Fuzzy image segmentation using shape information
FUZZY IMAGE SEGMENTATION USING SHAPE INFORMATION

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ABSTRACT

Results of any clustering algorithm are highly sensitive to features that limit their generalization and hence provide a strong motivation to integrate shape information into the algorithm. Existing fuzzy shape-based clustering algorithms consider only circular and elliptical shape information and consequently do not segment well, arbitrary shaped objects. To address this issue, this paper introduces a new shape-based algorithm, called fuzzy image segmentation using shape information (FISS) by incorporating general shape information. Both qualitative and quantitative analysis proves the superiority of the new FISS algorithm compared to other well-established shape-based fuzzy clustering algorithms, including Gustafson-Kessel, ring-shaped, circular shell, c-ellipsoidal shells and elliptic ring-shaped clusters.

Keywords: Image Segmentation, Shape Information.

1. INTRODUCTION

Image segmentation is an important research area because it plays a fundamental role in image analysis, understanding and coding [1]. Segmenting an image is the most challenging and difficult task because there exist different objects and a huge variations between them using a general framework. The effectiveness of a clustering algorithm [2], [3], [4] is solely dependent on the type of the features used and the information about the domain of the objects in that image. This raises an open question about which type of feature produces better results for which type of image. This provides a motivation to incorporate shape information into the segmentation process to address this limitation. Popular fuzzy shape-based image segmentation using clustering algorithms are Gustafson-Kessel (GK) [5], ring-shaped (FKR) [6], circular shell (FCS) [7], c-ellipsoidal shells (FCES) [8] and elliptic ring-shaped clusters (FKE) [9]. The GK algorithm does not explicitly consider specific shape information while FKR and FCS only take account of objects which are ring, compact spherical or combination of ring-shaped to some extent. To generalize the FCS and FKR algorithms, FCES and FKE were proposed respectively as alternative fuzzy shape-based clustering algorithms that increase the application area of shape-based segmentation algorithms as they are able to both detect and separate ring and elliptical objects or a combination of them. The main problem is that most natural objects are neither ring nor elliptical in shape, so existing segmentation algorithms cannot be applied to arbitrary objects in all types of images. To address this issue, a new shape-based algorithm namely fuzzy image segmentation using shape information (FISS) is presented which considers the generic shape information of the objects.

The foundation of the new FISS algorithm is to integrate generic shape information into the framework of the Gustafson-Kessel (GK) [5] algorithm with the express aim of being able to detect and separate all types of image objects. The motivation behind the usage of the GK algorithm is that it applies the cluster covariance matrix, which adjusts the local distance with respect to the shape of cluster. Since, the proposed FISS algorithm is able to segment arbitrary shaped object, it can help solve two of the most challenging problems in multimedia research, namely (1) the segmentation of video object planes (VOP) in real video for object-based video coding in MPEG-4, which has already been implemented but only for synthetic video and (2) fast object-based content retrieval in MPEG-7 [10].

This paper is organized as follows: Section 2 details the FISS algorithm including all relevant processes and the experimental results are analysed fully in Section 3. Finally, some conclusions are provided in Section 4.

2. THEORETICAL FOUNDATIONS OF FISS ALGORITHM

As mentioned previously, the shape-based fuzzy clustering algorithms FSC and FKR only consider ring shapes, while FKE and FCES incorporate elliptical shape-based information into the segmentation. To separate objects based on their shape demands integration of generic shape information into the segmentation process. This section introduces a new algorithm called fuzzy image segmentation using shape information (FISS) which comprises the following major component blocks, namely Initial Contour, Locating Intersection Point, Mathematical Modelling, and The FISS Algorithm.

2.1 Initial Contour

As the shape of an object is able to be represented by a series of contour points, to incorporate generic shape information into the image segmentation process, these points have to be provided for each object. For this reason, objects are initially segmented using the GK algorithm, as this automatically performs the local adaptation of the distance to the shape of the cluster [5]. A set of significant points of an object are generated using the convex hull of the respective initial segmentation and a Bezier curve (BC) approximation is then applied, which exploits the inherent global shape property of BC [11], and generates the requisite m contour points for these significant points.

