New Dynamic Enhancements to the Vertex-Based Rate-Distortion Optimal Shape Coding Framework
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Abstract—Existing vertex-based operational rate-distortion (ORD) optimal shape coding algorithms use a vertex band around the shape boundary as the source of candidate control points (CP) usually in combination with a tolerance band (TB) and sliding window (SW) arrangement, as their distortion measuring technique. These algorithms however, employ a fixed vertex-hand width irrespective of the shape and admissible distortion (AD), so the full bit-rate reduction potential is not fulfilled. Moreover, despite the causal impact of the SW-length upon both the bit-rate and computational-speed, there is no formal mechanism for determining the most suitable SW-length. This paper introduces the concept of a variable width admissible CP band and new adaptive SW-length selection strategy to address these issues. The presented quantitative and qualitative results analysis endorses the superior performance achieved by integrating these enhancements into the existing vertex-based ORD optimal algorithms.

Index Terms—Vertex-based shape coding, video coding.

I. INTRODUCTION
SHAPE CODING has become a popular topic in multimedia technology research as evidenced by the corpus of literature [1]–[9]. Shape coders are generally classified as being either bit-map or contour based [1], with examples of the former including the context based arithmetic encoder (CAE) [5], and new digital straight line segments based context coding (DSLSC) [6], while the baseline [7] and vertex-based polynomial approaches [1] are two popular examples of contour-based shape coding.

A concise treatise of the shape coding algorithms has been presented in [1] with both polygon and B-spline based operational rate-distortion (ORD) frameworks being developed which form the basis for a suite of algorithms [1]–[4] that all have as their aim, for some prescribed distortion, to optimally encode a shape contour in terms of the number of bits, by selecting the set of control points (CPs) that incurs the lowest bit rate and vice versa. The CP are selected from a set of vertices contained in the admissible control point band (ACB) around the shape contour. Algorithms [1], [2] employ a single admissible distortion (AD) for all boundary points, while those in [3], [4] embrace variable AD for each boundary point, though all use a fixed-width admissible CP band (FCB) with the width being arbitrarily selected. This does not guarantee an optimal solution of admissible CP unless the width is sufficiently large, and large FCB widths imply increased computational complexity. This paper addresses this issue by proposing a variable-width admissible control point band (VCB), where the width of each boundary point is individually determined from the admissible peak distortion and shape curvature information.

The sliding widow (SW) is also a core element of the ORD algorithms, as its length directly impacts upon the bit-rate. It was originally introduced [1] to force the encoder to follow the shape boundary so avoiding trivial solutions, as well as improving computational speed. In fact, the SW confines the search space for the next CP to only those points within the SW-length, so compromising global optimality in a bit-rate sense. For computationally efficient encoding, a small SW-length is desirable though decreasing the length increases the overall bit-rate overhead. While the SW is a deep-rooted idea, no formal mechanism has been developed for determining the most appropriate SW-length, so this paper also presents a shape adaptive sliding window (ASW) strategy based on the curvature of boundary points.

These enhancements can all be seamlessly embedded into the vertex-based ORD optimal shape coding framework, with the performance of these ORD algorithms integrating the enhancements being analyzed, and consistently shown to provide superior RD results in terms of both lower bit-rate and faster encoding speed compared with [1]–[4].

The rest of this paper is organized as follows. Section II provides a brief overview of existing vertex-based ORD optimal shape coding algorithms and identifies some limitations, while Section III details the underlying theoretical principles of the proposed enhancements. Section IV presents an analysis of the empirical results and performance comparison for the original ORD algorithms with the new enhancements embedded, while some concluding remarks are provided in Section V.

II. EXISTING VERTEX-BASED RATE-DISTORTION OPTIMAL SHAPE CODING FRAMEWORK

The algorithms [1]–[4] seek to determine and encode a set of CP to represent a particular shape within prescribed RD constraints. Assume boundary $B = \{b_0, b_1, \ldots, b_{N_B-1}\}$ is an ordered set of shape points, where $N_B$ is the total number of points and $b_0 = b_{N_B-1}$ for a closed boundary. $P = \{p_0, p_1, \ldots, p_{N_P-1}\}$ is an ordered set of CP used to approximate $B$, where $N_P$ is the total number of CP and $P \subseteq C$, where $C$ is the ordered set of vertices in ACB. In the core of these algorithms a weighted directed acyclic graph (DAG) is formed using the vertices in and the shortest path from the first vertex to the last vertex is searched. These algorithms however, possess some inherent limitations.
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A smaller SW clearly incurs a lower time penalty but a higher efficient coding, the ORD algorithms [3], [4] employ two admissible peak distortion bounds (\(T_{\text{max}}\) and \(T_{\text{min}}\)) to obtain the AD \(T[\mathcal{P}]\) at boundary point \(b_i\), by permitting a smaller distortion at boundary points having a lower image intensity gradient or shape curvature [4], and vice versa. A FCB is again employed however, with \(W_{\text{max}} = 1\) pel for example in [3], being invariant of the AD value so the philosophy of variable AD is not fully exploited in bit-rate minimization, with FCB accordingly no longer being optimal for either polygonal or curve-based approximations.

