results in certain cases while FKE does better for others. This is because for an object having arbitrary shape, FKE assumes these objects as elliptical, while FKR considers them as circular. This motivates a strategy to merge the initial segmented results produced by these two shape-based clustering algorithms, which is the basis of a new shape-based algorithm called fuzzy image segmentation combing ring and elliptic shaped clustering algorithms (FCRE) which is presented in this paper. This considers the FKR and FKE algorithms for initial segmentation, and the connectedness property of objects and fuzzy c-means (FCM) [3] using a combination of pixel intensity and normalized pixel locations. The bedrock of the new algorithm is to merge the initial segmented regions produced by FKR and FKE algorithms and then distribute any overlapping pixels using connectedness property and FCM using combination of pixel intensity and normalized pixel locations, with the aim of detecting and separating all types of objects in the image [8, 9].

The paper is organized as follows: Sections 2 and Section 3 detail the basic operations of the FKR and FKE clustering algorithms respectively, before the new FCRE algorithm is introduced in Section 4. Experimental results are analysed fully in Section 5, with some conclusions provided in Section 6.

2. FKR Algorithm

The FCM algorithm [3] is not capable to segment all objects in an image because different objects have a huge variation amongst them which demands to incorporate shape information into the FCM algorithm. To detect and separate the ring-shaped objects in the FKR algorithm [6] the circular shape information is incorporated into the bedrock of the FCM algorithm. The FKR algorithm works based on the following objective function:

$$J_q(\mu, r, v) = \sum_{j=1}^{n} \sum_{i=1}^{c} (\mu_{ij})^q d_{ij}^2 \tag{1}$$

where c and n are the number of clusters and data respectively. μ is a set of membership values μ_{ij}. v is a vector containing the values of cluster centres v_i. q is the fuzzifier $1 < q \le \infty$. $d_{ij} = d(x_j, v_i) - r_i$ where $d(x_j, v_i)$ is the Euclidean distance between a datum x_j and the centre of the i^{th} cluster v_i and r_i is the radius of i^{th} cluster. The objective function (1) is iteratively minimized using the following equations for μ and v respectively:

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{kj}} \right)^{\frac{2}{q-1}}} \tag{2}$$

$$r_i = \frac{\sum_{i=1}^{c} (\mu_{ij})^q d(x_j, v_i)}{\sum_{i=1}^{c} (\mu_{ij})^q} \tag{3}$$

$$v_i = \frac{\sum_{j=1}^{n} (\mu_{ij})^q \begin{pmatrix} x_{j1} - r_i \cos\theta \\ x_{j2} - r_i \sin\theta \end{pmatrix}}{\sum_{j=1}^{n} (\mu_{ij})^q} \tag{4}$$

The initial segmentation is performed using fuzzy k-means (FKM) algorithm [10]. The initial cluster centers and radius are calculated based on the initial data types (concentric data type: combination of ring-shaped pattern and compact spherical, eccentric: intersected patterns). The membership values, radius and cluster centers are updated through an iterative process until the change in μ_{ij} become less or equal to a specified threshold.

3. THE FKE ALGORITHM

The FKR algorithm cannot detect and separate all objects in an image because they are not all circular in shape. To address this issue, the *fuzzy clustering of elliptic ring-shaped clusters* (FKE) algorithm introduced the concept of considering elliptical shape information. Since an ellipse is a generalize form of a circle, the FKE algorithm can detect ring, elliptic and a combination of ring and elliptical shape-based objects. The distance D_{ij} between any data pattern x_j and the elliptic prototype $\left[v_i^{(1)}, v_i^{(2)}, r_i\right]$ can be defined by:

$$D_{ij} = \left\| x_j - v_i^{(1)} \right\| + \left\| x_j - v_i^{(2)} \right\| - r_i = d_{ij}^{(1)} + d_{ij}^{(2)} - r_i \tag{5}$$

where $d_{ij}^{(1)}$ and $d_{ij}^{(2)}$ are the Euclidean distance between the datum x_j and the two foci $v_i^{(1)}$ and $v_i^{(2)}$.