2.2 Locating the Intersection Point

For any segmentation strategy, the most important aspect is how the distance $d_{ij}$ of a datum is calculated. Fuzzy clustering algorithms concomitantly seek to both minimize the intra- and maximize the inter-cluster distance, so in order to segment an image with respect to a given shape contour, the distance $d_{ij}$...
between a datum \( S_j \) and its respective shape contour point has to be calculated. As the example in Figure 1 reveals, this means locating the intersection point \( S_{ij} \) on the contour, of a line \( (l_i) \) from datum \( S_j \) to the \( i^{th} \) cluster centre \( v_i \). This calculation is both complex and computationally expensive when the Cartesian coordinate system is used, because there is no specific analytic contour equation that can be applied to find the intersection point \( S_{ij} \) from the line connecting the cluster centre \( v_i \) and \( S_j \). One possible approach to using Cartesian coordinates is to calculate \( S_{ij} \) as follows:— i) Find two points on the contour of the curve that are closest and lie on opposite sides of the line \( (l_i) \) between the cluster centre \( v_i \) and \( S_j \). ii) Determine a point which lies between these two points and also on the boundary of the curve and line \( (l_i) \). This point is intersection point \( S'_{ij} \). Both the search for the two contour points and required intersection point defined above, require extensive calculations and a number of iterations which are computationally expensive. A more efficient option is to use the polar coordinate system, where a point is represented as \( (r, \theta) \), where \( r \) is the distance between the centre and the respective point and \( \theta \) is the angle between the horizontal line passing through the centre and the line between the centre and that point. For this reason, each datum and contour point has an angle with the cluster centre. The angle of the intersection point \( S_{ij} \) is equal to the angle of the corresponding datum \( S_j \). The steps to finding the intersection point are given in Algorithm 1.

**2.3 Mathematical Modelling**

The objective function of the FISS algorithm is defined using the GK algorithm which is based on the following objective function:

\[
J_k(\mu, V) = \sum_{j=1}^{n} \sum_{i=1}^{k} \left( \mu_{ij} \right)^p d_{ij}^2
\]

subject to \( \sum_{j=1}^{n} \mu_{ij} = 1 \) and \( d_{ij} = d(S_j, v_i) - r'_{ij} \)  

where \( r'_{ij} \) is the Euclidian distance between \( S_j \) and its corresponding intersection point \( S'_{ij} \) and \( \mu_{ij} \) is the membership value of \( j^{th} \) datum in the \( i^{th} \) cluster. \( n \) and \( c \) are the number of data points and clusters respectively, while \( q \) is a fuzzifier. The algorithm iteratively minimizes the objective function (1) using the following equations (3-6). The membership value \( \mu_{ij} \) is defined as follows:-

\[
\text{IF } d_{ij} = 0, \ \mu_{ij} = 1 \text{ and maintain } \sum_{j=1}^{n} \mu_{ij} = 1
\]

For all other values of \( d_{ij} \), \( \mu_{ij} = \left( \frac{\sum_{j=1}^{n} \left( \frac{d_{ij}}{d_{ij}} \right)^{2}}{q-1} \right) \)

**Algorithm 1:** Determining the intersection point between a datum and its corresponding cluster centre.

**Precondition:** cluster contour points, cluster centre \( v_i \) and data \( S_j \).

**Post condition:** The intersection point \( S_{ij}' \).

1. Convert contour points into polar form \( (r_{ij}', \theta_{ij}') \) with respect to corresponding cluster centre.
2. Convert data points into polar form \( (r_{ij}, \theta_{ij}) \) with respect to corresponding cluster centre.
3. Calculate the difference between \( \theta_{ij} \) and \( \theta_{ij}' \) \((\Delta \theta_{ij} = \theta_{ij} - \theta_{ij}')\)
4. The contour point with the minimum \( \Delta \theta_{ij} \) is intersection point \( S_{ij}' \) of the corresponding data \( S_j \) in the \( i^{th} \) cluster.