The FCB example in Fig. 1(b) corroborates this with the encoder mandating 29 bits (4 CP) for a polygonal shape approximation, while the corresponding VCB solution needs only 24 bits (3 CP) in Fig. 1(c) for \(T_{\text{max}} = 2\) and \(T_{\text{min}} = 1\) pel.

B. Impact of a Sliding Window

The overall computational complexity of polygon-based ORD algorithms is \(O\left(N^3_B\right)\), since the DAG mandates \(O\left(N^3_B\right)\) and the distortion measurement, which is an integral DAG loop calculation, also incurs \(O\left(N^3_B\right)\) time. If an SW of length \(L\) is applied, where \(L < N\), then the complexity drops to \(O\left(NBL^3\right)\), so the overall computational overhead is always lower when a SW is used, because it limits the CP search space. This conversely increases the bit-rate requirement, with the nexus between bit-rate and computational speed being illustrated in Fig. 2, where typical rates and CPU-times are plotted for various SW-lengths on the same shape and prescribed AD.

A smaller SW clearly incurs a lower time penalty but a higher bit-rate and vice versa, so the significance in selecting the most appropriate SW-length is a key design parameter for achieving better admissible bit-rate utilization and reduced computational costs.

III. PROPOSED ENHANCEMENTS TO ORD OPTIMAL SHAPE CODING FRAMEWORK

This section firstly elucidates the theoretical basis of the VCB, before formally developing bounds for the width of this band for every boundary point. A brief discussion upon the proposed enhancement to the distortion measurement technique is then provided, followed by SW-length selection strategy.

A. Variable Width Admissible CP Band

VCB is an ordered set of vertices formed around the shape, with every boundary point having some associated vertices in the band. As the AD for each boundary point differs, they will have different VCB widths and so the number of VCB points associated with each point will vary. Let \(W[j]\) be the width of VCB for the boundary point \(b_j\).

Bounds for the width of the admissible control point band: Lemmas 1 and 2 focus upon the ACB width for a polygonal and quadratic B-spline based coding, respectively.

Lemma 1: For a polygonal approximation, the width of the ACB for any boundary point is bounded by the peak AD for that point, i.e., \(W[j] \leq T[\mathcal{P}]\) for \(b_j\).
Proof (by Contradiction): Let there be such a vertex \( u \) associated with boundary point \( b_j \) where \( u \in C \) and the distance of \( u \) from \( b_j \) is greater than \( T[j] \), i.e., \( W[j] > T[j] \). Assume \( u \) is now selected as a CP. Since it is a polygonal approximation, the approximated shape will pass through this vertex (in fact, it will be an end point of one edge), so the distortion at this vertex will exceed the peak AD for \( b_j \), and for \( b_j \) such a vertex \( u \) can never be selected as a CP and lie within the VCB vertices associated with \( b_j \).

Lemma 2: For a quadratic B-spline based RD constrained shape approximation,

\[
\alpha[j] \leq \min \left\{ (3\delta + 4T_{\text{max}} + 2T[j]/6), (\rho\sqrt{2}/4) \right\},
\]

where \( \delta \) and \( \rho \) are respectively the longest chord length of the boundary and the largest run-length possible for the code employed, \( \alpha[j] \) is the difference between the corresponding AD and width of the admissible CP band, i.e., \( W[j] = \alpha[j] + T[j] \).