The objective function of the FKE algorithm is defined as:-

$$J_q(\mu, r, v^{(1)}, v^{(2)}) = \sum_{j=1}^{n} \sum_{i=1}^{c} (\mu_{ij})^q D_{ij}^2 \tag{6}$$

where $v^{(1)}$ and $v^{(2)}$ are the two foci of the ellipse.

The objective function (6) is iteratively minimized using the following equations for μ, r, $v^{(1)}$ and $v^{(2)}$ respectively:

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{D_{ij}}{D_{kj}} \right)^{\frac{2}{q-1}}} \tag{7}$$

$$r_i = \frac{\sum_{i=1}^{c} (\mu_{ij})^q \left(d_{ij}^{(1)} + d_{ij}^{(2)} \right)}{\sum_{i=1}^{c} (\mu_{ij})^q} \tag{8}$$

$$v_i^{(1)} = \frac{\sum_{j=1}^{n} (\mu_{ij})^q \begin{pmatrix} x_{j1} - \left(r_i - d_{ij}^{(2)}\right) \cos\theta_{ij}^{(1)} \\ x_{j2} - \left(r_i - d_{ij}^{(2)}\right) \sin\theta_{ij}^{(1)} \end{pmatrix}}{\sum_{j=1}^{n} (\mu_{ij})^q} \tag{9}$$

$$v_i^{(2)} = \frac{\sum_{j=1}^{n} (\mu_{ij})^q \begin{pmatrix} x_{j1} - (r_i - d_{ij}^{(1)}) \cos\theta_{ij}^{(2)} \\ x_{j2} - (r_i - d_{ij}^{(1)}) \sin\theta_{ij}^{(2)} \end{pmatrix}}{\sum_{j=1}^{n} (\mu_{ij})^q} \qquad (10)$$

The initialization of the FKE algorithm is performed using the FKM or FKR algorithms depending on the data type patterns i.e., concentric or eccentric.

4. THE FCRE ALGORITHM

As alluded to in Section 1, since different objects have different shapes, FKR produces better results for some objects and FKE for other objects in an image. So, for an arbitrary shape, no single algorithm is entirely appropriate for segmenting each object. This is the rationale behind independently merging the segmented results produced by FKR and FKE. The various steps involved in the *merging of individually segmented results* are detailed in Algorithm 1. R^{fkr} and R^{fke} represent the individual segmented regions produced by FKR and FKE respectively. To merge similar regions, their similarity is determined by summing the absolute differences of pixel intensity on a bitwise basis (Step 1)—so the smaller the difference, the greater the similarity between the regions. Region R_j^{fkr} is considered similar to R_k^{fke} if:-

$$Similar(R_j^{fkr}) = \min_{1 \le k \le \Re} \left\{ \sum \left| P_j^{fkr}(x,y) - P_k^{fke}(x,y) \right| \right\} \qquad (11)$$

where $P(x,y)$ is a pixel at location (x,y) and $P_j^{fkr}(x,y) \in R_j^{fkr} \wedge P_k^{fke}(x,y) \in R_k^{fke}$.

Similar regions are merged (Step 2) by computing the union of the relevant regions. The merging of two similar regions R^{fkr} to R^{fke} is defined as:-

$$R_j = \left\{ P(x,y) \middle| P(x,y) \in R_j^{fkr} \vee P(x,y) \in R_k^{fke} \right\} \qquad (12)$$

Since the merged region is formed by combining two similar regions produced by FKR and FKE, the result may contain some overlapping pixels which are treated as misclassified. The overlapping pixels between two merged regions R_i and R_j are expressed as:-

$$R_{ij}^O = \left\{ P(x,y) \middle| P(x,y) \in R_i \wedge P(x,y) \in R_j \right\} \qquad (13)$$

where $i \neq j \; AND \; 1 \le i, j \le \Re$.