The average radius of the \( i^{th} \) cluster, namely circular radius \( r_i \), is used to scale the shape of objects and is derived as:-

\[
r_i = \frac{\sum_{j=1}^{n} (\mu_{ij})^p d(S_j, v_i)}{\sum_{j=1}^{n} (\mu_{ij})^p}
\]

The \( i^{th} \) cluster centre \( v_i \) in (6) is calculated as:

\[
\begin{align*}
    f_x &= S_{ji} - d(S_j, v_i) \frac{S_{ji} - v_{i1}}{d(S_j, v_i)} + S_{ji1} - d(S_{ij1}, v_i) \frac{S_{ji1} - v_{i1}}{d(S_{ji1}, v_i)} \\
    f_y &= S_{ij2} - d(S_j, v_i) \frac{S_{ij2} - v_{i2}}{d(S_{ij2}, v_i)} + S_{ij21} - d(S_{ij21}, v_i) \frac{S_{ij21} - v_{i2}}{d(S_{ij21}, v_i)} \\
    v_i &= \frac{\sum_{j=1}^{n} (\mu_{ij})^p \left( \frac{f_x}{f_y} \right)}{\sum_{j=1}^{n} (\mu_{ij})^p} \quad \text{(6)}
\end{align*}
\]

where \( S_j = \begin{bmatrix} S_{ji} \\ S_{ij2} \end{bmatrix} \) and \( S_{ij} = \begin{bmatrix} S_{ji} \\ S_{ij2} \end{bmatrix} \) with the subscripts 1 and 2 respectively representing the \( x,y \) coordinates. Using the GK algorithm, the distance \( d^2(S_j, v_i) = (S_j - v_i)^T A(S_j - v_i) \) where \( A_i \) is the covariance matrix that helps to adapt the local distance according to the shape of a cluster.
2.4 Proposed FISS Algorithm

The complete FISS algorithm is detailed in Algorithm 2. To evolve the shape during the segmentation process, it is needed to consider the average radius of the shape. This is calculated by considering the minimization criteria of the objective function based on a circular shape. The ratio between the current and the previous radius of the circular shape is used as the scaling factor and is done by multiplying the ratio with the contour radius as shown in Steps 3-5 of Algorithm 2. The various steps in the FISS algorithm are summarised as follows:

**Algorithm 2: Fuzzy image segmentation using shape information (FISS).**

**Precondition:** The number of clusters $c$, the initial values of cluster centre $v_i$, the initial segmented regions $R_i$, the significant points $P$ and the number of points $m$ representing the shape contour.

**Post condition:** Final segmented regions $R_i$.

1. Generate $m$ points on the contour of the shape for $R_i$ using the Bezier curve.
2. Repeat Steps 3-9 for each iteration $l = 0, 1, \cdots$
3. Update $r_i$ by (5)
4. Calculate $Ratio = \frac{r_i}{r_{i-1}}$
5. Scale contour by multiplying with $Ratio$.
6. Find intersection point using Algorithm 1
7. Update $\mu_j$ using (3) and (4).
8. Update $V_i$ using (6).
9. FOR all $i, j$
   
   IF $\|\mu_j - \mu_j^{l+1}\| < \varepsilon$ THEN STOP
   ELSE GOTO 3

3. EXPERIMENTAL RESULTS

The FKR, FKE, GK, FCS, FCES and new FISS algorithms were all implemented using Matlab 6.1. Different natural and synthetic gray-scale images were randomly selected for the experimental analysis, comprising different number of regions (objects) having different shapes (obtained from IMSI, own collection and the Internet). As each image is rectangular, segmentation based upon using the location of pixels of interest will always arbitrarily divide the image into a given number of clusters, unless the background is removed, hence all the background pixels in an image were manually removed by setting them to zero. Any zero-valued foreground object pixels were replaced by 1, which had no effect upon visual perception and avoided the possibility of foreground pixels merging with the background.