Proof: Fig. 3(a) shows a uniform quadratic B-spline curve produced by CP \( S_1, S_2, \text{ and } S_3 \). This is actually a Bezier curve (BC) generated by \( S_1', S_2' \& S_3' \), where \( S_1' = (1/2)(S_1 + S_2) \) and \( S_2' = (1/2)(S_2 + S_3) \), with \( h \) being the minimum distance of the middle CP \( S_2 \) from the BC. It thus follows from \([10]\) that

\[
2h \leq \max \{ ||S_1S_2||, ||S_2S_3||\}, \text{ where } ||S_2S_3|| \text{ is the length of edge } S_2S_3', \text{ so } 4h \leq \max \{ ||S_1S_2||, ||S_2S_3||\}.
\]

In Fig. 3(b), three CP \( P, Q, \text{ and } R \) are employed to encode a shape segment that includes the boundary point \( b_j \) which has an AD \( T[j] \). Assuming \( PQ \geq QR \), the distance of the B-spline curve from \( Q \) is always \( \leq (1/4)||PQ|| \). The maximum length of \( PQ \) is \( \delta + T_{\text{max}} + T_{\text{max}} + \alpha_{\text{max}} + \alpha_{\text{max}} = \delta + 2T_{\text{max}} + 2\alpha_{\text{max}} \) where \( \alpha_{\text{max}} \) is the maximum value of \( \alpha \). So

\[
\delta + 2T_{\text{max}} + 2\alpha_{\text{max}} \geq 4\alpha_{\text{max}} \text{ Hence, } \alpha_{\text{max}} \leq (\delta/2) + T_{\text{max}}.
\]

Now the corresponding \( \alpha[j] \) for boundary point \( b_j \) is given by

\[
4\alpha[j] \leq \delta + T_{\text{max}} + \alpha_{\text{max}} + T[j] + \alpha[j],
\]

so

\[
\alpha[j] \leq \frac{1}{8} (3\delta + 4T_{\text{max}} + 2T[j]).
\]

The encoding strategy adopted can limit the length of an edge since if \( \rho \) is the length of codebook, it is able to encode a maximum length of \( \rho\sqrt{2}/4 \) pel (through the diagonal) so

\[
\alpha[j] \leq \rho\sqrt{2}/4. \tag{3}
\]

From (2) and (3), \( \alpha[j] \leq \min \{ (3\delta + 4T_{\text{max}} + 2T[j]/6), \rho\sqrt{2}/4 \} \).

B. Sliding Window Length Selection Strategies

As discussed earlier, a SW improves computational complexity while concomitantly compromising global optimality in a bit-rate sense. Thus, rather than using an arbitrary SW-length, there are cogent arguments for applying an appropriate SW-length within the ORD coding framework. This section presents an ASW-length strategy that adjusts the SW-length automatically based on a shape’s cornerity. As the cornerity of points on a shape do not change abruptly, but rather gradually follow a trend to and from the local maximum value \([11]\), its value can be used to determine the most appropriate SW-length. By monitoring the relative cornerity of shape points, an adaptive algorithm has been developed so boundary points with a higher cornerity induce a smaller SW and vice versa. This ensures more CP for the boundary-segment with sharp changes and corners and fewer CP for segments where the rate of change in shape is more gradual (longer window lengths). Let \( K[j] \) be the cornerity of the \( j \)th boundary point, and \( K_{\text{max}} \) and \( K_{\text{min}} \) respectively be the maximum and minimum cornerity values of the vertices on a boundary. The SW-length for the \( j \)th boundary point is then given by (4), shown at the bottom of the page.

The ASW strategy is effective for both cursive and non-cursive shapes, while for the special case of a straight line, when \( K_{\text{max}} = K_{\text{min}} \) the SW-length is always \( L_{\text{max}} \), since both \( K_{\text{max}} \) and \( K_{\text{min}} \) will be zero. The encoder may be constrained by either bit-rate or AD, with in both cases, for logarithmic

\[
L[j] = \begin{cases} 
L_{\text{max}} & \text{if } K_{\text{max}} = K_{\text{min}} \\
L_{\text{min}} + \frac{L_{\text{max}} - L_{\text{min}}}{e^{K_{\text{max}} - e^{K_{\text{min}}}}} \cdot (e^{K_{\text{max}} - e^{K[j]}}), & \text{else.}
\end{cases}
\]

Fig. 3. (a) Distance between a quadratic B-spline curve and its CP and (b) Maximal width of the admissible CP band calculation.
cornerity calculation method [11]. In addition, the accurate distortion measurement technique in [9] has been applied in lieu of the shortest absolute distance (SAD), distortion band (DB) or tolerance band (TB) metrics, since it has been proven in [9] that in certain circumstances, both SAD and DB have limitations in their measurements and as the TB is in fact a generalization of the DB [3], it too inherits the same restrictions as the DB.