To derive the final segmented result, the overlapping pixels need to be distributed between the merged regions. This requires all misclassified pixels to be removed (Step 3) from the corresponding merged regions using the following equations:-

$$R_i^{'} = \left\{ P(x,y) \middle| P(x,y) \in R_i \wedge P(x,y) \notin R_{ij}^O \right\} \qquad (14)$$

$$R_j^{'} = \left\{ P(x,y) \middle| P(x,y) \in R_j \wedge P(x,y) \notin R_{ij}^O \right\} \qquad (15)$$

Algorithm 1: ***Fuzzy image segmentation combing ring and elliptic shaped clustering algorithms*** (FCRE)

Precondition: Initially segmented regions R^{fkr} and R^{fke}.
Post-condition: The segmented regions R.
1. Determine similar regions of R^{fkr} and R^{fke} using (11).
2. Merge these similar regions using (12).
3. Calculate the overlap between the two merging regions using (13) and remove overlapping pixels using (14) and (15).
4. Distribute 8-connected objects of the overlap to merging regions R_i and R_j using 8-connectivity.
5. Redistribute any remaining overlapping pixels by a clustering algorithm using combination of pixel intensity and normalized pixel locations.

All misclassified pixels now distributed to the corresponding merged pair using 8-connectivity (Step 4), to ensure all weak object connections are considered. If there are still any remaining non-connected pixels, these are then redistributed by FCM using a combination of pixel intensity and normalised pixel location (Step 5) in order to consider both pixel intensity and pixel location. The complete algorithm is formulised in Algorithm 1.

5. Experimental Results

The FKR, FKE and the new FCRE algorithms were all implemented using Matlab 6.1 (The Mathworks Inc.). Different natural and synthetic gray-scale images were randomly selected for experimental analysis, comprising different number of regions (objects) having various degrees of surface variation and with different shapes (*obtained* from IMSI[1], own collection and the Internet). To segment only the foreground objects in an image, the background was manually removed by setting it to zero. Any zero-valued foreground object pixels were replaced by 1, which had no effect upon visual perception and avoided the possibility of foreground pixels merging with the background. Pixel locations in the form of the *(x, y)* coordinates were normalized within the range [0, 255]

[1] IMSI's Master Photo Collection, 1895 Francisco Blvd. East, San Rafael, CA 94901-5506, USA.

in order to constrain them to the same range pixel intensity for 8-bit gray-scale images.

To quantitatively appraise the performance of all the various fuzzy clustering algorithms, the efficient objective segmentation evaluation method, *discrepancy based on the number of misclassified pixels* [2] was used. Two types of error, namely Type I, $errorI_i$ and Type II, $errorII_i$ are computed, the former being the percentage error of all i^{th} region pixels misclassified into other regions, while the latter is the error percentage of all region pixels misclassified into i^{th} region. Representative samples of the manually segmented reference regions together with their original images are shown in Figures 1(a)-1(b) and 2(a)-2(b). To provide a better visual interpretation of the segmented results, both the reference and segmented regions are displayed using different colours rather than their original gray-scale intensities.

The *dog* image in Figure 1 (a) has two regions: the camel (R_1) and the dog (R_2). The segmented results of FKR, FKE and FCRE are shown in Figure 1 (c)-(e). If the segmented results in Figure 1 (c)-(d) are compared with the manually segmented reference regions in Figure 1 (b), it is visually apparent a large number of pixels of region (R_1) have been misclassified into (R_2) for both FKR and FKE and vice versa for Figure 1(c).

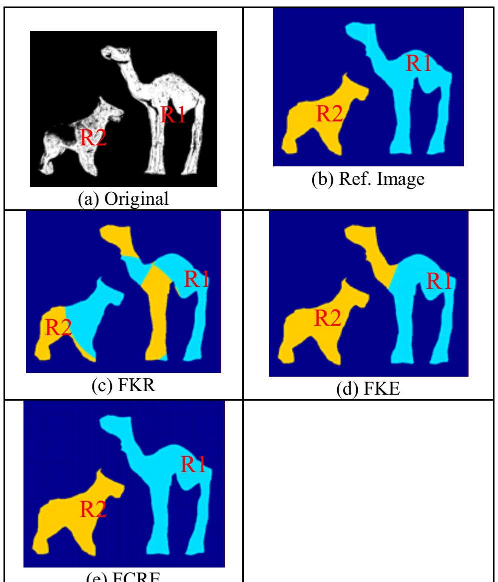


Figure 1: (a) Original *dog* image, (b) Manually segmented reference of (a). Figures (c) – (d) the segmented results of (a) using FKR and FKE respectively. (e) The segmentation results using FCRE.