To quantitatively appraise the performance of all the various fuzzy clustering algorithms, the efficient objective segmentation evaluation method, discrepancy based on the number of misclassified pixels [12] was used. Two types of error, namely Type I, $errorI_i$ and Type II, $errorII_i$ are computed, the former being the percentage error of all $i^{th}$ region pixels misclassified into other regions, while the latter is the error percentage of all region pixels misclassified into $i^{th}$ region. Representative samples of the manually segmented reference regions together with their original images are shown in Figures 2(a)-2(b) and 3(a)-3(b). To provide a better visual interpretation of the segmented results, both the reference and segmented regions are displayed using different colours rather than their original gray-scale intensities.

The sun image in Figure 2(a) has two regions: the sun ($R_1$) and the branch of the tree ($R_2$). The segmented results of FKR, FKE, GK, FCS, FCES and FISS are shown in Figure 2(c)-(h). If the segmented results in Figure 2(c)-(e) are compared with the manually segmented reference regions in Figure 2 (b), it is visually apparent a considerable amount of pixels of region ($R_1$) have been misclassified into ($R_2$) for FKR, FKE and GK while a portion of ($R_1$) are misclassified into ($R_2$) for FCS and FCES shown in Figures 2(f)-(g). This is because both regions are neither circular nor elliptic in shape, which is why FKR, FKE, GK, FCS and FCES generate so many misclassified pixels (Figure 2(c)-(g)). In contrast, the FISS algorithm perfectly segmented the objects shown in Figure 2 (h) because of the strategy employed to incorporate the shape in the segmentation. The corresponding average Type I and Type II errors for FKR,
FKE, GK, FCS, FCES and FISS are given in Table 1, which confirms the improvement of FISS with an average error of 0%. A second sample image is shown in Figure 3(a), which contains three different regions having different shapes: the reptile ($R_1$), the bird ($R_2$) and the branch of the tree ($R_3$). The segmented results for FKR, FKE, GK, FCS, FCES and FISS are shown in Table 1:

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Type I</th>
<th>Type II</th>
<th>Mean</th>
<th>Bird</th>
</tr>
</thead>
<tbody>
<tr>
<td>FKR</td>
<td>21.2</td>
<td>0</td>
<td>10.6</td>
<td>46.3</td>
</tr>
<tr>
<td>FKE</td>
<td>20.4</td>
<td>0</td>
<td>10.2</td>
<td>24.6</td>
</tr>
<tr>
<td>GK</td>
<td>22.5</td>
<td>0</td>
<td>11.2</td>
<td>3.1</td>
</tr>
<tr>
<td>FCS</td>
<td>0</td>
<td>3.8</td>
<td>1.9</td>
<td>26.1</td>
</tr>
<tr>
<td>FCES</td>
<td>0</td>
<td>11.98</td>
<td>6</td>
<td>27.5</td>
</tr>
<tr>
<td>FISS</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 1: Percentage errors for the dog image segmentation in Figure 3 and Figure 3.

Figure 3 (c)-(h) respectively. For the results produced by FKR, FKE, GK, FCS and FCES shown in Figure 3 (c)-(g), a number of pixels of $R_1$ are misclassified into $R_2$ and vice versa. The FISS algorithm almost completely segmented all the regions due to considering the shape of objects (Figure 3(h)). This improvement is confirmed in Table 1, which shows the average error percentages of FISS, FCES, FCS, GK, FKE and FKR were 1.7%, 27.5%, 26.1%, 3.1%, 24.6% and 46.3% respectively. To evaluate the generalization of the proposed FISS algorithm, the experiment was performed using 158 different images. The FISS algorithm provides best result for 80 images while FKR, FKE, GK, FCS and FCES algorithms produce better results for 4, 23, 50, 17 and 30 respectively. This result also proves the superiority of the proposed algorithm.

4. CONCLUSIONS
This paper has presented a new shape-based image segmentation algorithm called *fuzzy image segmentation using shape information* (FISS) which incorporates generic shape information. Both a qualitative and quantitative analysis has been conducted comparing the performance against existing shape-based algorithms FKR, FKE, GK, FCS and FCES. FISS consistently has provided segmentation superiority. The main advantage of the algorithm is that it exhibits significant potential to be applied in many different segmentation applications including video object plane (VOP) for object-based video coding in MPEG-4. Since the algorithm is based on clustering the initial number of clusters needs to be provided.

5. REFERENCES