The first series of experiments performed upon the Neck region focused upon the peak distortion for a prescribed set of admissible bit-rate values with the respective numerical results produced by different algorithms being summarized in Table I. These confirm VCB-based algorithms produce lower distortions than their FCB counterparts, so for example with an admissible bit-rate of 75 bits, B-spline-FCB and B-spline-VCB respectively produced $T_{\text{max}}$ values of 3.17 and 2.0 pel as a direct consequence of VCB providing a larger and more dynamic set of potential admissible CP.

The next series of experiments concentrated upon the bit-rate and distortion for a prescribed set of AD settings. The respective results produced by the various ORD algorithmic combinations are shown in Fig. 4(a)–(d) for a peak distortion bound of $T_{\text{max}} = 3$ pel, $T_{\text{min}} = 1$ pel with the corresponding numerical results given in Table II. These reveal that the Polygon-FCB and Polygon-VCB approximations with $T_{\text{max}} = 2$, $T_{\text{min}} = 1$ pel required 109 and 99 bits, respectively, with a similar trend being

\begin{table}[h]
\centering
\begin{tabular}{|l|cccc|}
\hline
\textbf{Algorithms} & \textbf{75 bits} & \textbf{90 bits} & \textbf{105 bits} \\
\hline
Polygon-FCB & 3.61 & 2.45 & 2.24 \\
Polygon-VCB & 3.17 & 2.24 & 2.0 \\
B-spline-FCB & 3.17 & 2.24 & 2.0 \\
B-spline-VCB & 2.0 & 2.0 & 1.42 \\
\hline
\end{tabular}
\caption{Maximum Distortion $T_{\text{max}}$ (pel) Required for the Neck Region for Different Algorithms for Various Admissible Bit-Rates With $T_{\text{min}} = 1$ pel}
\end{table}
observed in respect of the bit requirements for all VCB based algorithms, namely a lower value than for the FCB-based algorithms due to the richer and more dynamic set of potential admissible CP. The various encircled CP in Fig. 4(b) and (d), lie outside the FCB yet approximate a significant portion of the shape, so lowering the bit-rate in the VCB case.

To substantiate the improved performance of the VCB-based strategies, additional experiments were conducted using the MPEG-4 shape distortion metric which is defined as the ratio of the number of erroneously represented pels of an approximating shape to the total number of pels in the original shape. Again the various enhancements presented in this paper provided better RD performance compared to MPEG-4 CAE [5] technique, when \( D_n \) is calculated for the first 100 frames of the Kids.sif sequence which contains multiple objects in a frame. The corresponding RD results for various polygon-based algorithms are plotted in Fig. 5(a) and clearly reveal the superior performance achieved by the VCB enhancements over both the FCB based framework and the CAE, especially at higher distortion values.

In addition to analyzing the lossy vertex-based ORD optimal shape coding framework, comparative results for a number of standard and contemporary lossless shape coders are presented in Table III. Both Polygon-VCB and DSLSC [6] in intra mode provided comparable bit-rate results for lossless compression and are evidently much more efficient than CAE encoders in terms of intra- and motion-compensated inter-modes for most shapes. It needs to be emphasized that while the framework in [1]–[4] is based on a tradeoff mechanism which is optimal in the ORD sense, the enhancements presented in this paper are specifically designed to secure greater efficiency for lossy compression, while DSLSC and CAE [5] are both lossless entropy coders. Obviously, the VCB-based paradigm also affords lossless compression whenever.

Fig. 5(b) demonstrates the performance of the ASW-length strategy, by displaying the decoded shape using Polygon-VCB for \( T_{\text{max}} = 2, T_{\text{min}} = 1 \) pel, where it is palpable the approximated shape has retained all its sharp corners by using more CP in these regions, while concomitantly using fewer CP in the flatter regions so vindicating using shape cornerity at the boundary points in the adaptation process.

V. CONCLUSION

This paper has presented a series of innovative enhancements to existing vertex-based ORD optimal shape coding algorithms. It has proposed a dynamic and flexible variable-width admissible control point band (VCB) instead of the conventional fixed-width band as the source of potential control points to fully utilize the variable admissible distortion (AD) in minimizing the bit-rate. For both polygonal and B-spline based approximations, theoretical bounds on the VCB-width have been established. The paper has also investigated automatically determining a shape adaptive length of a sliding widow suitable for RD constraints, with empirical results confirming the improvements in RD performance and distortion measurement accuracy over the existing ORD optimal shape coding algorithms.
REFERENCES