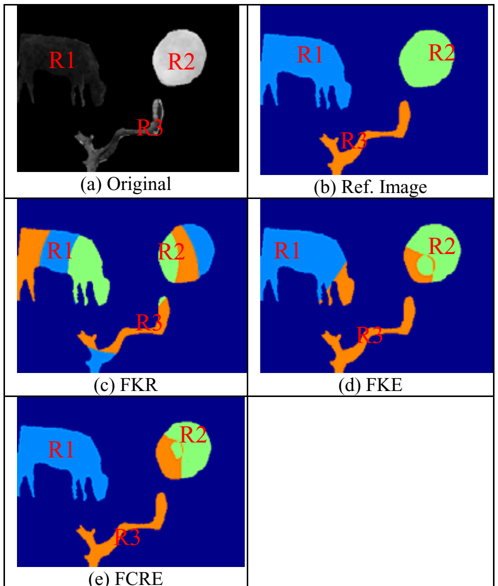
This is because both regions are neither exclusively circular nor elliptic in shape. In contrast, two regions have been correctly classified by the FCRE algorithm in Figure 1(e) because of the strategy employed to merge the initially segmented regions produced by FKR and FKE and then distribute the misclassified pixels using connectedness property and, where appropriate FCM using a combination of pixel intensity and normalised pixel location. This endorses the superiority of FCRE algorithm over FKR and FKE algorithms. The corresponding average Type I and Type II errors for FKR, FKE and FCRE are given in **Error! Not a valid bookmark self-reference.**, which again confirms the improvement of FCRE with an average error of 0%.

Table 1: The Average error percentages for the camel region (R_1) segmentation in Figure 1

Algorithm	Error		
	Type I	Type II	Mean
FKR	48.9	47.9	48.4
FKE	47.2	0	23.6
FCRE	0	0	0

A second sample image *(cow)* is shown in Figure 2(a), which contains three regions: the cow (R_1), the sun (R_2) and the branch of the tree (R_3), each having different pixel intensities and shapes. The segmented results for FKR, FKE and FCRE are shown in Figure 2 (c)-(e) respectively. For the results produced by FKR and FKE in Figure 2 (c)-(d), a considerable many of pixels of (R_1) and R_2 are misclassified into the branch of the tree (R_3) and vice versa. This is because the two of the objects are neither ring nor elliptically shaped. Conversely, the FCRE algorithm accurately segmented the cow (R_1) and the tree branch (R_3) but some pixels of (R_2) were misclassified into (R_3) due to considering the merging strategy of initial segmented results produced by FKR and FKE algorithms and also by considering the connectedness property of the objects in Figure 2(e). This improvement is confirmed in Table 2, which shows the average percentage error is 7.33% for the FCRE algorithm compared with 8.33% and 45.6% for the FKE and FKR algorithms respectively.

The experiments have been performed upon 83 images containing different number of regions up to five. The FCRE algorithm produces better results for 41 images while the FKR and FKE algorithms perform better segmentation for only 12 and 30 images respectively, endorsing the superiority of the new algorithm.



(a) Original (b) Ref. Image (c) FKR (d) FKE (e) FCRE

Figure 2: (a) Original *cow* image, (b) Manually segmented reference of (a). Figures (c) – (d) the segmented results of (a) using FKR and FKE respectively. (e) The segmentation results using FCRE.

Table 2: The average error percentages for the *cow* image segmentation in Figure 2

Algorithm	Error						
	Type I			Type II			Average
	R1	R2	R3	R1	R2	R3	
FKR	73.1	72.3	30.3	31.9	32.1	33.9	45.6
FKE	13	20.9	0	0	0	16	8.33
FCRE	0	31.8	0	0	0	11.9	7.3

6. Conclusions

This paper has presented a new shape-based image segmentation algorithm called *fuzzy image segmentation combing ring and elliptic shaped clustering algorithms* (FCRE) which merges the initial segmented results produced by both the FKR and FKE algorithms. Both a qualitative and quantitative analysis has been conducted comparing the performance against existing shape-based algorithms FKR and FKE. The FCRE algorithm shows the superior performance over existing algorithms for different objects having different shapes.

7. References

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